Quantum Reading of Unitary Optical Devices

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Abstract. We address the problem of quantum reading of optical memories, namely the retrieving of classical information stored in the optical properties of a media with minimum energy. We present optimal strategies for ambiguous and unambiguous quantum reading of unitary optical memories, namely when one's task is to minimize the probability of errors in the retrieved information and when perfect retrieving of information is achieved probabilistically, respectively. A comparison of the optimal strategy with coherent probes and homodyne detection shows that the former saves orders of magnitude of energy when achieving the same performances. Experimental proposals for quantum reading which are feasible with present quantum optical technology are reported.

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In the engineering of optical memories (such as CDs or DVDs), a tradeoff among several parameters must be taken into account. High precision in the retrieving of information is surely an indefeasible assumption, but also energy requirements, size and weight can play a very relevant role for applications. Clearly using a low energetic radiation to read information reduces the heating of the physical bit, thus allowing for smaller implementation of the bit itself. In the problem of *quantum reading* [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] of optical memories one's task is to exploit the quantum properties of light to retrieve some classical digital information stored in the optical properties of a given media with minimum energy. We focus on the case where information is encoded into linear and energy-preserving unitary optical devices [2, 6, 10], while most of the previous literature focused on the case of non unitary - e.g. lossy - devices.

In this hypothesis two different scenarios can be distinguished. A possibility is the on-the-fly retrieving of information (e.g. multimedia streaming), where one requires that the reading operation is performed fast - namely, only once, but a modest amount of errors in the retrieved information is tolerable. In this context, denoted as ambiguous quantum reading, the relevant figure of merit is the probability P_e to have an error in the retrieved information. On the other hand for highly reliable technology, perfect retrieving of information is an issue. Then, unambiguous quantum reading, where one allows for an inconclusive outcome (while, in case of conclusive outcome, the probability of error is zero) becomes essential. Here, the relevant figure of merit is clearly the failure probability P_f of getting an inconclusive outcome.

We discuss [2, 6, 10] optimal strategies for both scenarios, which exploit fundamental properties of quantum theory such as entanglement, allowing for the ambiguous (unambiguous) discrimination of linear and energy-preserving unitary devices with probability

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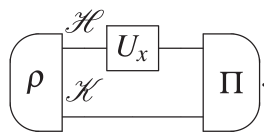
of error P_e (probability of failure P_f) under any given threshold, while minimizing the energy requirement. The most general strategy for performing quantum reading consists in preparing a bipartite probe ρ (we allow for an ancillary mode), applying locally the unknown device and performing a bipartite POVM Π on the output state, namely



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Since the optimal POVMs and the corresponding error (failure) probabilities for ambiguous (unambiguous) discrimination of two states are well known, the problem of quantum reading can be formulated as an optimization over probe only. For any set of two optical devices $\{U_1, U_2\}$ and any threshold q in the probability of error (failure), find the minimum energy probe ρ^* that allows to ambiguously (unambiguously) discriminate between U_0 and U_1 (with equal priors) with probability of error P_e (probability of failure P_f) not larger than q, namely

$$\rho^* = \arg \min_{\rho \text{ s.t. } P(\rho, U_1, U_2) \leq q} E(\rho).$$

where $P(\rho, U_0, U_1)$ can either be given by P_e or P_f and $E(\rho) := \mathrm{Tr}[\rho N]$ is the energy (expectation value of number operator N) of probe ρ. Notice that the discrimination of devices $\{U_1, U_2\}$ can be easily recasted to that of devices $\{I, U := U_1^{-1} U_2\}$. In the following we will write Fock states $|n, m\rangle$ in the basis where U is diagonal.

We are now ready to present our main results [2, 6, 10]. It is possible to prove that without loss of generality the optimal probe ρ^* for (ambiguous or unambiguous) quantum reading can be taken pure and no ancillary modes are required. For the quantum reading of beamsplitters, the optimal probe ρ^* is given by a superposition of a NOON state and the vacuum, namely

$$|\psi^*\rangle = \alpha \frac{|0, n^*\rangle + |n^*, 0\rangle}{\sqrt{2}} + \sqrt{1 - \alpha^2}\, |0, 0\rangle,$$

where $|\alpha| = \sqrt{\frac{1-K}{1-\cos \delta n^*}}$, $n^* = \arg\min_{\lfloor x^* \rfloor, \lceil x^* \rceil} E(\psi^*)$, $x^* = \min[x \geq 0 | \delta x = \tan(\delta x/2)]$, $K = \sqrt{4q(1-q)}$ for ambiguous quantum reading while $K = q$ for unambiguous quantum reading, and the eigenvalues of the scattering matrix of the beamsplitter are $e^{\pm i\delta}$.

A comparison of the optimal strategy for ambiguous quantum reading with the optimal coherent strategy (using coherent probes and homodyne detection), reminiscent of the one implemented in common CD readers, is provided in Fig. 1. The Figure clarifies that the former strategy saves orders of magnitude of energy, moreover allowing for perfect discrimination with finite energy. We underline that experimental proposals [6] for ambiguous and unambiguous optimal quantum reading were provided for the single-photon case - namely, $n^* = 1$. They are feasible with present quantum optical technology, in terms of single-photon source, linear optics and photodetectors.

In this work we addressed the problem of quantum reading of linear and energy-preserving unitary optical memories. We showed that the probe can be taken pure and no

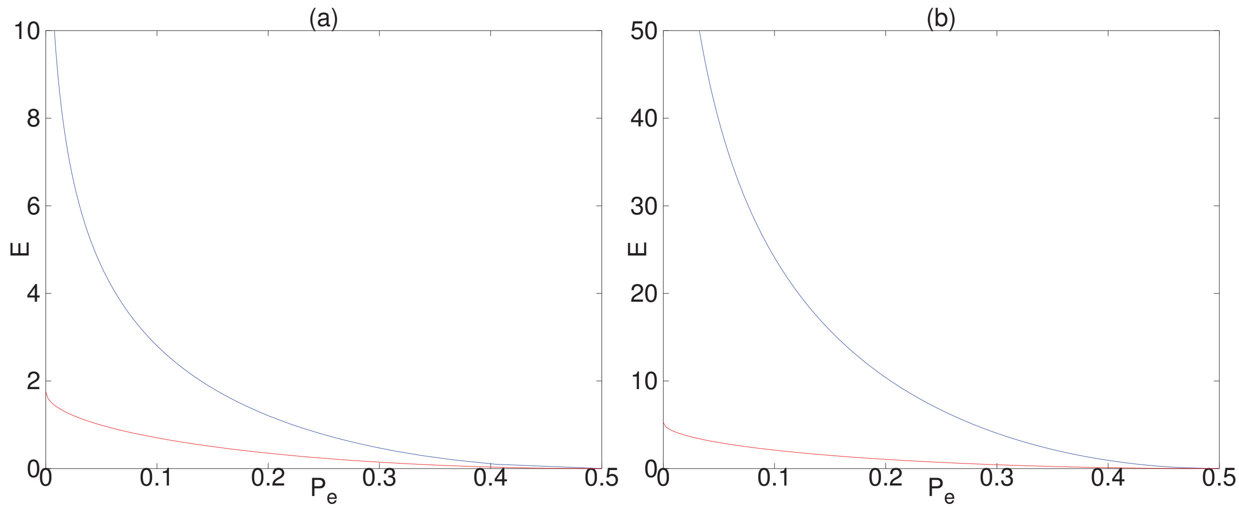

FIGURE 1. (Color online) Optimal tradeoff between the energy E and the probability of error P_e in the discrimination of I and $U = \exp(i(\delta a_1^\dagger a_1 - \delta a_2^\dagger a_2))$ ($\delta = \pi/4$ in (a) and $\delta = \pi/12$ in (b)). The upper line represents the discrimination with coherent probe and homodyne detection, while the lower line represents the optimal discrimination.

ancillary modes are needed. For quantum reading of beamsplitters, we presented optimal strategies for ambiguous (unambiguous) quantum reading, where the probe is given by a superposition of a NOON state and the vacuum. We compared the optimal quantum strategy with a coherent strategy, showing that the former saves orders of magnitude of energy when compared with the latter, and we discussed experimental feasibility.

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