1.2 EXPERIMENTAL CHARACTERIZATION OF THE TRANSFER MATRIX OF A QUANTUM DEVICE

It is unquestionable that the current main technological focus is related to the problem of handling the *information*, either in transmission, processing, storing, or computing. The marriage of Quantum Physics and Information Technology -originally motivated by the need for miniaturization- has recently opened the way to the realization of radically new information-processing devices, with the possibility of guaranteed secure cryptographic communications, and of tremendous speedups of some computational tasks. The scientists have thus learned the lesson that the *uncertainty* that is intrinsic of Quantum Mechanics -which has always been considered just as a major limitation- can be actually turned into a powerful horse, which we can harness and ride.

Among the many problems posed by the new information technology [1] there is the need of characterizing the new quantum devices, making the "radiography" of their functioning. Again, Quantum Mechanics, on first sight seems to make things more difficult, but at the end it provides us with a powerful tool to achieve the task easily and efficiently: this tool is the so-called *quantum entanglement*, the basis of the *quantum parallelism* of the future computers.

How do we usually characterize the operation of a device? Actually, here we are interested only in *linear devices*, since quantum dynamics is intrinsically linear. Any linear device –either quantum or classical (examples are: an optical lens or a good amplifier)- can be completely described by a *transfer matrix* which gives the output vector by matrix-multiplying the input vector. Now the problem is: how to reconstruct the full transfer matrix of the device? This can be done by running a *basis* of possible inputs, and measuring the corresponding outputs. In quantum mechanics the inputs are density operators, here denoted by ρ, and the role of the transfer matrix is played by the so-called *quantum operation* of the device, here denoted by E. Thus the output state ρ_{out} (a part from a possible normalization) is given by the quantum operation applied to the input state as follows

$$\rho_{out} = \mathrm{E}(\rho_{in}).$$

Here we don't use the usual matrix-multiplication notation, since the linear transformation E corresponds to multiplication both on the right and on the left. Also, technically, the set of states ρ actually belongs to a *space of operators*: this means that if we want to characterize E completely, we need to run a complete orthogonal basis of quantum states $|n\rangle$ ($n = 0,1,2\ldots$), along with their linear combinations $\frac{1}{\sqrt{2}}(|n'\rangle + i^k|n''\rangle)$, with $k = 0,1,2,3$ and i denoting the imaginary unit. However, the availability of such a set of states in the lab is, by itself, a very hard technological problem. For example, for an optical device, the states $|n\rangle$ are those with a precise number n of photons, and, a part from very small n -say at most $n = 2$- they have never been achieved in the lab, whereas achieving their superposition is still a dream for experimentalists, especially if $n \rangle 1$ (a kind of *Schrödinger kitten* states).

The *quantum parallelism* intrinsic of entanglement now comes to help us, running all possible input states in parallel by using only a single entangled state as the input [2]. We need to prepare two identical systems into an entangled state, say $|\Psi\rangle$, and input only one of them into the device, leaving the other system untouched as in Fig.1.

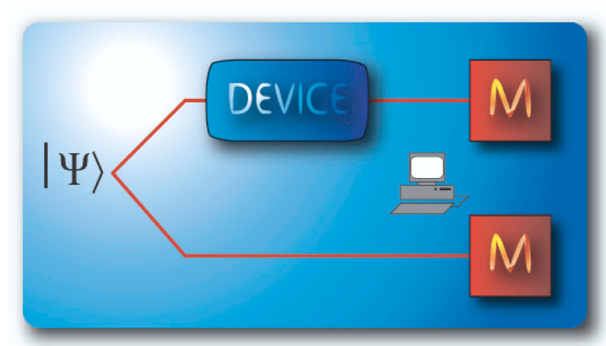

Fig. 1
General experimental scheme for measuring the transfer matrix of a quantum device. Two identical quantum systems are prepared in a (maximally) entangled state $|\psi\rangle$, one of the two systems undergoes the quantum operation E of the device, whereas the other is left untouched. At the output one makes a quantum tomographic reconstruction of the states by measuring jointly two random observables from a quorum (see the text).

This setup leads to an output state

$$R_{out} = \mathrm{E} \otimes \mathrm{I}(|\psi\rangle\langle\psi|)$$

where I denotes the identical operation. It is a result of linear algebra that R_{out} is in one-to-one correspondence with the quantum operation E, as long as the state $|\psi\rangle$ is

full-rank, i. e. it has non-vanishing components on the whole state-space of each system, such as, for example, a so-called *maximally entangled* state. Now, the good news is that full-rank entangled states can be easily generated in Quantum Optics by *parametric downconversion* of vacuum (full-rank states of *qubits* can be generated by means of networks of controlled-NOT gates).
Therefore, the problem of availability of all possible input states is solved: we just need a single entangled state $|\psi\rangle$, which works as all possible inputs in a sort of quantum parallelism!
Now, how to characterize the entangled state R_{out} at the output? We obviously need to perform many measurements on an ensemble of equally prepared quantum systems, since, due to the *no-clonining theorem* [3] we cannot determine the state of a single system. For this purpose a technique for the full determination of the quantum state has been introduced and developed by our group since 1994. The method, now generally called *Quantum Tomography* [4], has been initially introduced for the state of a single-mode of radiation -the so-called *Homodyne Tomography*- and thereafter it has been generalized to any quantum system. The basis of the method is just performing measurements of a suitably complete set of observables called *quorum*. Therefore, for our needs, we just have to measure jointly a quorum of observables on the two entangled systems at the output, in order to determine the output state R_{out}, and hence the quantum operation E.
A computer simulation for measuring the transfer matrix of a quantum device in the quantum optical regime is given in Fig. 2.
Here the device is a field *amplitude displacer*, and the input entangled state comes from parametric downconversion of vacuum. All experimental parameters are the same of the experiment in Ref. [5]: however, for the tomographic reconstruction of the quantum operation, the control of the pump-phase of the downconverter must be improved with respect to the original experiment. A sketch of the setup of the experiment is given in Fig. 3.
This research has been carried out within the project: *Quantum Information Transmission And Processing: Quantum Teleportation And Error Correction*, cosponsored by the Italian Ministero dell'Università e della Ricerca Scientifica e Tecnologica.

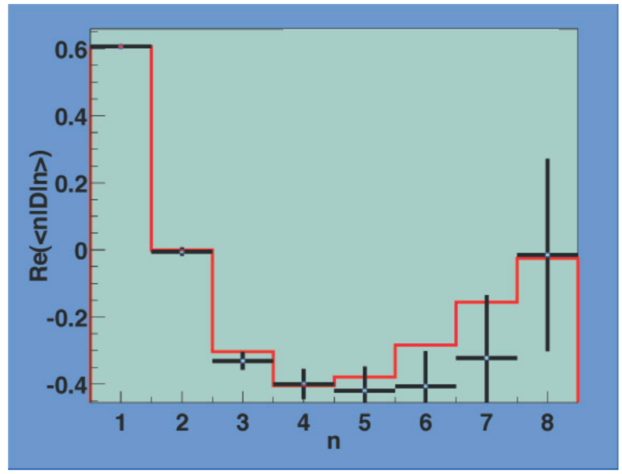

Fig. 2
Computer simulation of an homodyne tomographic experiment for measuring the quantum operation E of a device that displaces the amplitude of a radiation mode in the complex plane by z = 1, i.e. $E(\rho) = D(1)\rho\, D(1)^{\dagger}$. The real part of the diagonal elements $\langle n|D(1)|n\rangle$ is plotted, with error bars, compared to the theoretical values (thick red line). Similar results are obtained for all upper and lower diagonals of the quantum operation matrix D(1). The reconstruction has been achieved by using an entangled input state $|\psi\rangle$ corresponding to parametric downconversion of vacuum with mean thermal photon number $\bar{n} = 5$, and quantum efficiency at homodyne detectors $\eta = 0.8$ (1.5x10[6] data have been used). The experimental parameters are the same of the experiment in Ref. [5].

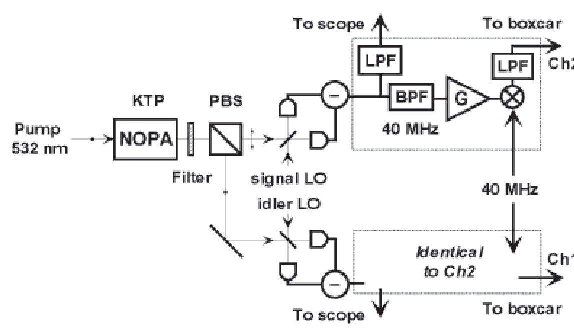

Fig. 3
A sketch of the setup of the experiment in Ref. [5]. NOPA: non-degenerate optical parametric amplifier; LOs: local oscillators; PBS: polarizing beam splitter; LPFs: low-pass filters; BPF: band-pass filter; G: electronic amplifier. Electronics in the two channels are identical.

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