

**D'Ariano *et al.* Reply:** The aim of our Letter [1] was to show that saturation effects in a traveling wave laser amplifier may reduce the output noise due to spontaneous emission, and such reduction can be achieved for sizable gains and realistic values of all physical parameters (direct detection is considered). In the linear regime, spontaneous emission gives a 3 dB noise figure ( $\mathcal{R} = 2$ ), which is the value achieved by linear phase sensitive amplifiers (PIA), and is usually referred to as the SQL for noise in amplifiers.

Nilsson *et al.* [2] now address a separate issue that was not considered in our Letter, and which deserves careful attention. Their point is as follows: "For on-off communications it is the BER—not the SNR—which represents the quantity of interest, and two are generally unrelated for non-Gaussian noise and/or nonlinear devices. Thus, in principle, it may happen that the saturable amplifier is no more valuable than the PIA in improving the transmitted BER, although it beats the SQL." In fact, the effect of saturation on BER is *a priori* not obvious, as the photon probability distribution is amplified asymmetrically, resulting in a shorter right (high-number) tail and a longer left (low-number) tail. The right tail of the "off" distribution is cut, whereas the left tail of the "on" distribution is stretched, and the resulting effect on the intersection area—which gives the BER—depends on the detailed analytical form of saturation.

Actually, the point made by Nilsson *et al.* is not meaningful for the communication scheme we considered, with *ideal detection*, and for which there is no way of improving the BER, even with a noiseless photon number amplifier (PNA) [3]. The issue must be posed in terms of comparing the SQL-breaching saturable amplifier with the PIA in improving the BER for *nonideal detection*, where there is actual room for BER reduction. We are currently considering this point for a numerical study of an amplifier with a small saturation number. (The Fokker-Planck regime we analyzed, although realistic for a conventional amplifier, is not very interesting for truly "quantum" communications, as the input number of photons is too large:  $\langle \hat{n} \rangle \sim 10^2$  at 0 dB m.) We expect that the saturable amplifier will beat the PIA—at least in some cases—just because the saturable amplifier has a small noise figure, and an amplifier with a "very good" noise figure, i.e., one approaching the ideal PNA, would beat the PIA. As a simple example, consider the case of a communication scheme based on 0-1 number eigenstates. For a detector with sensitivity threshold at  $n = 2$  photons, one has BER  $\mathcal{B} = 1$ : Using a PNA with gain  $\mathcal{G} \geq 2$  one achieves  $\mathcal{B} = 0$ , whereas a PIA would get  $\mathcal{B} = 1/2$  for  $\mathcal{G} = 2$ . On the other hand, for a thresholdless detector with quantum efficiency  $\eta < 1$  the BER is  $\mathcal{B} = 1 - \eta$ : Using a PNA one obtains  $\mathcal{B} = (1 - \eta)^{\mathcal{G}}$ —which can be reduced at will by increasing the gain—whereas a PIA achieves

$\mathcal{B} = 1 - \frac{1}{2}(\eta + 1)$  for  $\mathcal{G} = 1 + \eta^{-1}$ , i.e.,  $\mathcal{B} \simeq 1/2$  for small  $\eta$ .

For the above reasons we think that the SNR is still a reasonably useful concept for nonlinear amplifiers, at least when the noise figure is very good and the device is not pathologically nonlinear (nonpolynomial). On the other hand, why is the SNR usually considered instead of the BER? (This is what happens in the experiments quoted by Nilsson *et al.*) A main reason is that when the BER is very low (as in our case, and as is often the case) it is very difficult to either measure or computer simulate it.

A final clarification is in order, in response to the objection raised by Nilsson *et al.* on the use of our "on-off" definition [Eq. 2 of our Letter] of the SNR for not small modulations. The considered optical amplifiers provide amplification of dc inputs, without the need of modulation around a high-derivative working point. Hence, the objection does not pertain to the case of dc inputs, where by definition the input "modulation" between the 0-1 dc levels is twice the central value, and can never be considered as small. For dc signals, others use a SNR defined only in terms of the on ensemble averages, i.e., with signal  $S \doteq \langle \hat{O} \rangle_{\text{on}}$  and noise  $\mathcal{N} \doteq \langle \Delta \hat{O}^2 \rangle_{\text{on}}$ . For the PIA both definitions give the SQL for sufficiently large gains. More generally, the different numerical values arise from a different SNR at the output. In the on-off definition, the dc contribution of the amplified vacuum is subtracted from the signal, and the noise is averaged together with the amplified vacuum: These two contributions nearly cancel out for sufficiently high signals. Remarkably, saturation enhances the former contribution as compared to the latter, with the consequence that the on definition would lead to even lower (i.e., better) noise figures than ours.

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