Quantum calibration

"New Frontiers in Physics", Istitituto Lombardo (November 27 2003)

Giacomo Mauro D'Ariano
QUIT Group: http://www.qubit.it

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Center for Photonic Communication and Computing
Northwestern University, Evanston IL 60208

Founded by: EC (ATESIT), INFM (PRA-CLON), MIUR (Cofin), US (MURI)

QUIT (Quantum Information Theory)

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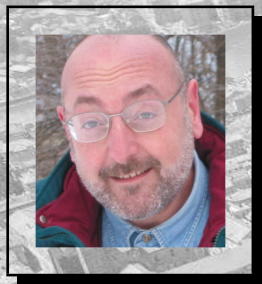
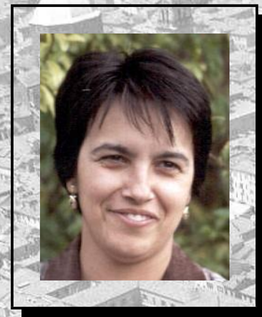


G. M. D'Ariano (full prof.)



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M. F. Sacchi (postdoc)

P. Perinotti (postdoc)

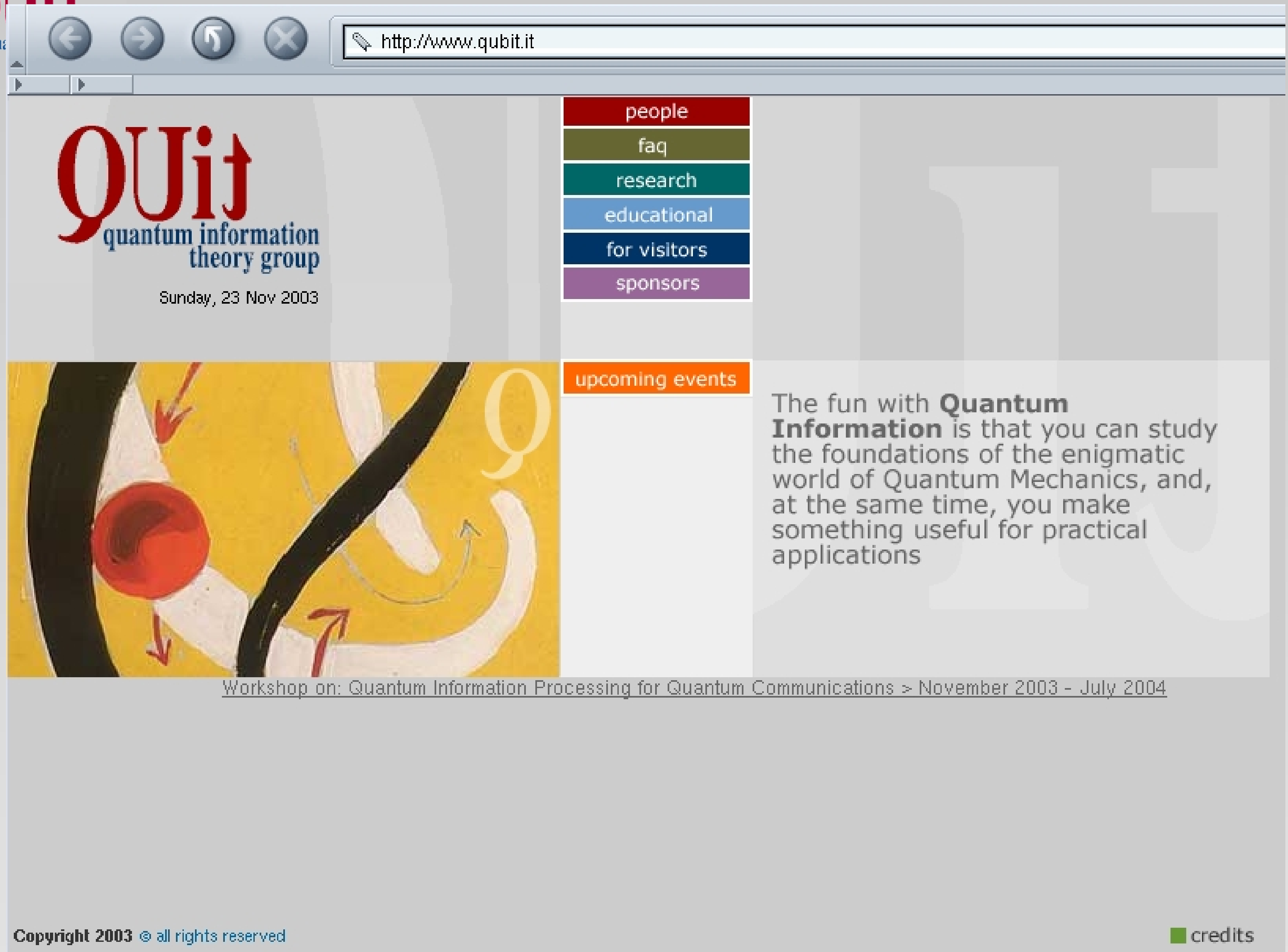
O. Rudolph (ATESIT)

G. Chiribella (PhD student)

P. Lo Presti (PhD student)

F. Buscemi (PhD student)

M. Medici (secretary)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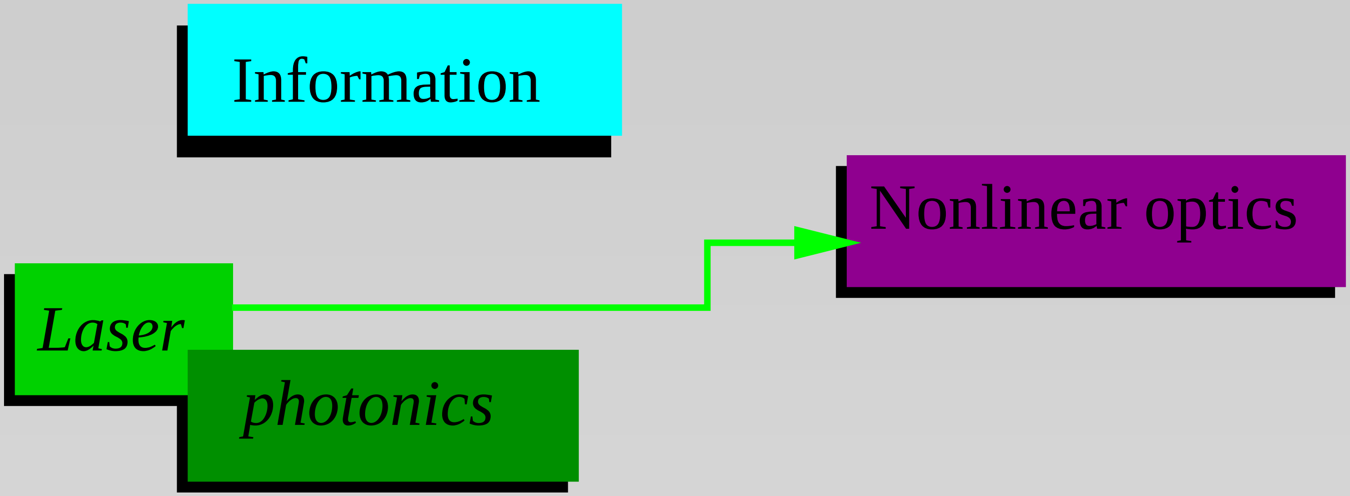

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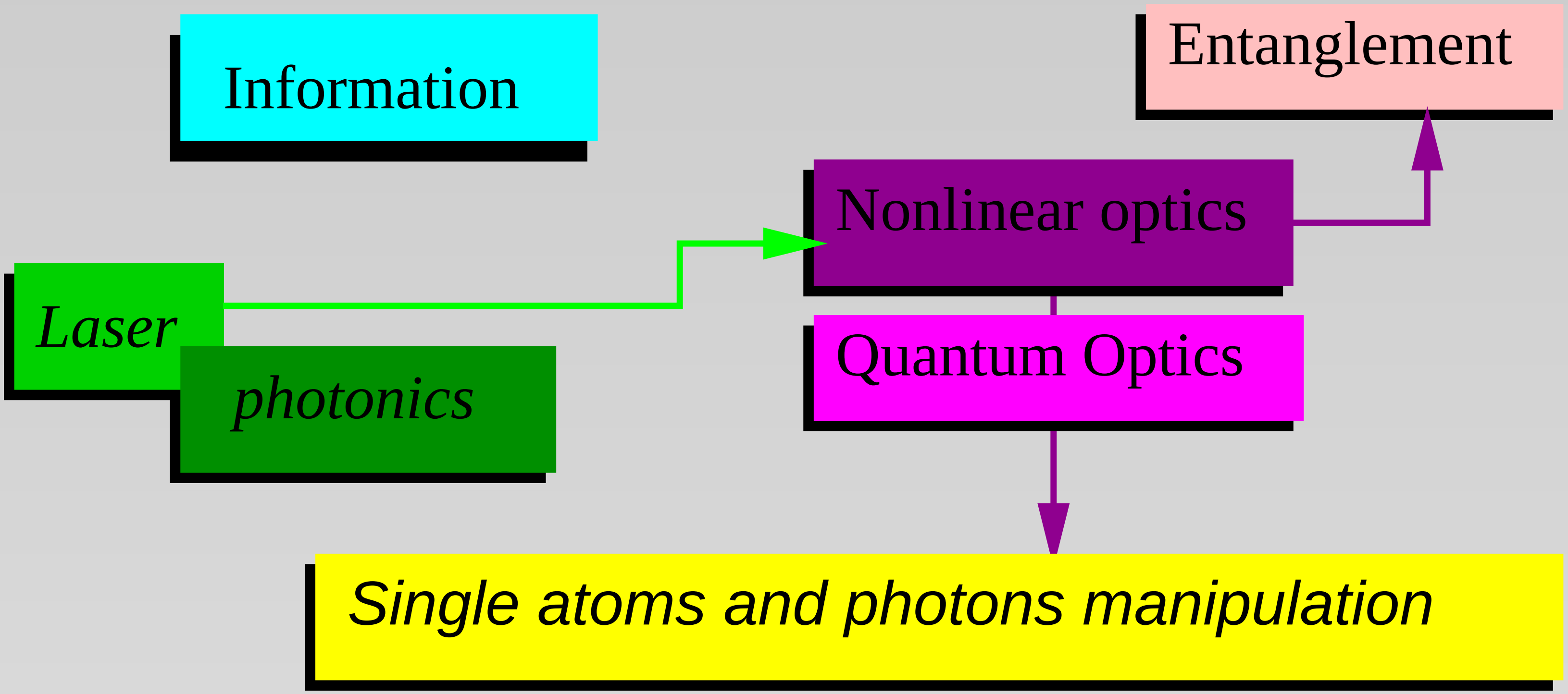

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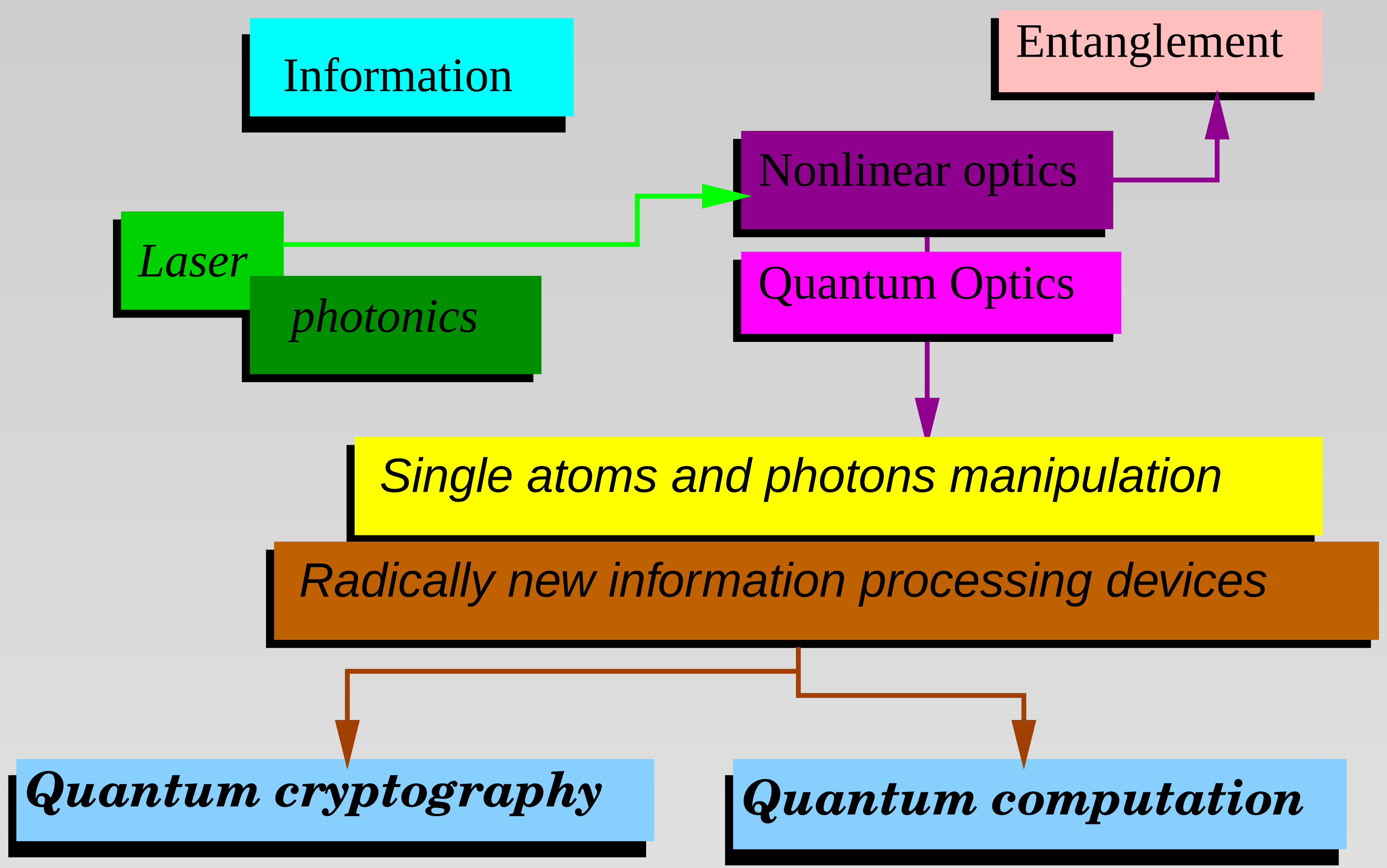
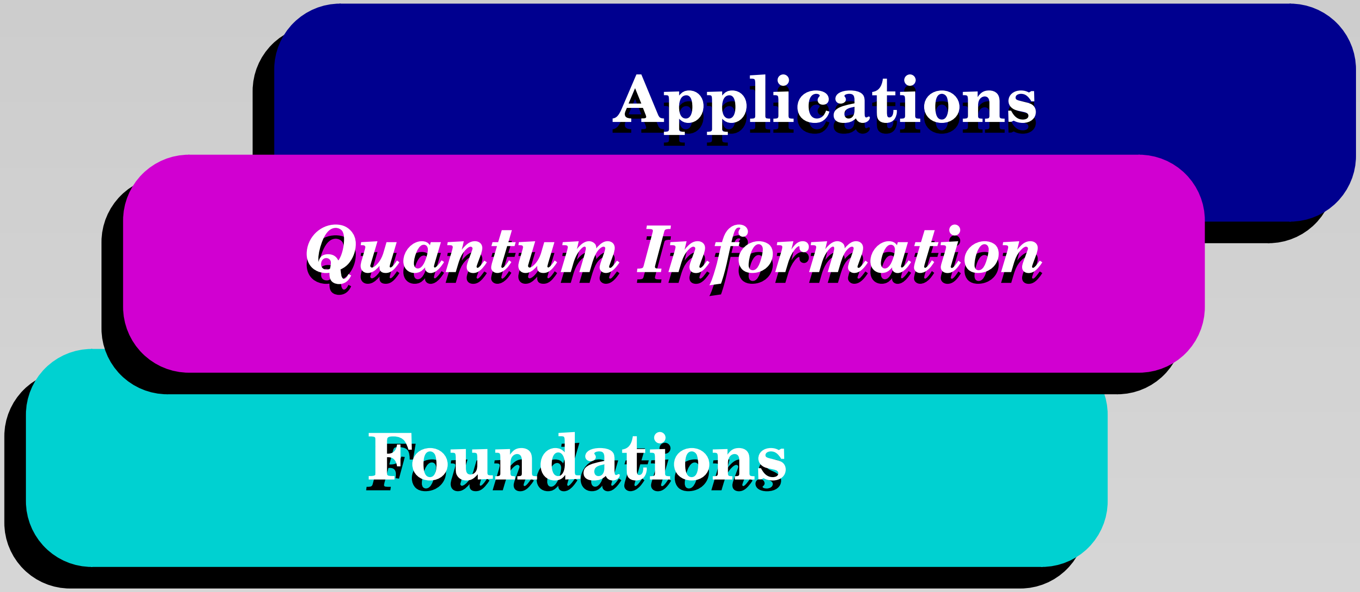


Quantum Information

Applications

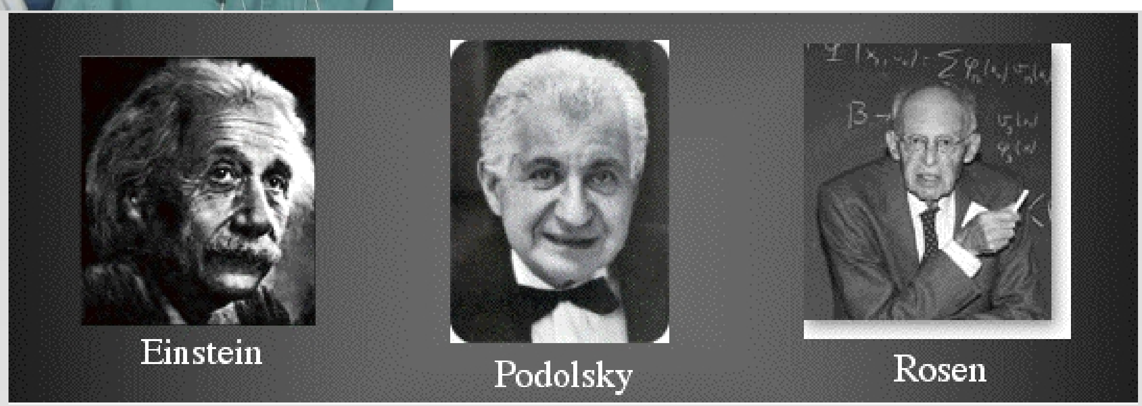
Quantum Information

Foundations


Applications
Quantum Information
Foundations

Entanglement

Quantum Mechanics: foundations



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Quantum Mechanics: foundations

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Quantum Information: applications

Applications

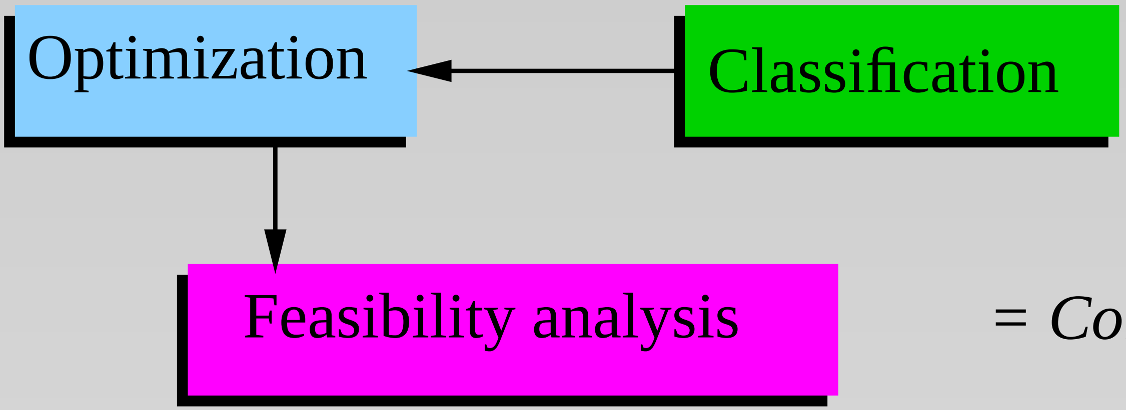
Quantum Information

Foundations

Quantum Information: applications

Optimization ← Classification = *Purely mathematical*

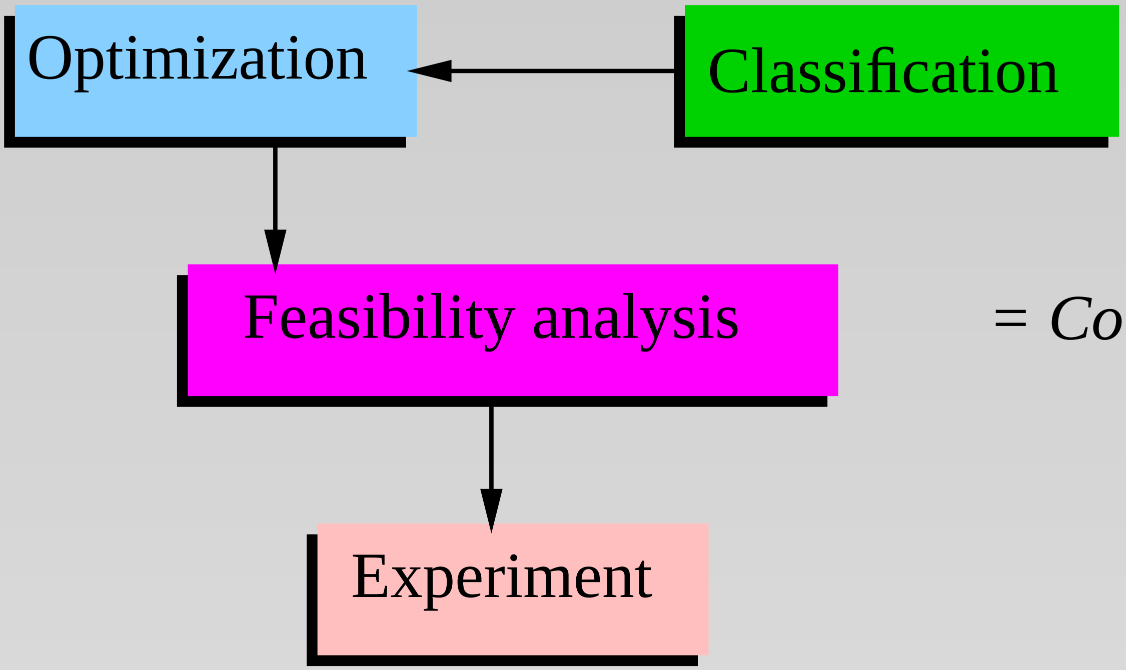
Quantum Information: applications



= *Purely mathematical*

= *Computer simulation, theory*

Quantum Information: applications



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Our general research program

To construct a **radically new generation of quantum devices** for quantum information technology

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Programmable quantum detectors [*]

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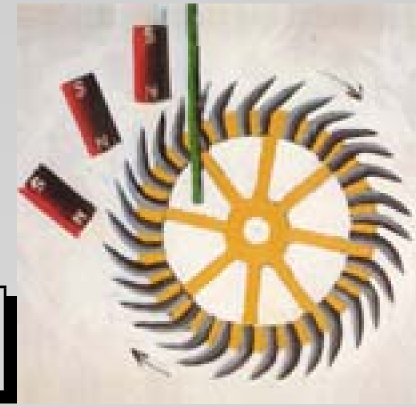
Universal quantum detectors



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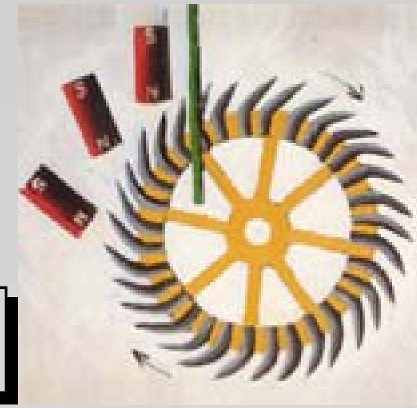

Quantum tomographers

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... and to design devices which **emulate optimally** ***impossible machines***

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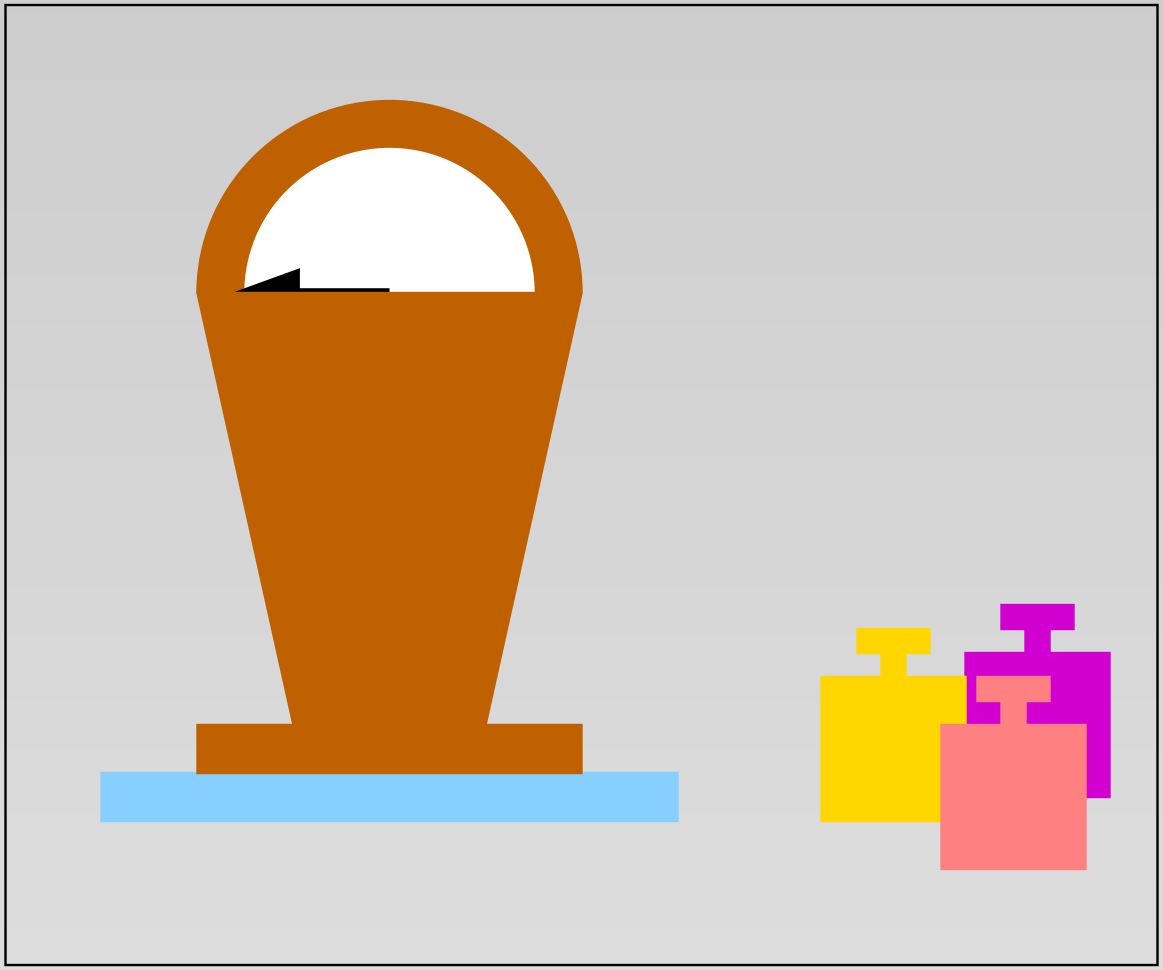
Quantum calibration

Calibration of a scale



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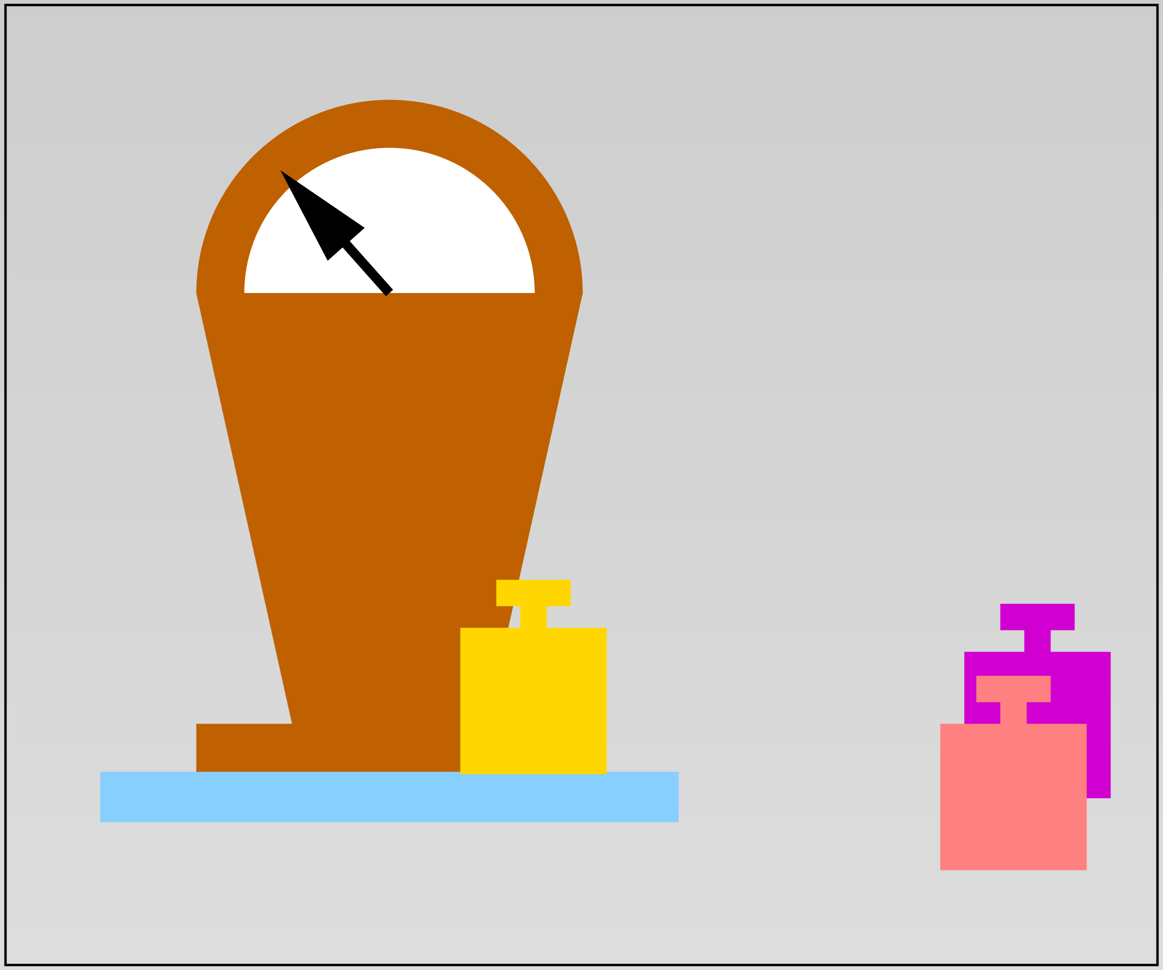

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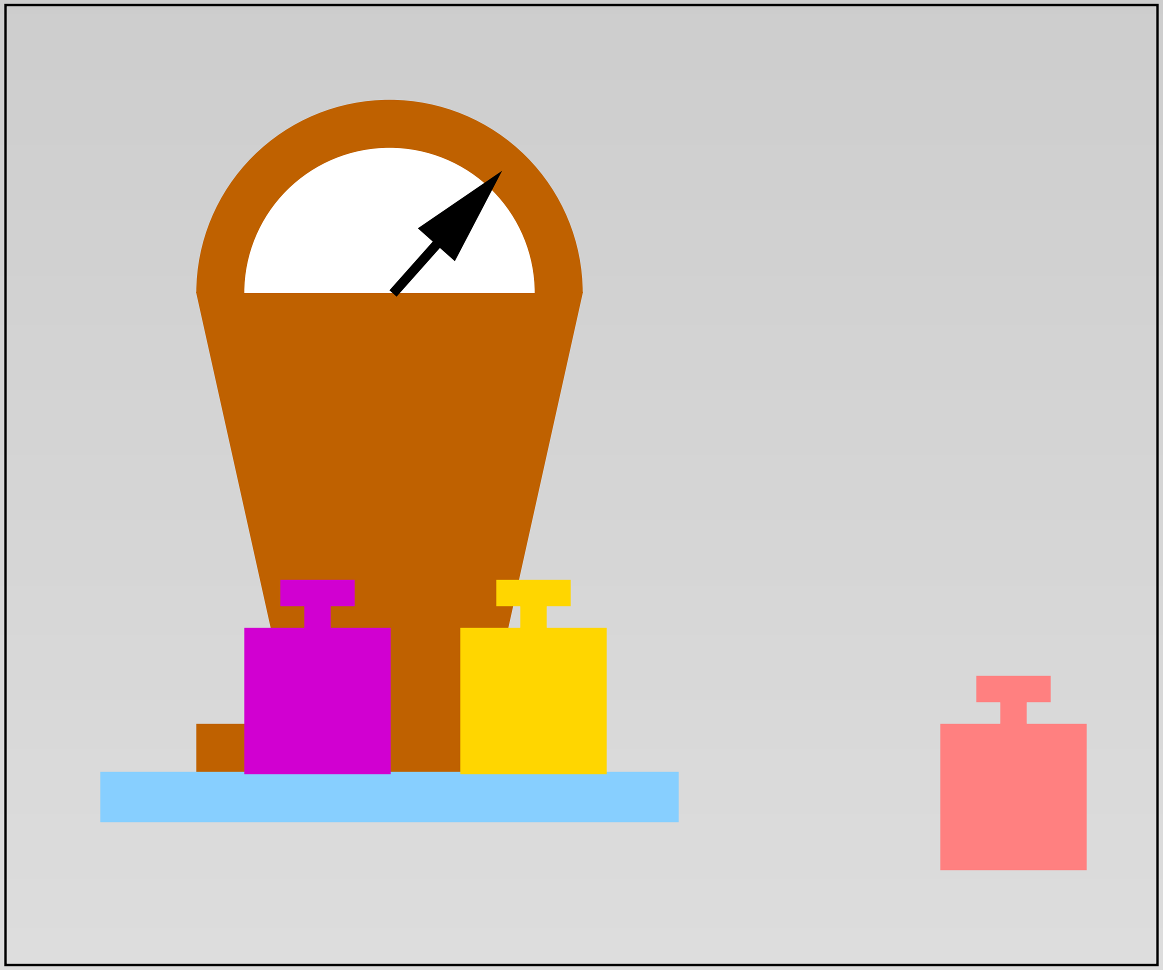

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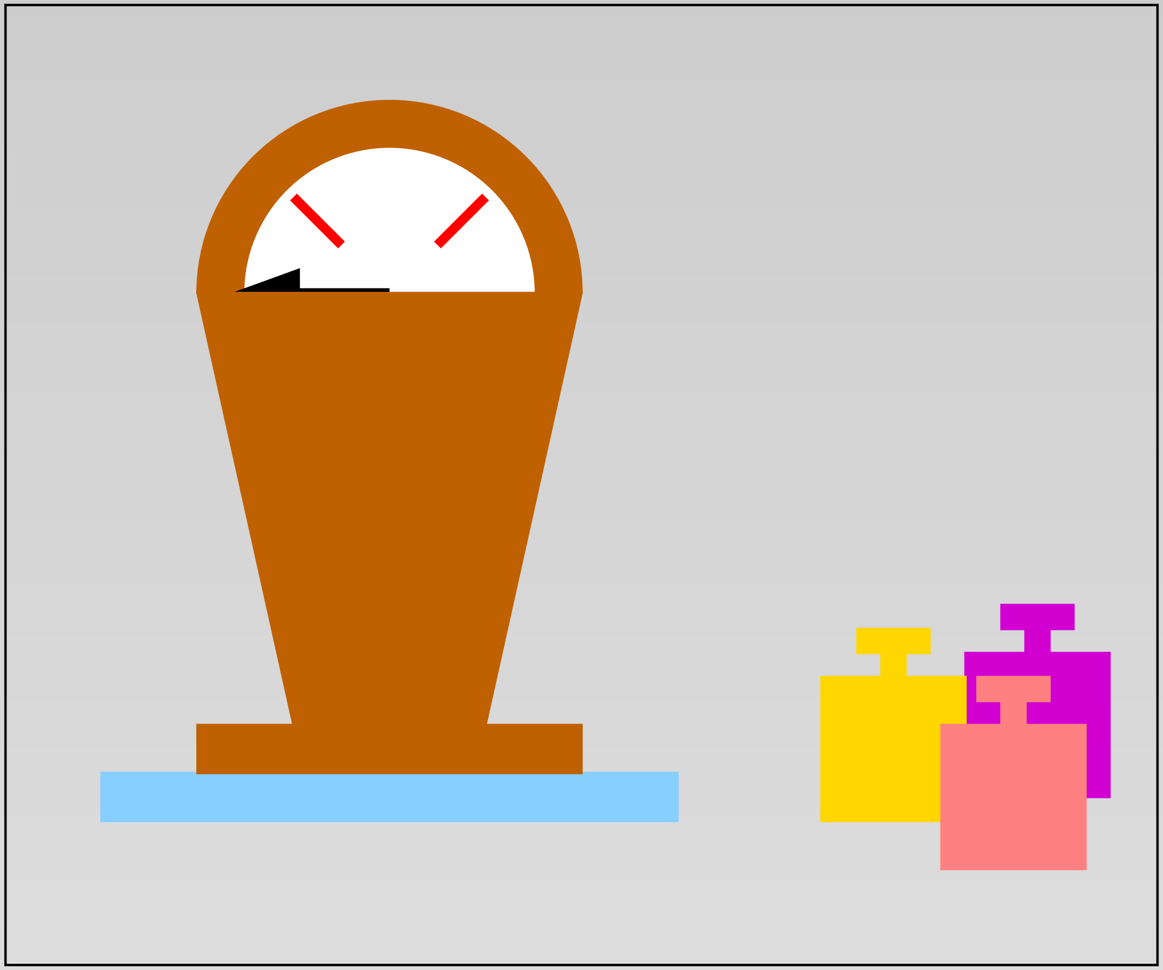

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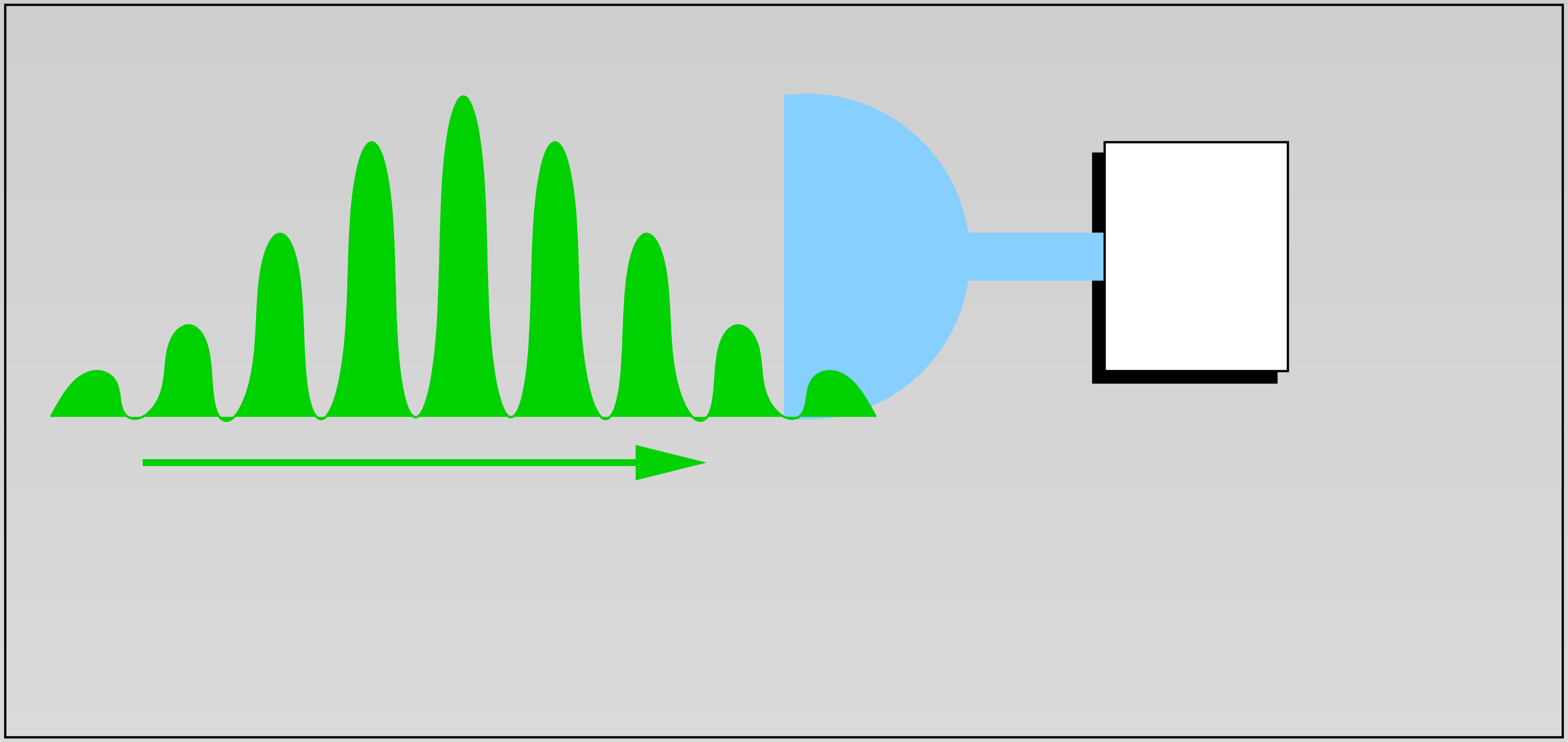

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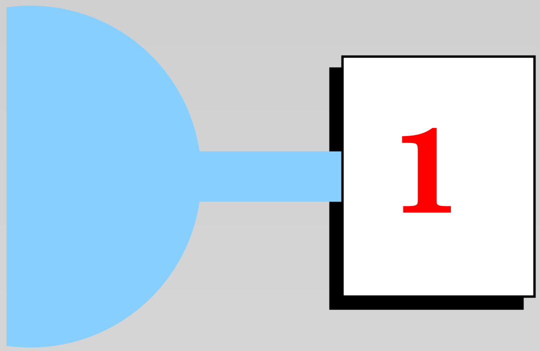

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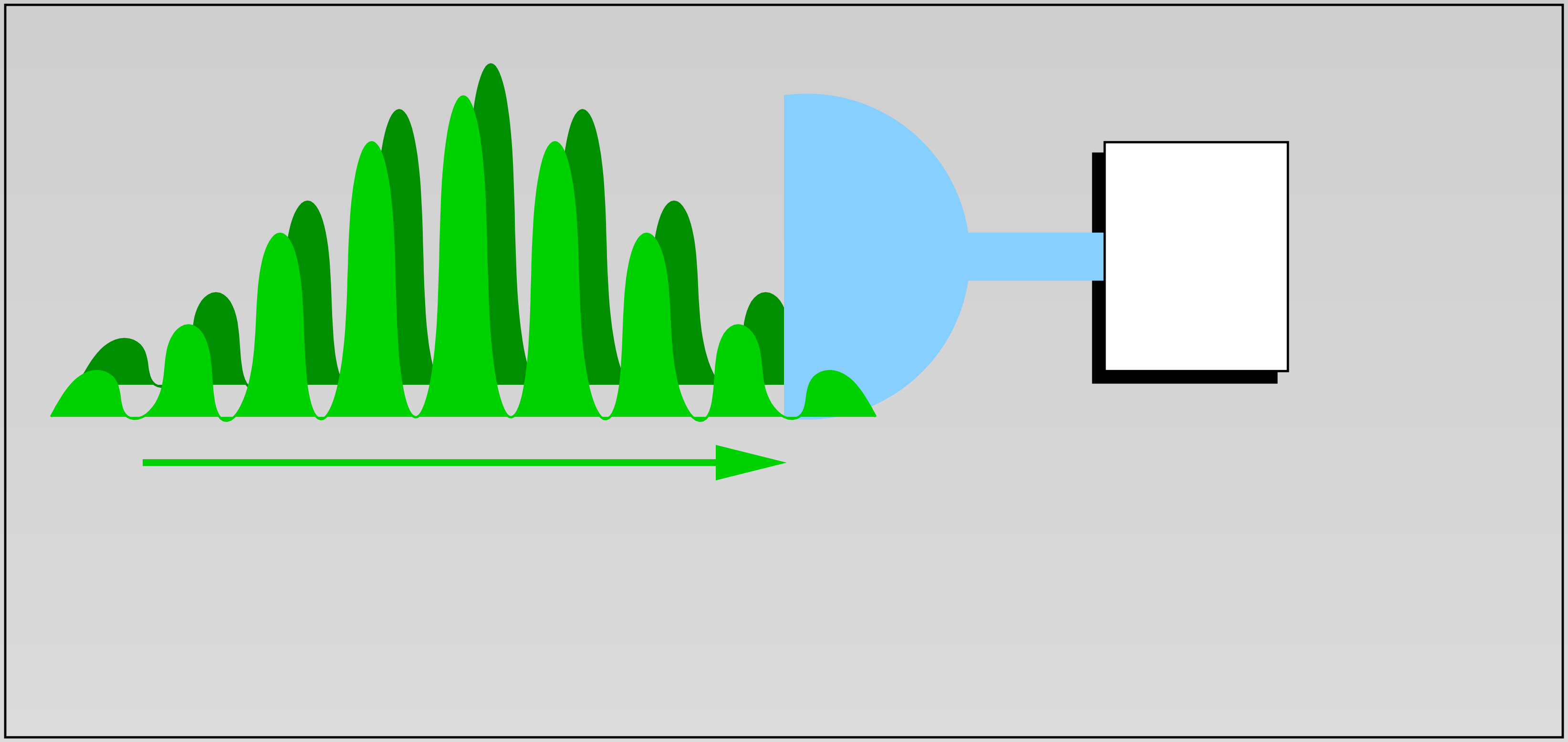

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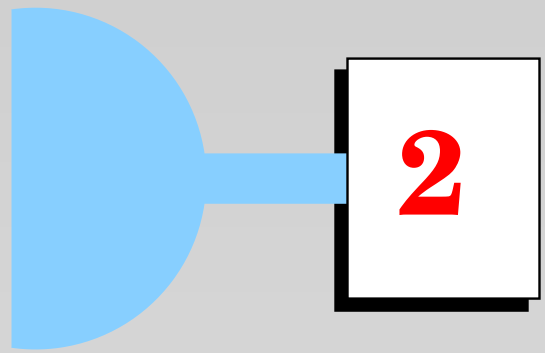

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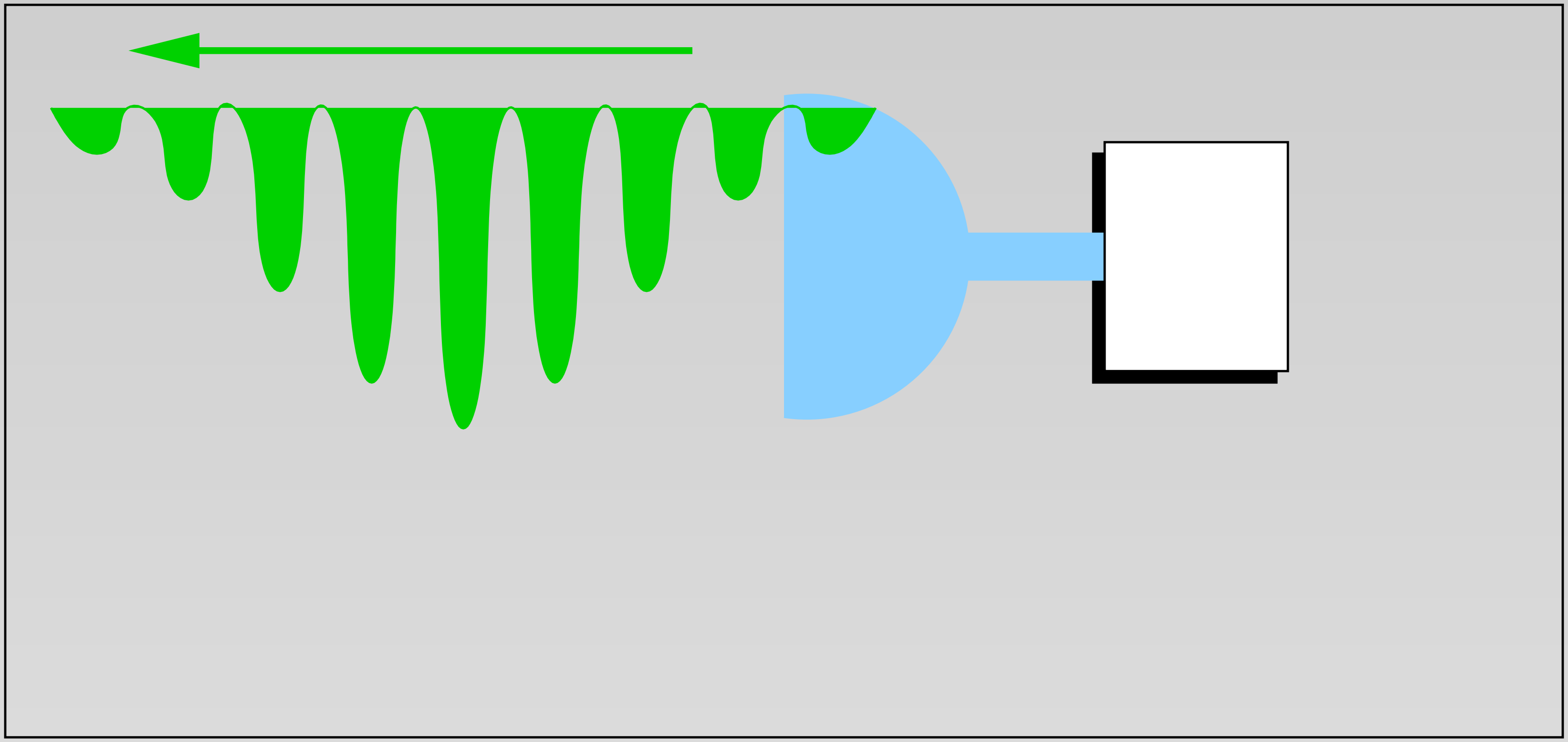

What does an apparatus measure?

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If we have an apparatus which performs a quantum measurement, how can we know what and how much it measures?

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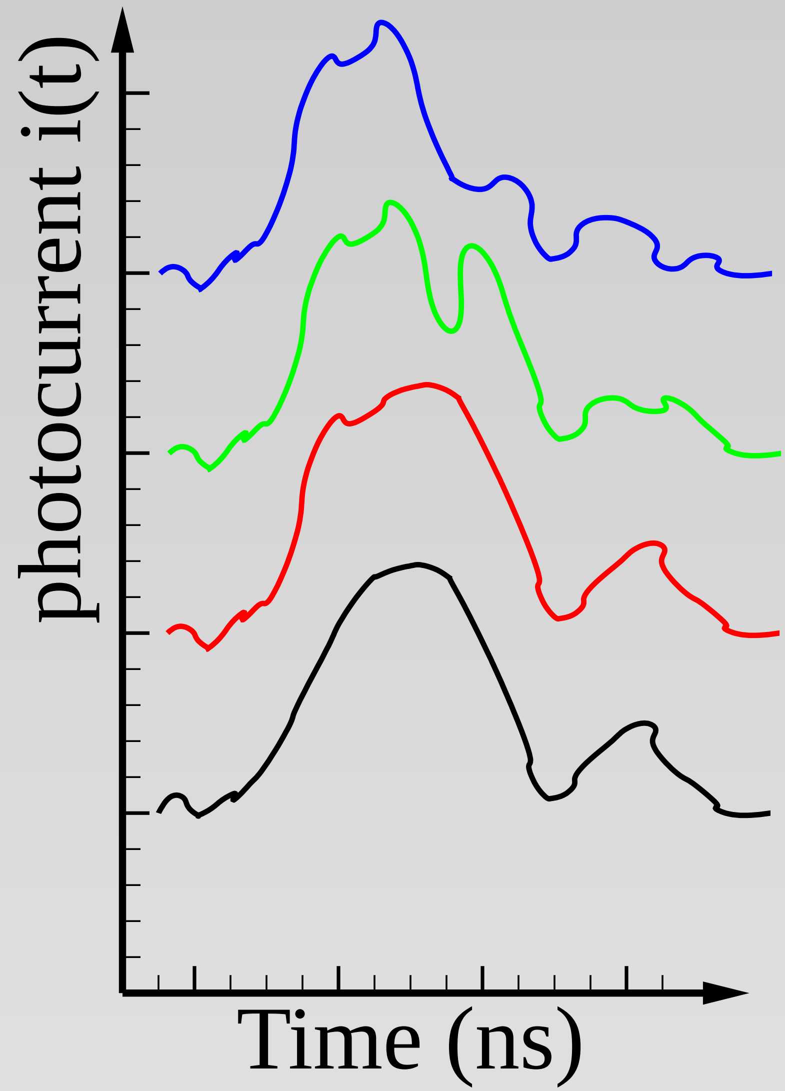
> *It is the theory which decides what we can observe!*
>
> **—Einstein to Heisenberg**

How many photons are detected?



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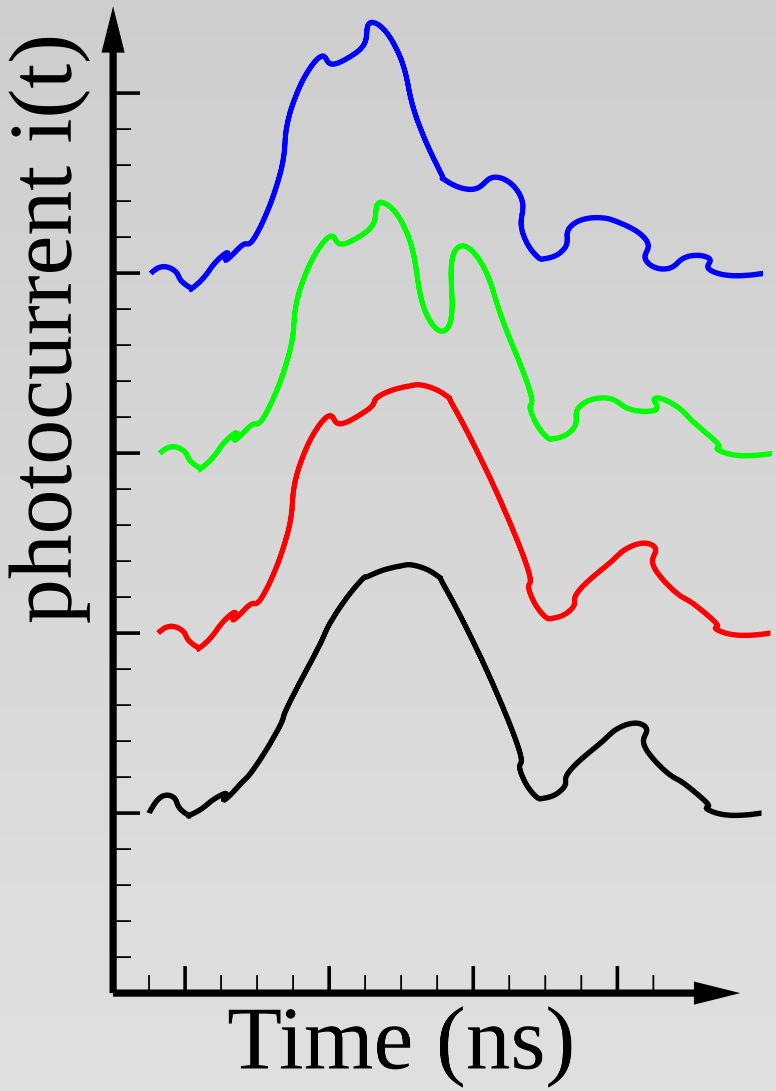

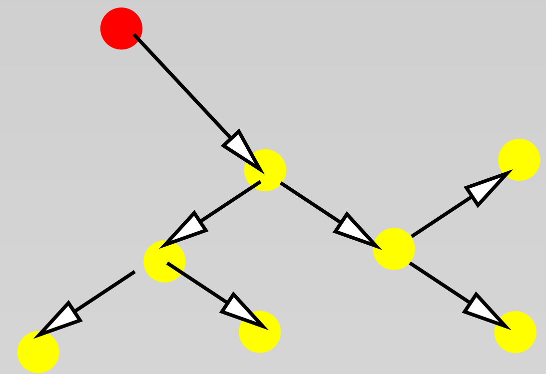

Problem

Is it possible to calibrate a photo-detector, and more generally any quantum measuring apparatus, without using the theoretical statistical mechanics description of its functioning?

How we describe a measuring apparatus

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A measuring apparatus with possible "outcomes" $\{n = 1, 2, \ldots\}$ is described by a set of operators (called POVM)

$$\mathbf{P} = \{P_n\},$$

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In order to have $p(n)$ a probability the operators P_n must satisfy the constraints

$$P_n \geq 0, \qquad \sum_n P_n = I.$$

How can we calibrate a measuring apparatus

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How can we calibrate a measuring apparatus

In principle, we can calibrate a quantum measuring apparatus without knowing its functioning by determining experimentally its POVM $\{P_n\}$.

Quantum tomography

- Quantum tomography is a method to estimate the ensemble average $\langle H \rangle$ of any arbitrary operator H by measuring a ***quorum* of observables** $\{O_l\}$.

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- There are general method for unbiasing instrumental noise, adaptive techniques, maximum-likelihood strategies, etc.

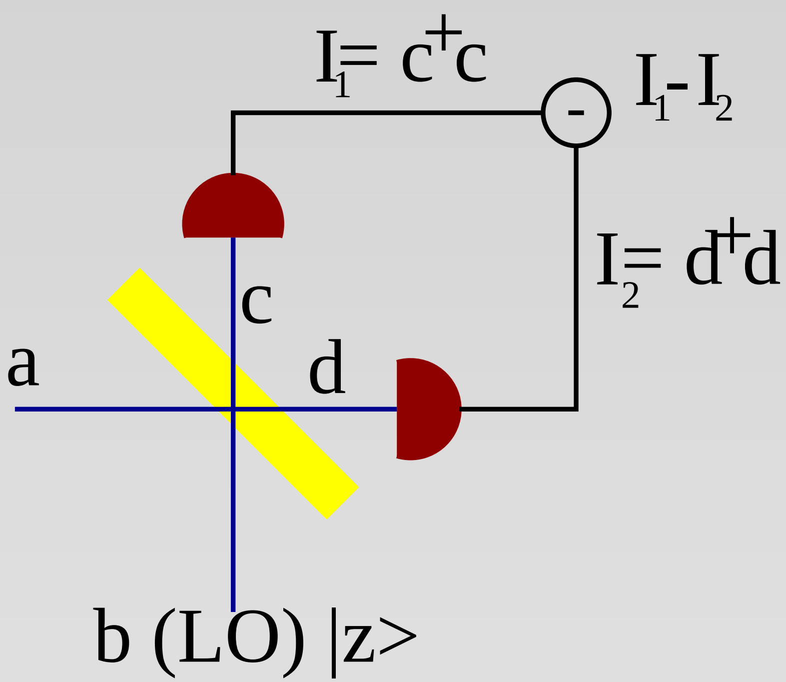
[G. M. D'Ariano, Scuola "E. Fermi", (IOS Press, Amsterdam 2002) pag. 385.]

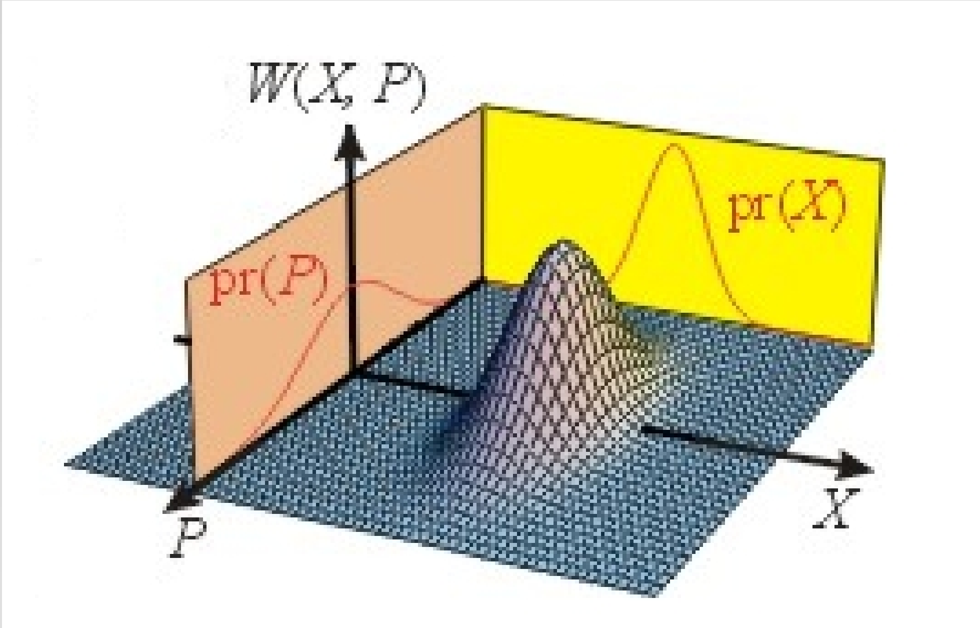
Homodyne tomography

- In quantum optics for each field mode a **quorum** $\equiv$ {**quadratures**}

$$\boxed{\mathsf{X}_\phi = \tfrac{1}{2}\left(a^\dagger e^{i\phi} + a e^{-i\phi}\right) \equiv Q\cos\phi + P\sin\phi.}$$

$$\langle H\rangle = \int_0^\pi \frac{\mathrm{d}\,\phi}{\pi}\langle E_H(X_\phi;\phi)\rangle\ , \qquad E_H(x;\phi) = \tfrac{1}{4}\int_{-\infty}^{+\infty}\mathrm{d}\,k\,|k|\mathrm{Tr}[H e^{ikX_\phi}]e^{-ikx}.$$





Pauli tomography

Pauli matrices I , σ_x , σ_y , σ_z orthonormal basis for the qubit operator space:

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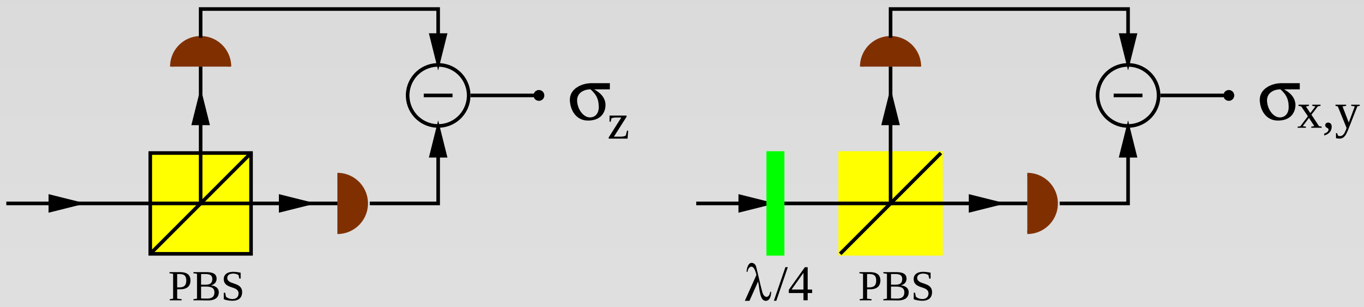
- In Quantum Optics the qubits are encoded on polarization of single photons:

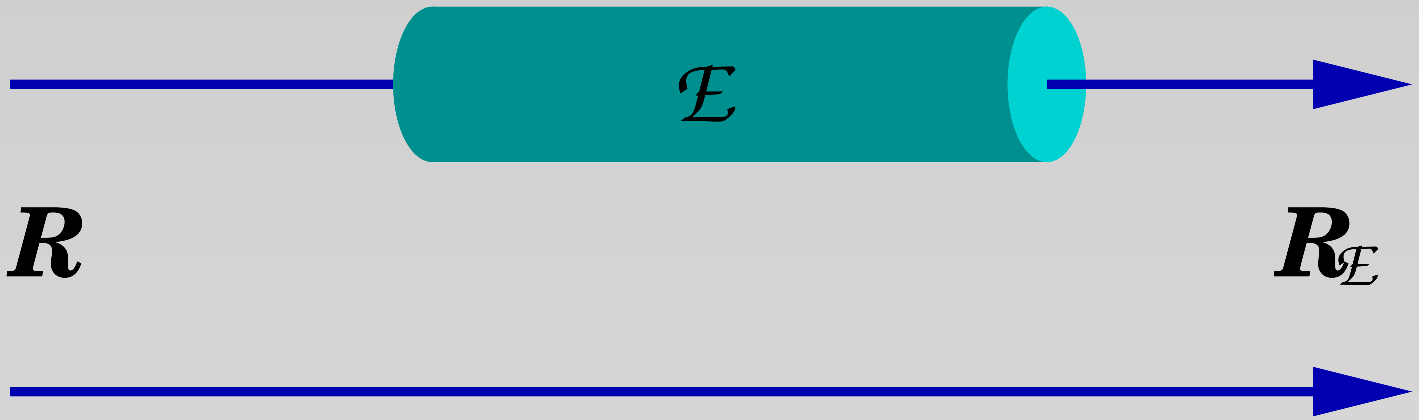
$$\sigma_z = h^\dagger h - v^\dagger v,$$

$$|\uparrow\rangle \equiv |1\rangle_h|0\rangle_v, \qquad |\downarrow\rangle \equiv |0\rangle_h|1\rangle_v,$$



Faithful states

- A bipartite state R is **faithful** when acting with a device on R as in figure the output $R_{\mathscr{E}}$ carries a complete information about the operation $\mathscr{E}$ of the device



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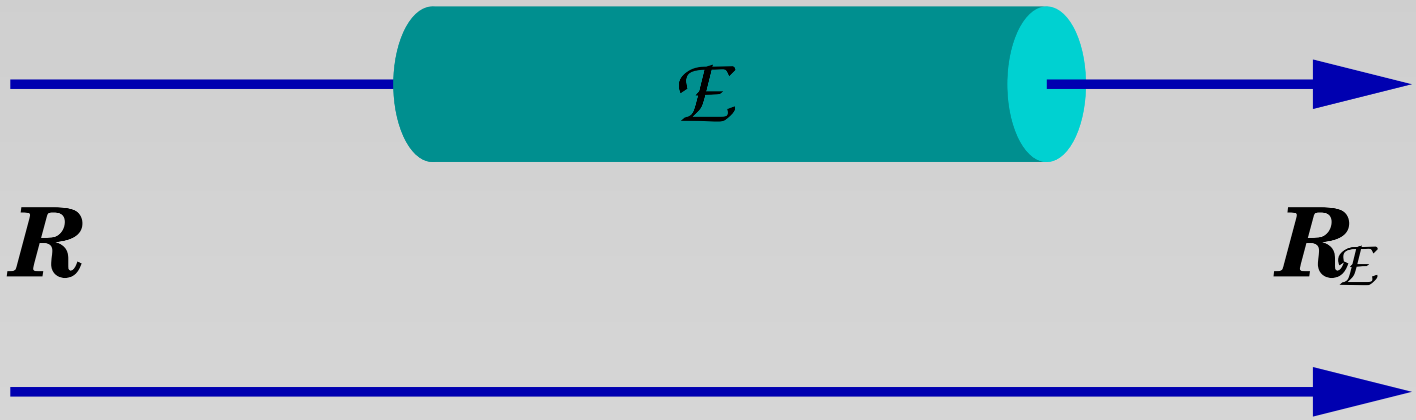

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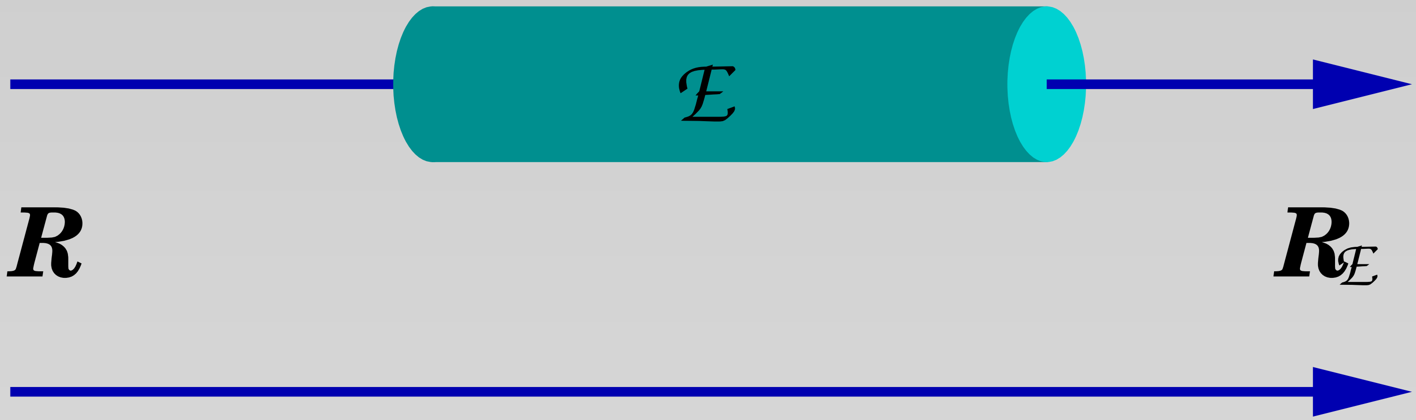

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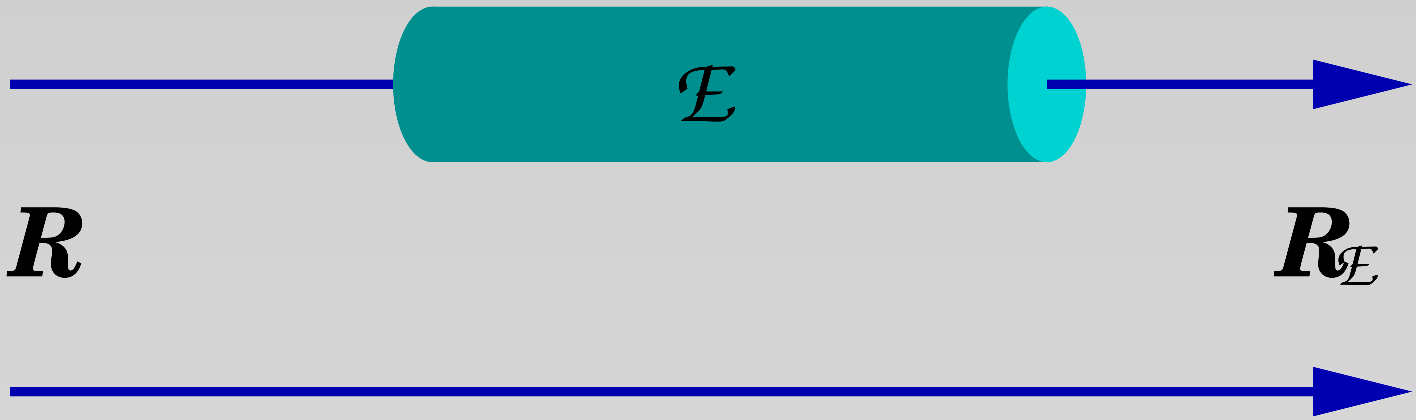

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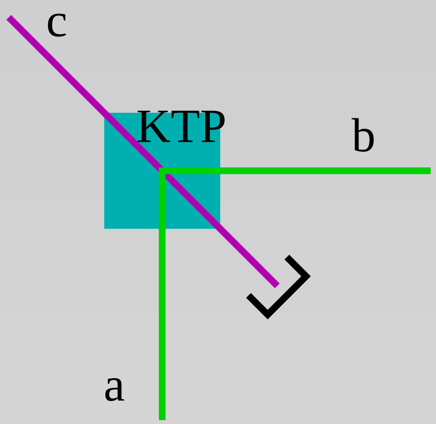
  There are **pure faithful states: the entangled states** (quantum parallelism of entanglement!)

[G. M. D'Ariano and P. Lo Presti, Phys. Rev. Lett. **91** *047902-1 (2003)]*

Entangled states in quantum optics

- Nonlinear Quantum Optics: **parametric downconversion of vacuum**

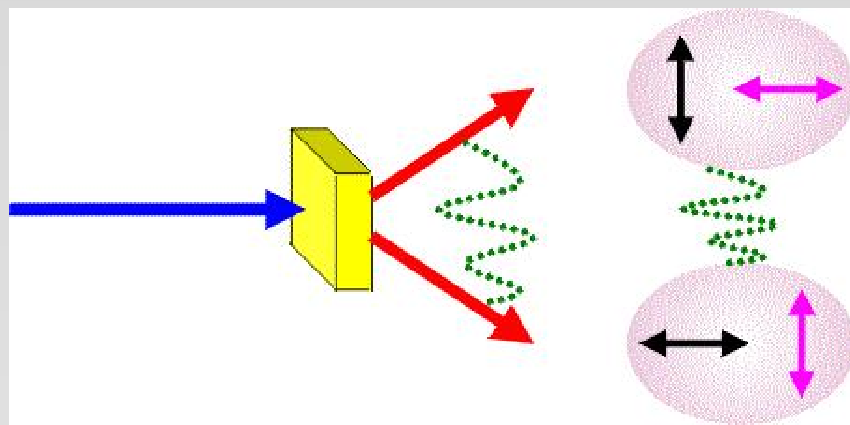


Hamiltonian $\boxed{H \propto c a^\dagger b^\dagger + h.c.}$

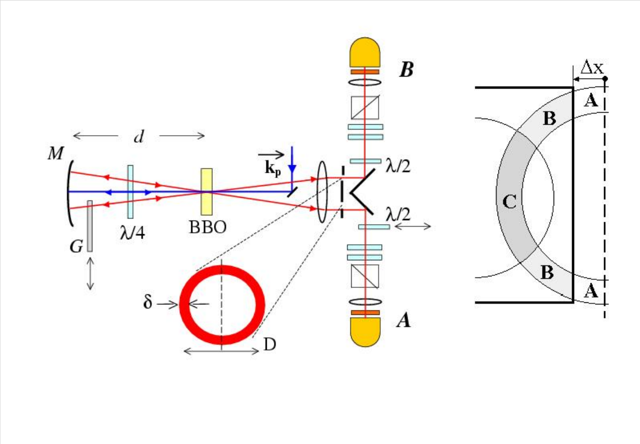
where $\omega_c = \omega_a + \omega_b$

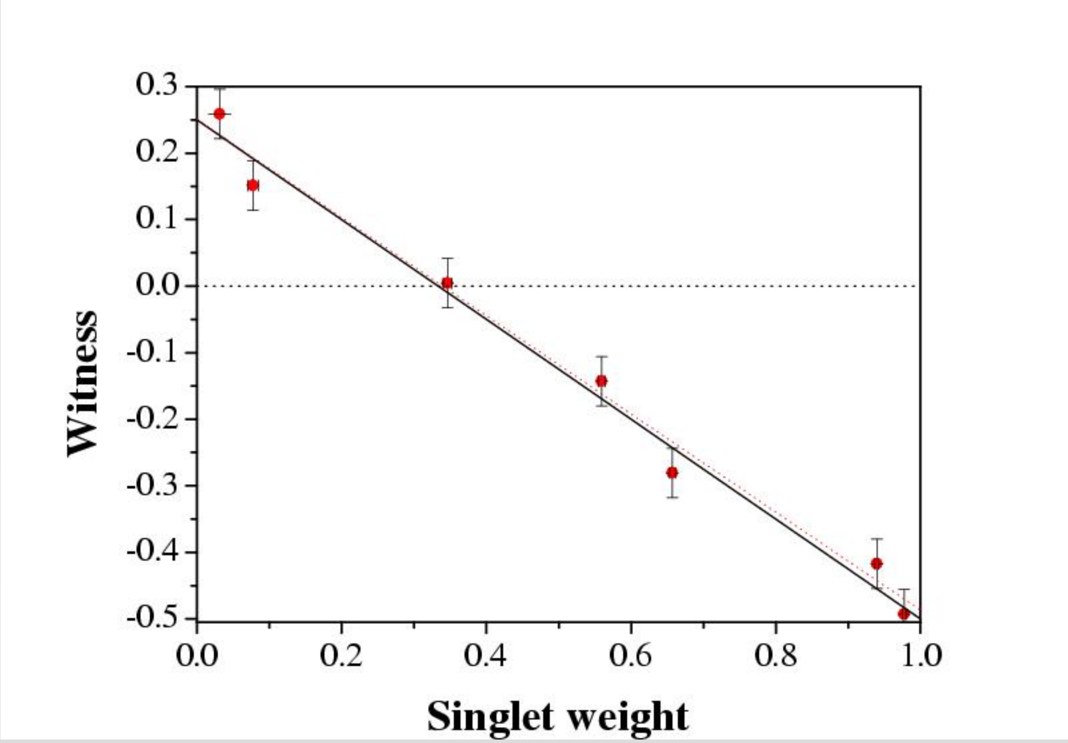
- From input vacuum in a and b and classical pump c produces the **twin-beam**

$$\boxed{|\Psi\rangle\!\rangle = (1 - |\xi|^2)^{\frac{1}{2}} \sum_{n=0}^{\infty} \xi^n |n\rangle \otimes |n\rangle}$$



Production of faithful states





Faithful states

- Essentially **any garbage state is faithful** (invertibility is a *dense* condition).

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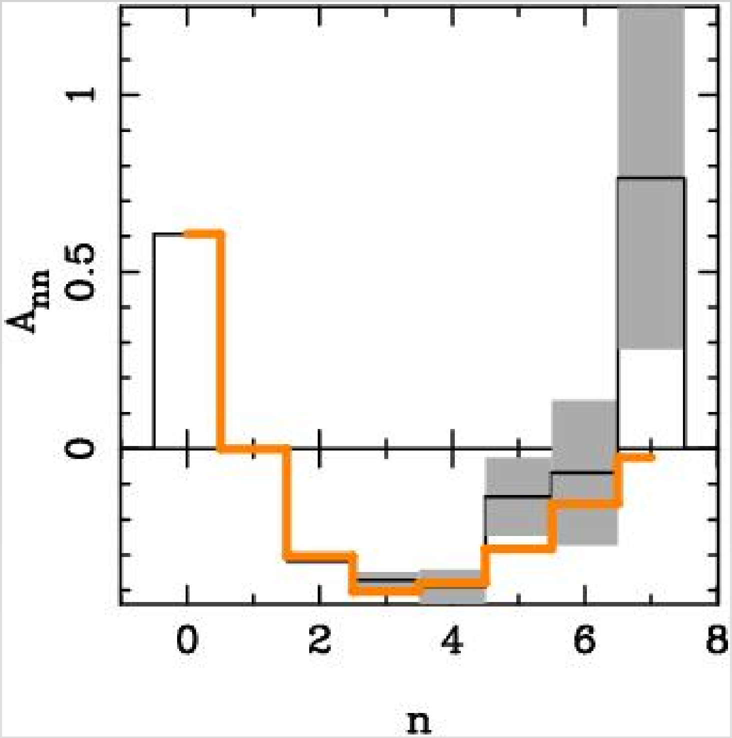
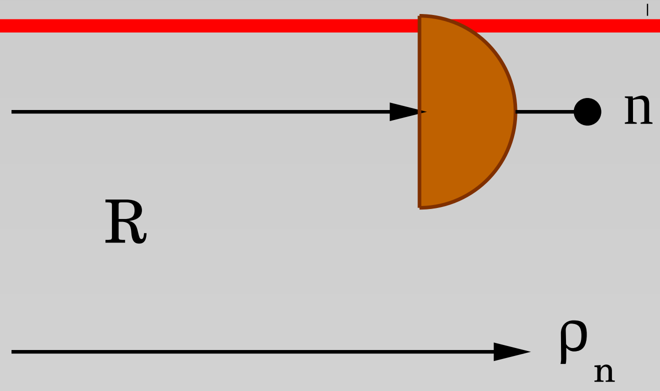
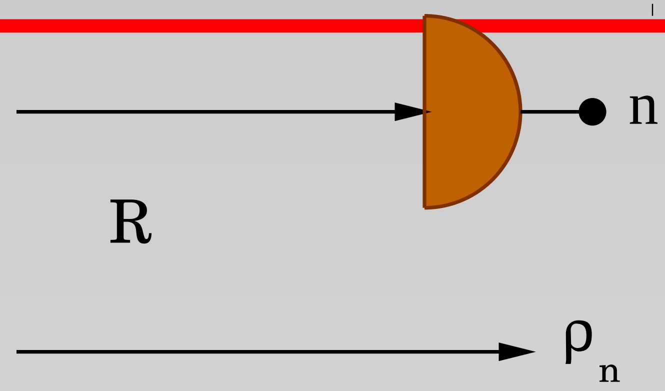
- However, the knowledge of the map $\mathscr{E}$ from the measured output state $R_{\mathscr{E}}$ will be affected by increasingly large statistical errors for input state R approaching a non-faithful one.



Quantum Calibration of a measuring apparatus



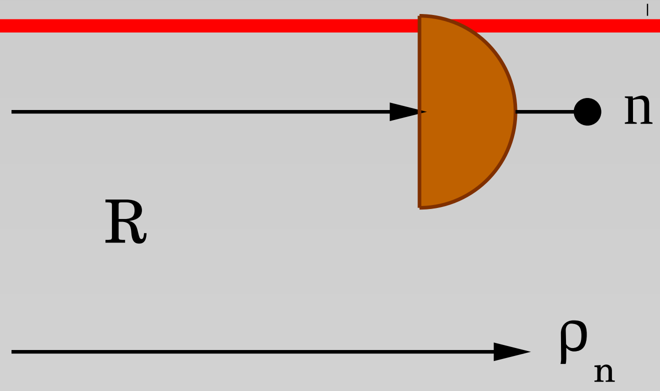
Quantum Calibration of a measuring apparatus



- The calibration is achieved by determining experimentally the POVM using a faithful state as in figure.

$$\boxed{\mathsf{P}_n = p(n)[\mathscr{R}^{-1}(\rho_n)]^\intercal].}$$

Quantum Calibration of a measuring apparatus



- The calibration is achieved by determining experimentally the POVM using a faithful state as in figure.

$$\boxed{P_n = p(n)[\mathscr{R}^{-1}(\rho_n)]^{\intercal}].}$$

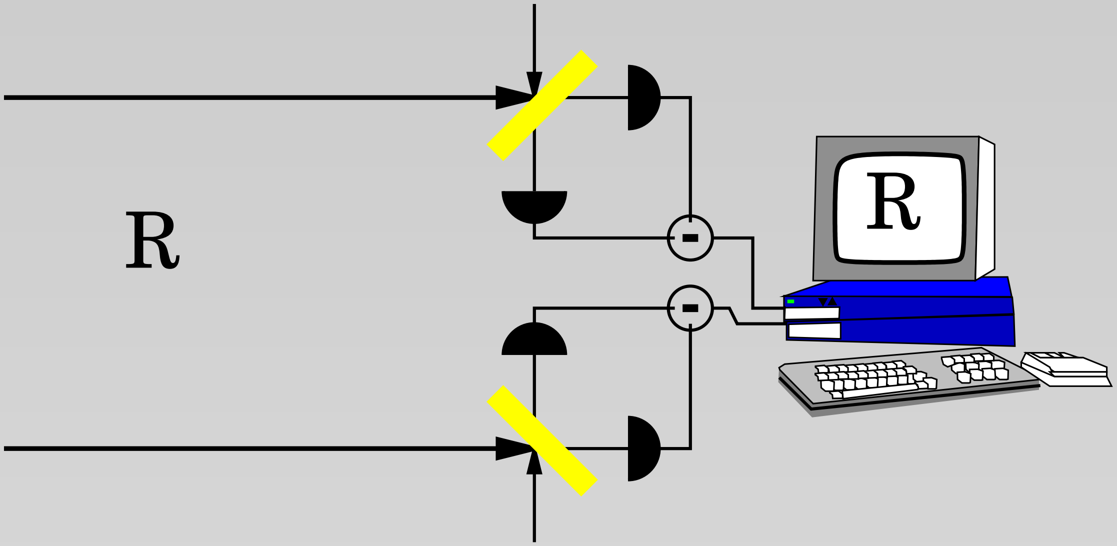
- $p(n)$ probability of the outcome n,

- ρ_n conditioned state, to be determined by quantum tomography,

- $\mathscr{R}$ associated map of the faithful state R.

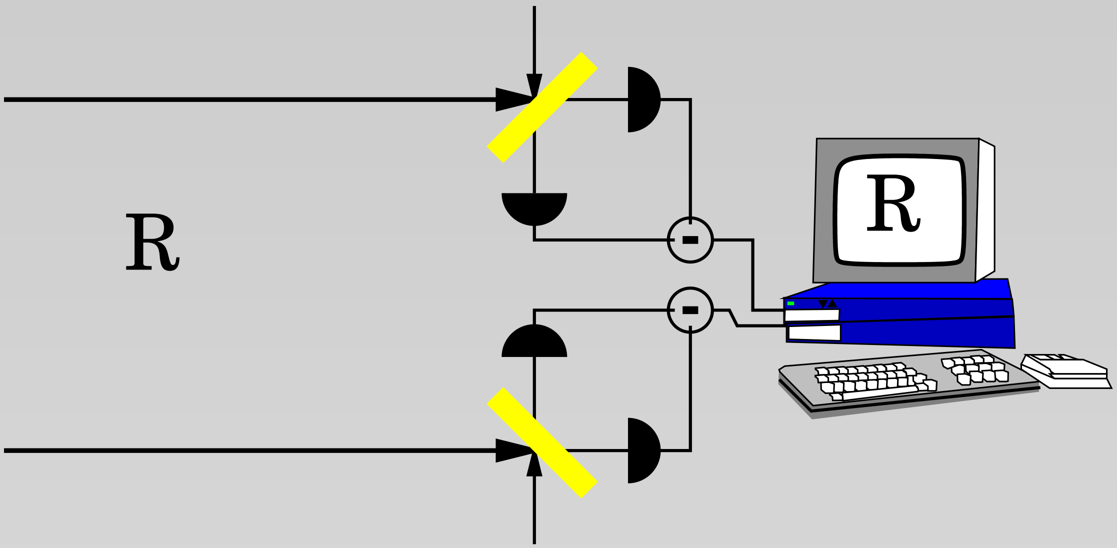
Quantum Calibration of a measuring apparatus

- *Precalibration*

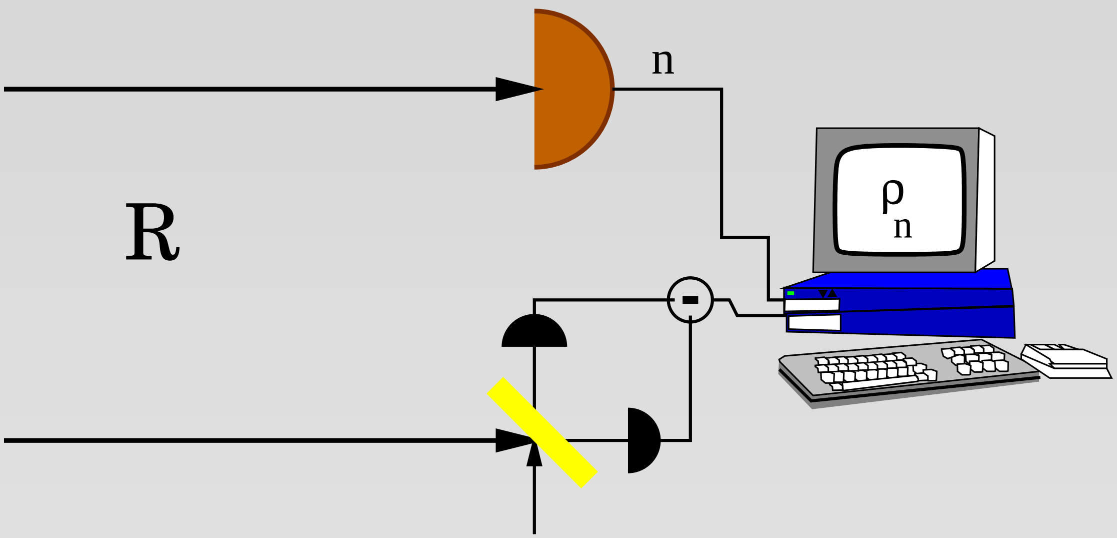


Quantum Calibration of a measuring apparatus

- *Precalibration*



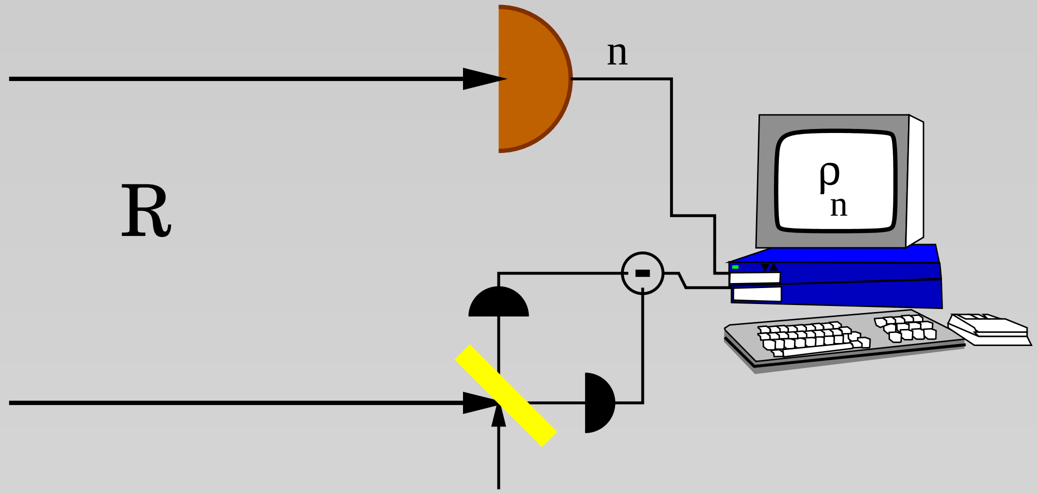
- *Calibration*



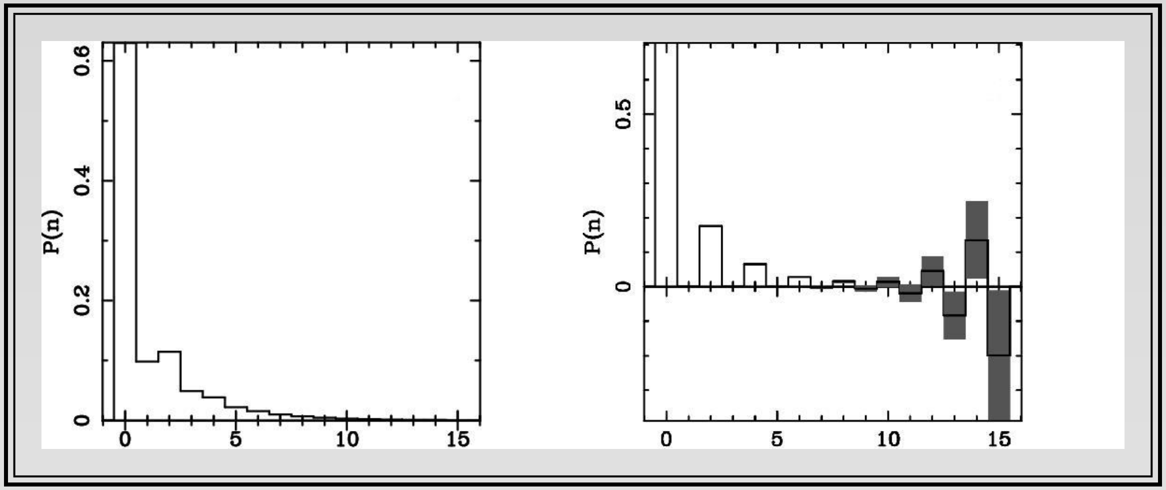
Quantum Calibration of a measuring apparatus

In principle we need only two tomographers and a single faithful state to calibrate any measuring apparatus.

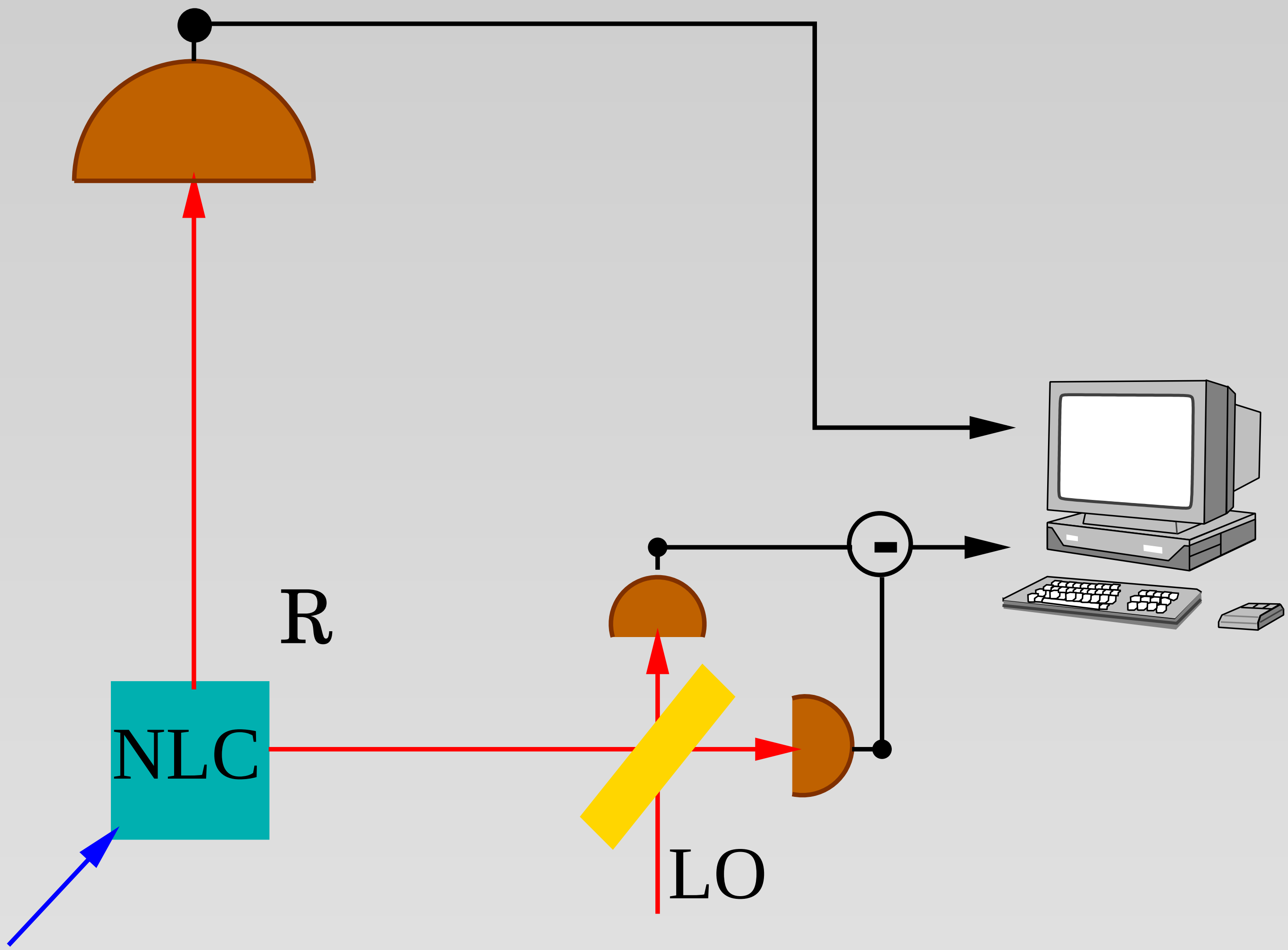
Quantum Calibration of a measuring apparatus



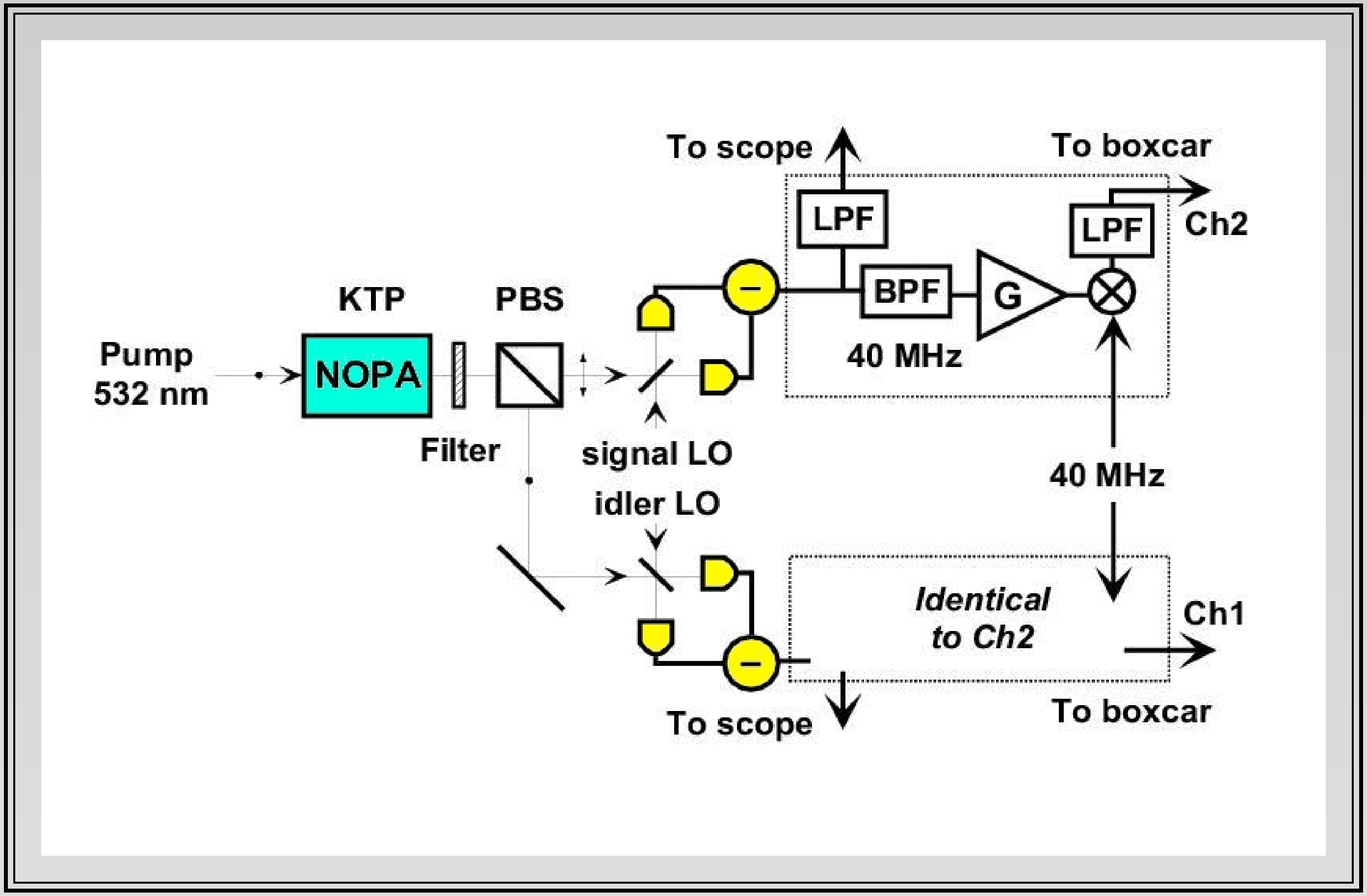
- Using a "calibrated observable" the measurement is "unbiased" (at expense of some increasing statistical error).



Quantum calibration of a photodetector



Tomography of a twin-beam

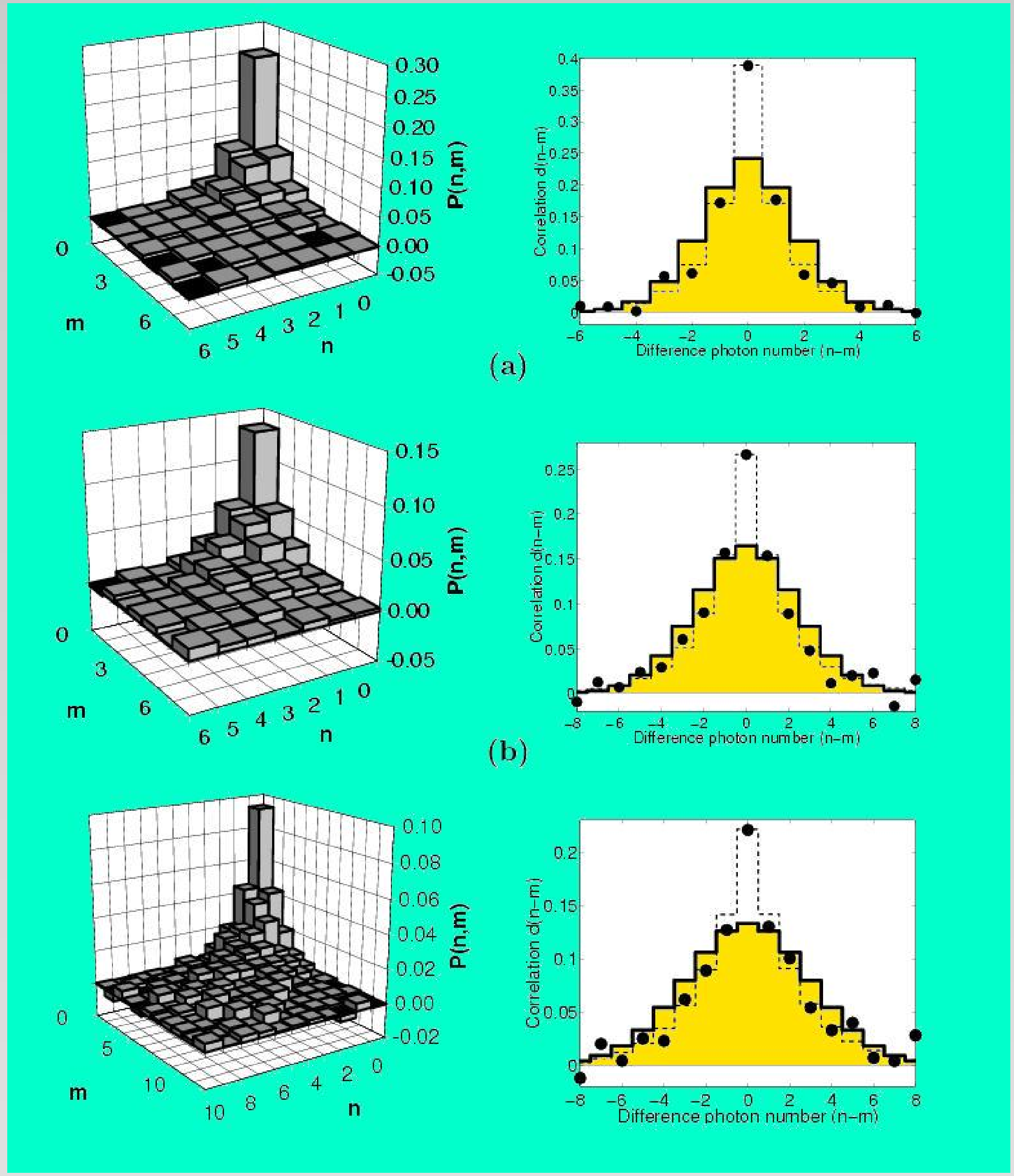


Measurement of the joint photon-number probability distribution of a twin-beam: schematic of the experimental setup. NOPA, non-degenerate optical parametric amplifier; LOs, local oscillators; PBS, polarizing beam splitter; LPFs, low-pass filters; BPF, band-pass filter; G, electronic amplifier. Electronics in the two channels are identical.

[M. Vasilyev, S.-K. Choi, P. Kumar, and G. M. D'Ariano, Phys. Rev. Lett. **84** *2354 (2000)]*

Results

Left: Measured joint photon-number probability distributions for the twin-beam state. Right: Difference photon number distributions corresponding to the left graphs (filled circles, experimental data; solid lines, theoretical predictions; dashed lines, difference photon-number distributions for two independent coherent states with the same total mean number of photons and $\overline{n} = \overline{m}$.) (a) 400000 samples, $\overline{n} = \overline{m} = 1.5$, $N = 10$; (b) 240000 samples, $\overline{n} = 3.2$, $\overline{m} = 3.0$, $N = 18$; (c) 640000 samples, $\overline{n} = 4.7$, $\overline{m} = 4.6$, $N = 16$. The measured distributions exhibit up to 1.9 dB of quantum correlation between the signal and idler photon numbers.



Homodyne calibration of a photodetector

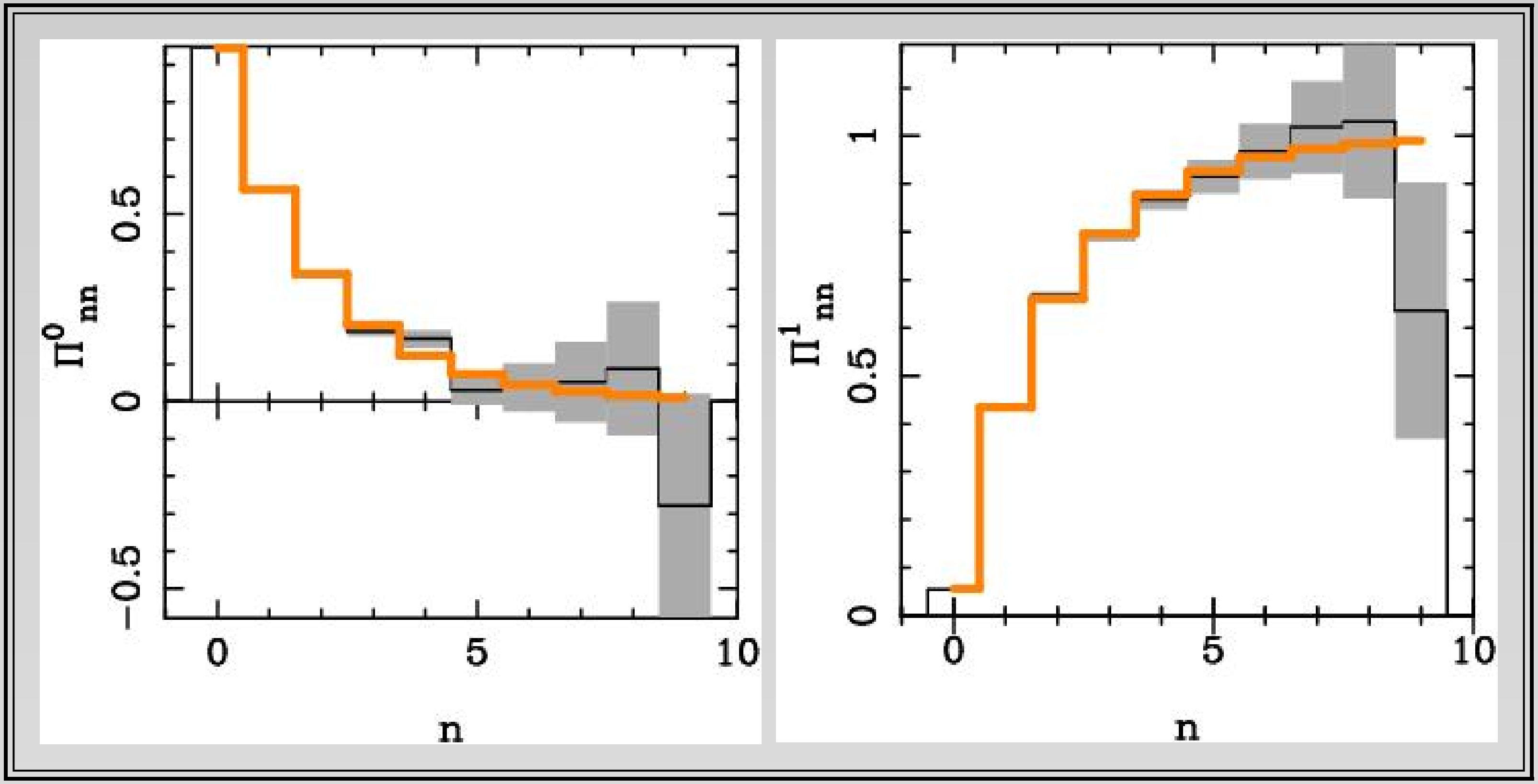


Figure 1: Homodyne tomography of an On/Off photo-detector with quantum efficiency $\eta = 0.4$ and thermal noise photon number $\nu = 0.1$. The reconstruction is obtained by pattern-function averaging of $1.5 \cdot 10^6$ data, for homodyne quantum efficiency $\eta = 0.9$ and twin beam thermal photon $\bar{n} = 3$.

Homodyne calibration of a photodetector

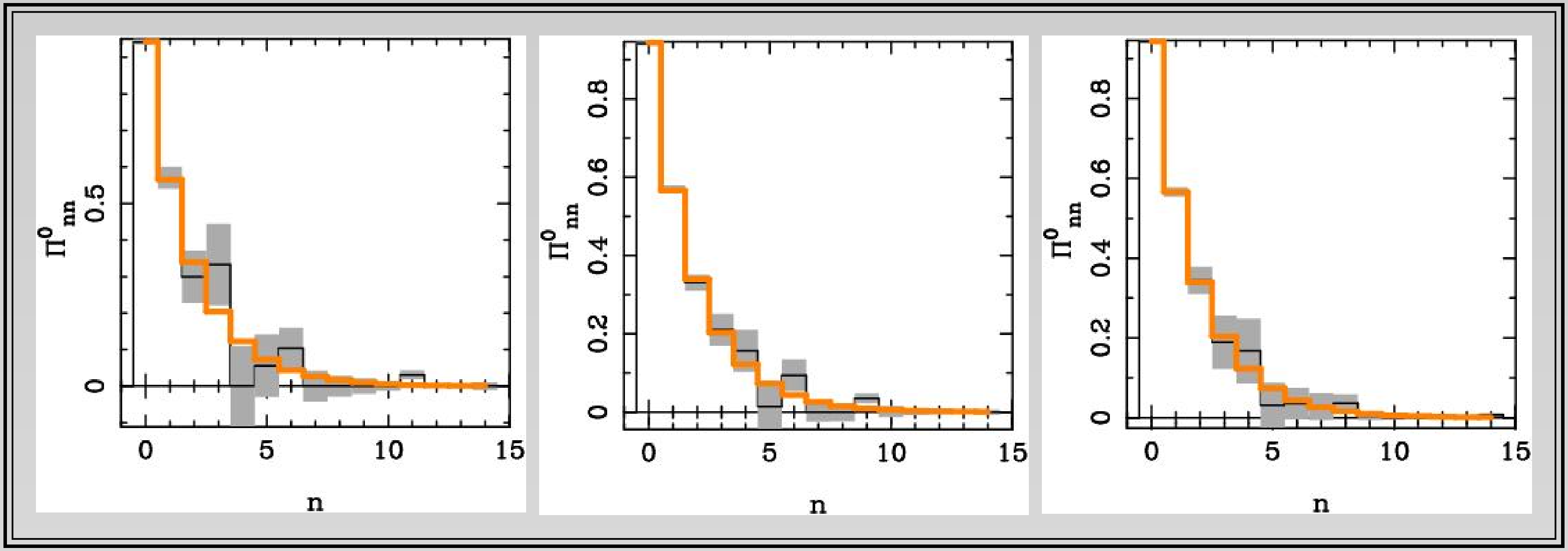
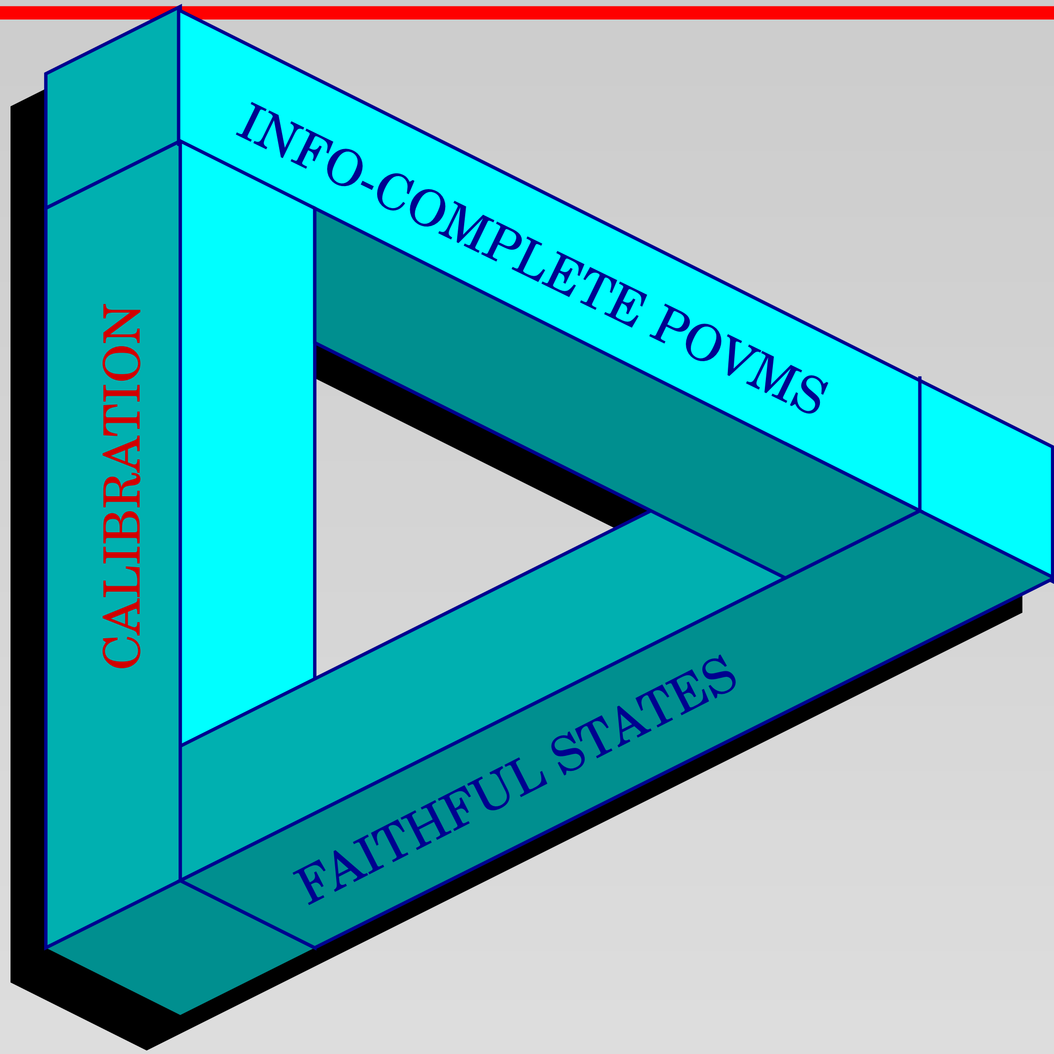
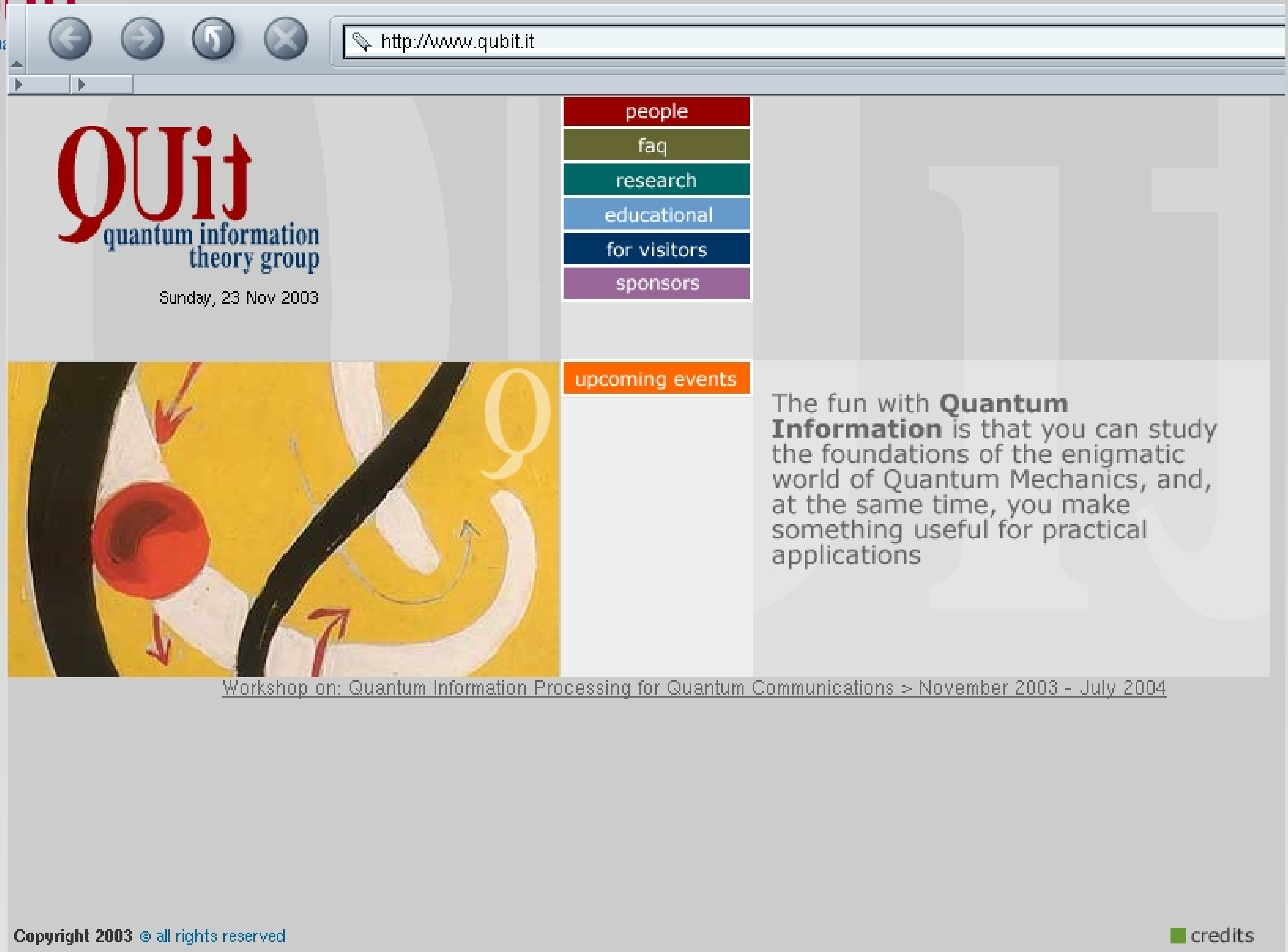


Figure 2: Homodyne tomography of an On/Off photodetector with quantum efficiency $\eta = 0.4$ and thermal noise photon number $\nu = 0.1$, with $\bar{n} = 3$ photons in the twin-beam. The ML estimation of the diagonal of the only Off POVM element are reported for different values of sample size N and homodyne quantum efficiency η_H. Left: $N = 10^5$, $\eta_H = 0.7$; Middle: $N = 10^4$, $\eta_H = 0.9$; Right: $N = 10^6$, $\eta_H = 0.7$.

Quantum Mechanics: physical axioms?



Informationally complete POVM's = **calibrators**: *"the quantum standards of the International Bureau of Weights and Measures à Paris"* — Chris Fuchs.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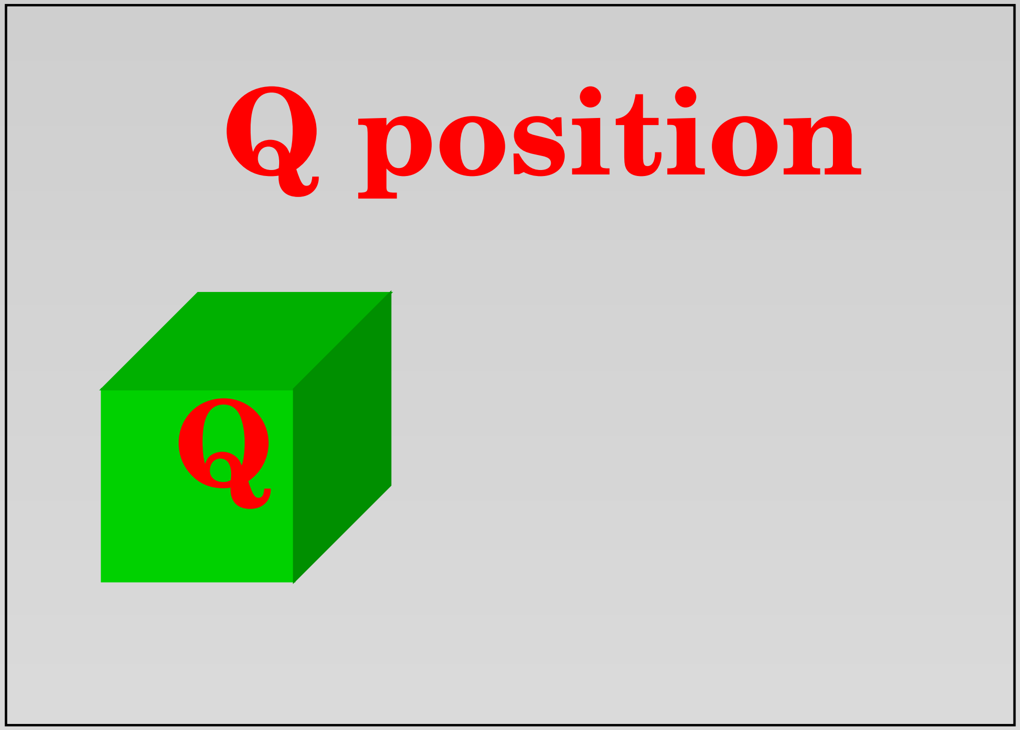
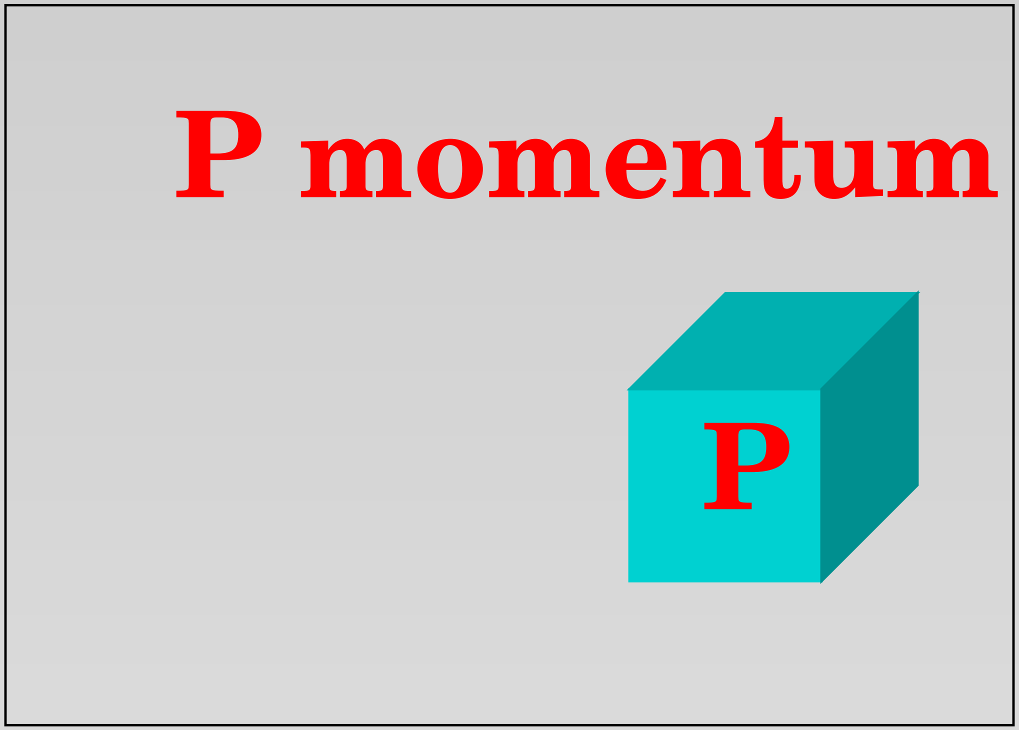
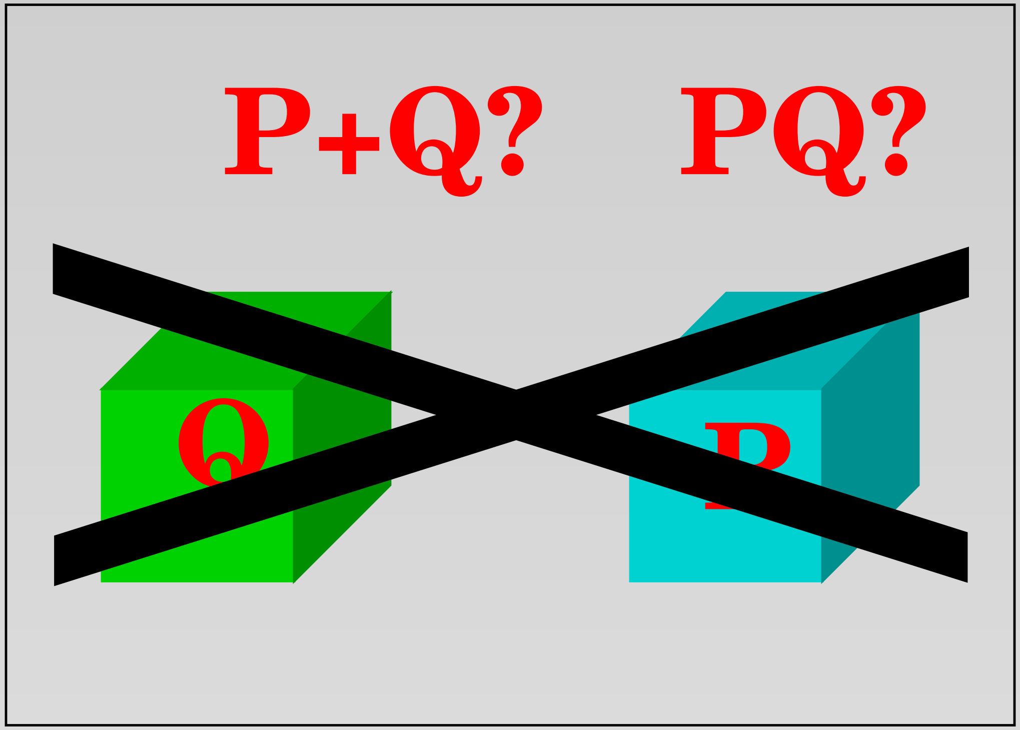
Quantum observables and measurement devices



Quantum observables and measurement devices



Quantum observables and measurement devices



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