

SECURE COMMUNICATION USING COHERENT STATES

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We demonstrate that secure communication using coherent states is possible. The optimal eavesdropping strategy for an M -ary ciphering scheme shows that the minimum probability of error in a measurement for bit determination can be made arbitrarily close to the pure guessing value $P_e = 1/2$. This ciphering scheme can be optically amplified without degrading the security level. New avenues are open to secure communications at high speeds in fiber-optic or free-space channels.

Secure communication protocols protected by physical laws instead of mathematical complexities, such as the BB84 quantum protocol for key distribution,¹ seem to have encountered a bottleneck that hampers their utilization in real networks. The same no-cloning theorem that guarantees security forbids the signal amplification that is necessary in long-haul communication links. No alternate quantum scheme using quantum repeaters or entangled states, having practical applicability within a reasonable time span, has been envisaged. The question "is it possible to create a system with available technology that could provide unconditional security in long-distance communication?" has a positive answer. It relies on the use of quantum noise inherent in the coherent states of light, as demonstrated in this paper.

In the presence of noise, it has been shown information theoretically² that new shared secret keys can be created between two users. A specific protocol has been proposed by Yuen,^{3,4} which has been called the YK protocol. A particular implementation of this YK protocol has been demonstrated,⁵ in which detector noise is utilized and hence it is not unconditionally secure. However, the YK protocol can be made unconditionally secure by utilizing the fundamental, unavoidable quantum noise in a quantum signaling scheme.⁶ The other known secure scheme is, of course, the BB84,¹ or its variant that involves features of B92,⁷ in which coherent states instead of Fock states are employed. The implementations of the YK protocol,⁵ and BB84, both suffer from the intrinsic limitation that very weak signals with no more than one photon per mode have to be used, making them severely rate-limited in a lossy channel. This problem can be alleviated in a new protocol, in which mesoscopic coherent states are employed to overcome loss and to allow ordinary amplification, switching and routing. In addition to the use of the

fundamentally-unavoidable quantum noise in coherent states, a crucial new ingredient in this protocol is the explicit use of a shared secret key for the cryptographic objective of key expansion. A shared secret key is also needed in the BB84 and the YK protocols for the purpose of user and message authentication amidst the protocol execution.

In the ciphering scheme for this new protocol the sender (Alice) uses an explicit secret key (a short key \mathbf{K} , appropriately expanded into a longer key \mathbf{K}' by use of another encryption mechanism such as a stream cipher) to modulate the parameters of, in general, a multimode coherent state. Coherent states span an infinite-dimensional Hilbert space which we refer to as a *qumode*. A qumode can be associated with any physical property of light such as polarization, phase, frequency or time. For the free-space implementation to be presented, the qumodes are the two orthogonal modes of polarization. In this case, Alice uses the running key \mathbf{K}' to specify a polarization basis from a set of M uniformly spaced two-mode bases spanning a great circle on the Poincaré sphere. Each basis consists of a polarization state and its antipodal state at an angle π from it, representing the 0 and 1 bit value for that basis. The message \mathbf{X} is encoded as $\mathbf{Y}_{\mathbf{K}'}(\mathbf{X})$. This mapping of the stream of bits onto points of the Poincaré sphere is the *key* to be shared by Alice and the receiver (Bob). Because of his knowledge of \mathbf{K}' , Bob is able to make a precise demodulation operation producing the plaintext \mathbf{X} . Bob applies \mathbf{K}' to the received sequence of arbitrary polarization states to return them to the linear orthogonally-polarized condition, representing the two original bits of the message \mathbf{X} .

We present analysis that covers both polarization as well as phase modulation of optical signals. In the case of *polarization* states, information is encrypted and encoded on two orthogonal polarization modes of radiation with annihilation operators a_1 and a_2 . A coherent state $|\psi_0\rangle$ with amplitude α in mode a_2 ($|\psi_0\rangle = |0\rangle \otimes |\alpha\rangle$) is "rotated" by a unitary transformation U_{φ_b} ($\varphi_b = 0$ or π) to create the bit of a message. This rotation is performed by, e.g., $U_{\varphi_b} = \exp[(\varphi_b/2)(a_1^\dagger a_2 - a_1 a_2^\dagger)]$, giving

$$|\psi_b\rangle = U_{\varphi_b}|0\rangle \otimes |\alpha\rangle = |\alpha \sin \frac{\varphi_b}{2}\rangle \otimes |\alpha \cos \frac{\varphi_b}{2}\rangle. \quad (1)$$

For *phase* encoding within a given polarization state, one could start by splitting a coherent state $|\alpha\rangle$ into a two-mode coherent state $|\Psi_0\rangle = |\alpha/\sqrt{2}\rangle_1 \otimes |\alpha/\sqrt{2}\rangle_2$. Bit encoding can be represented by the operation

$$|\Psi_b\rangle = e^{-iJ_z\varphi_b}|\Psi_0\rangle = |e^{-i\varphi_b/2}\alpha/\sqrt{2}\rangle_1 \otimes |e^{i\varphi_b/2}\alpha/\sqrt{2}\rangle_2, \quad (2)$$

where $J_z = (a_1^\dagger a_1 - a_2^\dagger a_2)/2$. Demonstration of the security of the ciphering scheme in both kinds of physical encoding, polarization or phase, can be treated with the same formalism and leads to the same result.

Let us analyze the phase ciphering, applied by the same modulator generating the bit sequence. The ciphering angle ϕ_ν could have ν as a discrete

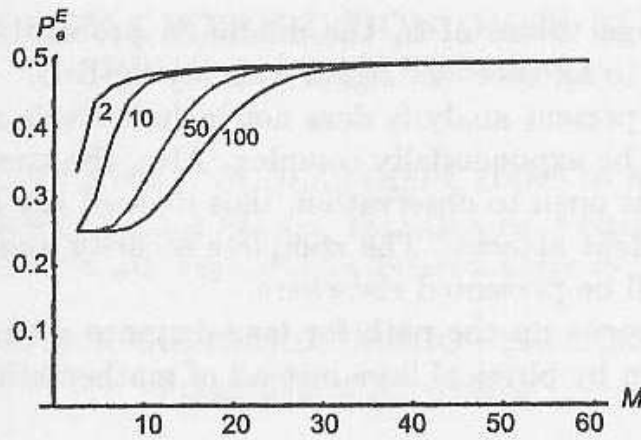


Figure 1. P_e^E as a function of M for $|\alpha|^2 = 2, 10, 50, 100$.

or a continuous variable determined by some general distribution. A ciphered two-mode state is $|\Psi_{b\nu}\rangle = e^{-iJ_z(\varphi_b + \phi_\nu)}|\Psi_0\rangle$ and the corresponding density operator for all possible choices of ν is ρ_b . The problem is to find the minimum probability of error P_e^E that an eavesdropper (Eve) can achieve in bit determination, given that ciphered states are used.

No restriction is imposed on the physical devices available to Eve, including perfect detectors and unlimited computational power. A close-to-source attack is considered, in which no losses have yet occurred that normally would during propagation of the signal. These are the ideal conditions for an eavesdropper.

The optimal POVM for discriminating between ρ_0 and ρ_1 is given by Helstrom's binary discrimination procedure⁹ applied to $\Delta\rho = \rho_1 - \rho_0$. Calling Π_1 and Π_0 ($\Pi_1 + \Pi_0 = I$) the projectors over eigenstates with the positive and negative eigenvalues of $\Delta\rho$, the probability of error P_e^E is

$$P_e^E = \text{Tr} [p_1 \Pi_0 \rho_1 + p_0 \Pi_1 \rho_0], \quad (3)$$

where p_1 and p_0 are *a-priori* probabilities to find a state in ρ_1 or ρ_0 , respectively. In one of Yuen's scheme,⁸ closest values of a given k are associated with distinct bits from the bit at position k . For example, $\phi_k = \pi [k/M + (1/2)(1 - (-1)^k)]$, $k = 0, 1, \dots, M - 1$. In this encoding protocol, two closest neighboring states represent distinct bits. Figure 1 shows the minimum probability of error as a function of the number of ciphering levels M . P_e^E goes very fast to the asymptotic pure-guessing limit of $1/2$ as M increases. The quantum noise present in the coherent state of light is what makes these states indistinguishable to an eavesdropper.

Bob, on the other hand, by knowing the key has much more complete information on the state of light that has been sent and can extract the information with great precision. His probability of error is

$$P_e^B = \frac{1}{2} \left(1 - \sqrt{1 - e^{-2|\alpha|^2}} \right). \quad (4)$$

For sufficiently large values of α , the minimum probability of error P_e^B is negligible, leading to an excellent signal recovery by Bob.

Note that the present analysis does not include Eve's attacks using trial keys, which would be exponentially complex. Also, the stream cipher output in the system is not open to observation, thus its seed key is not open to the usual known-plaintext attacks. The complete security analysis for the most general attacks will be presented elsewhere.

This scheme opens up the path for long-distance secure communication based on protection by physical laws instead of mathematical complexities.

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