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Principles for Quantum Theory

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

QMS: decoherence and empirical estimates

June 29 - July 1st 2015

Principles for Quantum Theory

- The experience in Quantum Information has led us to look at Quantum Theory (QT) under a completely new angle
- QT is a *theory of information*

Giulio Chiribella

Paolo Perinotti

Selected for a Viewpoint in *Physics*

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Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification*

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Book from CUP soon

Principles for Quantum Theory

- The experience in Quantum Information has led us to look at Quantum Theory (QT) under a completely new angle
- QT is a *theory of information*

Giulio Chiribella

Paolo Perinotti

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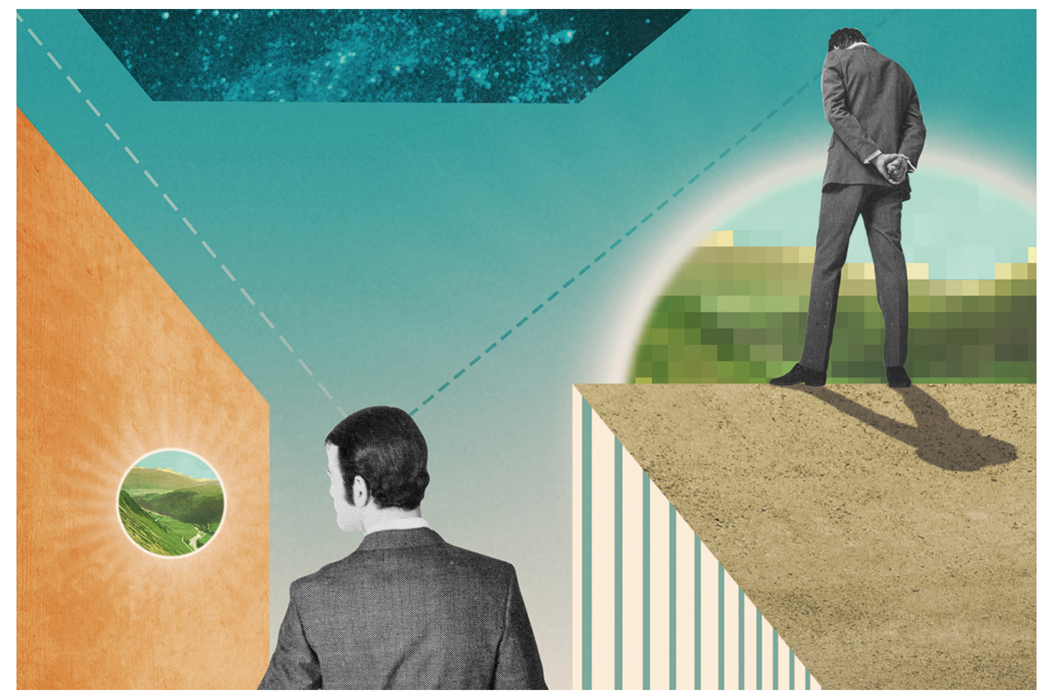


(Image: Julien Pacaud)

WE HAVE become accustomed to the universe blowing our minds – perhaps too accustomed. Quantum weirdness – things like particles being in two places at once, or appearing to share a telepathic link – has been baffling us for more than a century now. The physicist Richard Feynman once said that nobody really understands the quantum world. Or as others have put it: if you think you understand it, then you definitely don't. So it is tempting to throw up our hands and say human brains can never grasp it.

But maybe we shouldn't be so defeatist. Isn't it just possible that we simply haven't yet got to the bottom of how quantum mechanics really works? That's what Giacomo Mauro D'Ariano of the University of Pavia in Italy, and his colleagues Giulio Chiribella and Paolo Perinotti think – and they have been doing something ...

Operational Probabilistic Theory

The framework

Logic $\subset$ Probability $\subset$ OPT

joint probabilities + connectivity

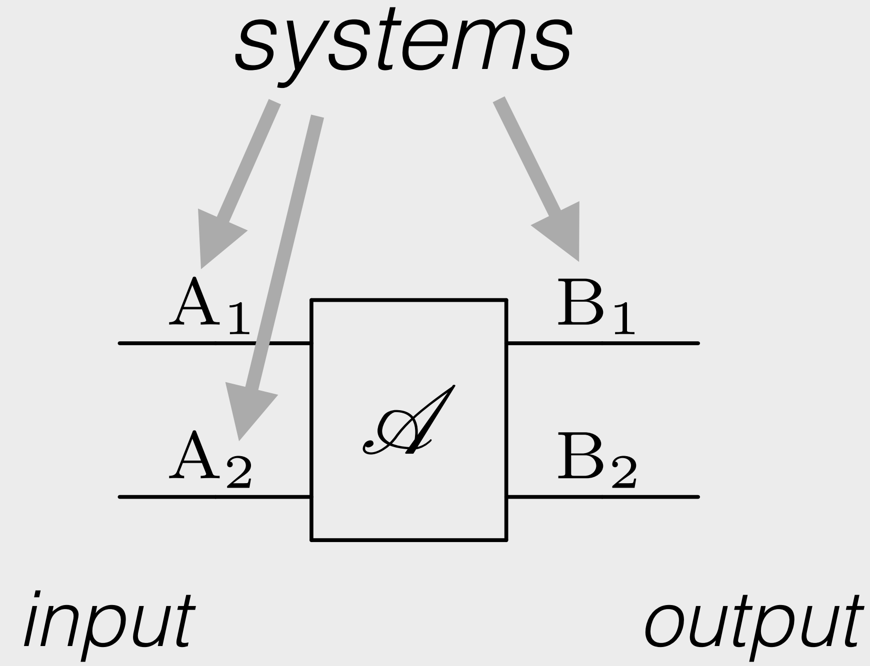
$$p(i,j,k,\ldots|\text{circuit})$$

Marginal probability

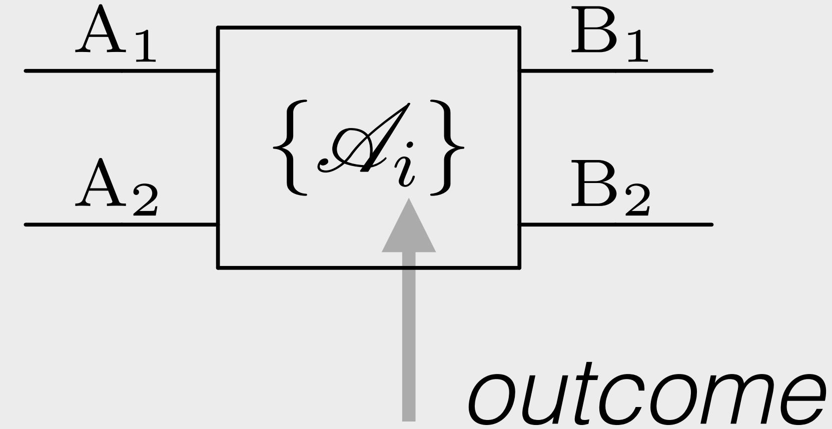
$$\sum_{i,k,\ldots} p(i,j,k,\ldots|\text{circuit}) =$$

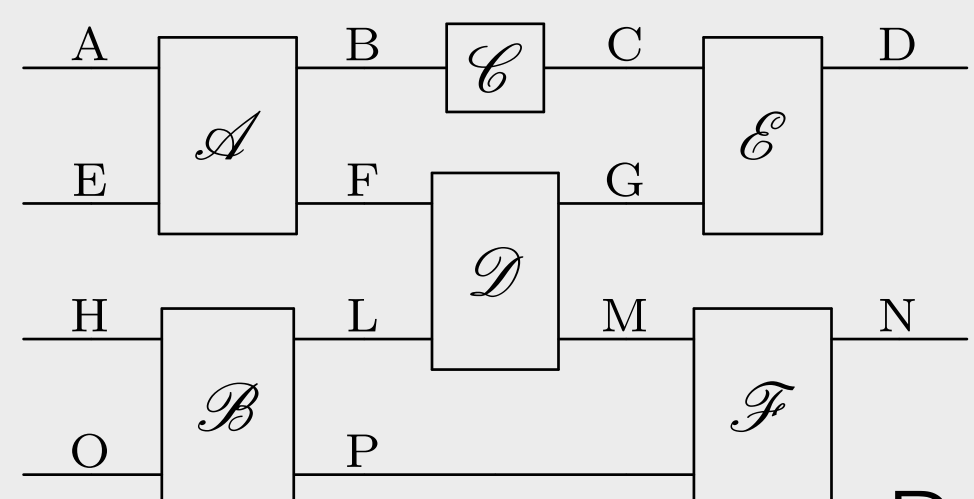
$$p(j|\text{circuit})$$

Event



Test





DAG

Operational Probabilistic Theory

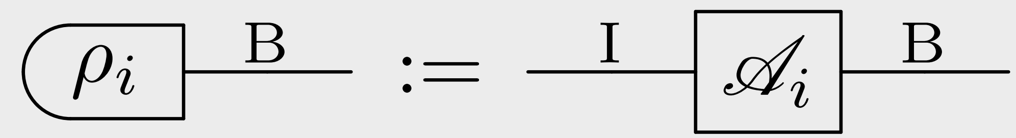
The framework

Logic ⊂ Probability ⊂ OPT

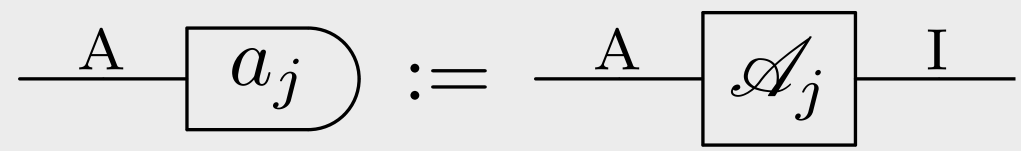
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Notice: the probability of a "preparation" generally depends on the circuit at its output.

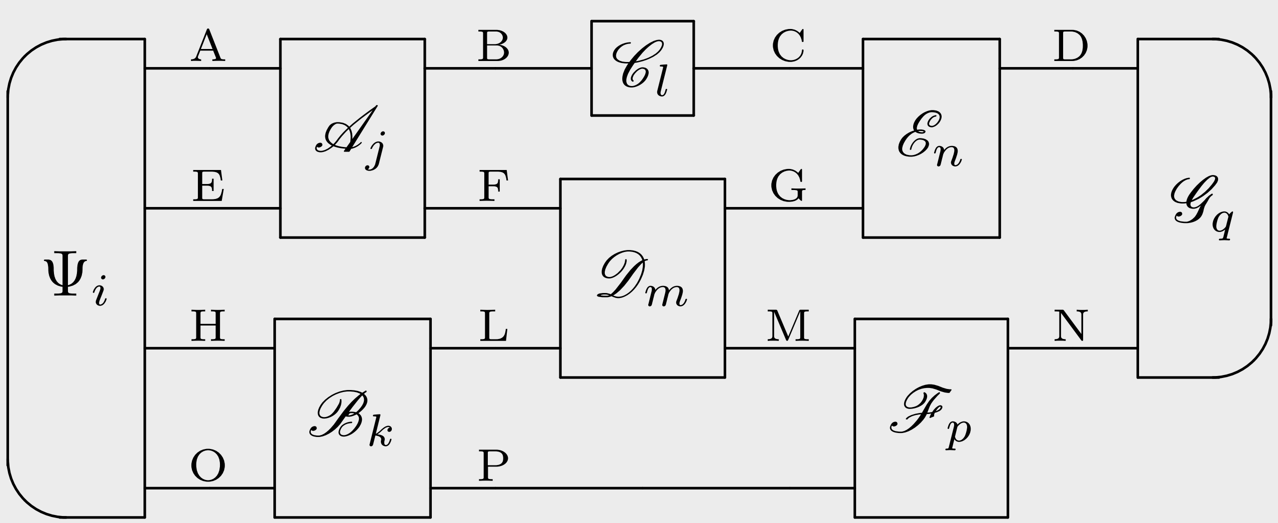


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

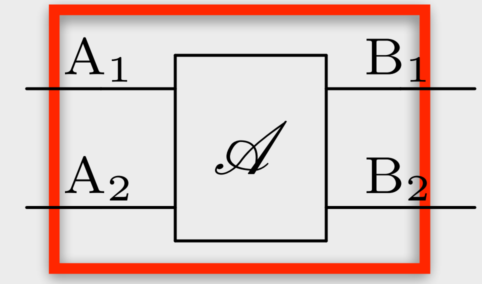
The framework

Logic ⊂ Probability ⊂ OPT

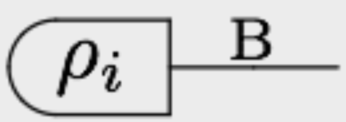
joint probabilities + connectivity

Probabilistic equivalence classes

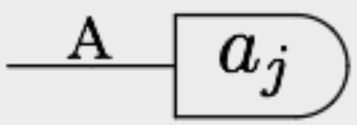
Notice: the probability of a transformation generally depends on the circuit at its output!!



transformation

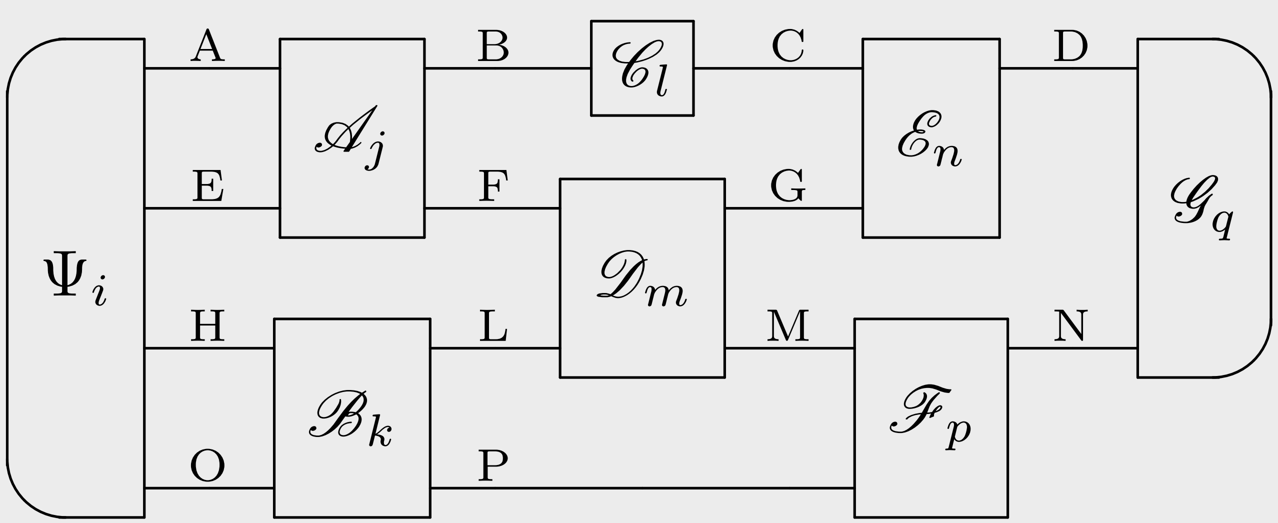


state



effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

The framework

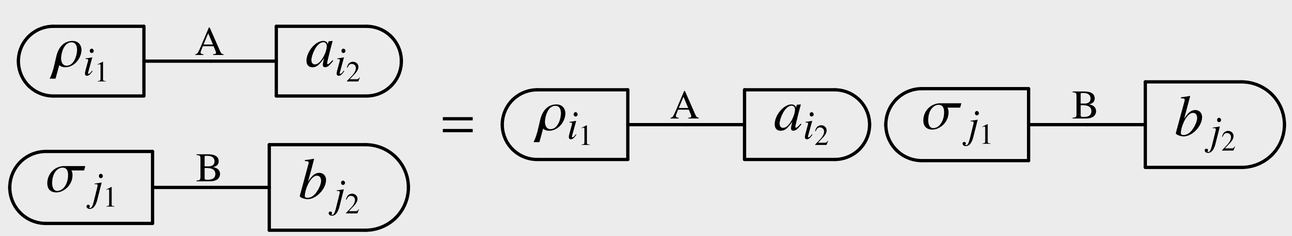
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

Probabilistic equivalence classes

monoidal category theory

Multiplication of closed circuits



$$= p(i_1, i_2)\, q(j_1, j_2)$$

Operational Probabilistic Theory

Sequential composition (associative)

$$\overset{\mathrm{A}}{\text{---}}\boxed{\{\mathscr{A}_x\}_{x\in\mathrm{X}}}\overset{\mathrm{B}}{\text{---}}\boxed{\{\mathscr{B}_y\}_{y\in\mathrm{Y}}}\overset{\mathrm{C}}{\text{---}} \quad =: \quad \overset{\mathrm{A}}{\text{---}}\boxed{\{\mathscr{B}_x\circ\mathscr{A}_y\}_{(x,y)\in\mathrm{X}\times\mathrm{Y}}}\overset{\mathrm{C}}{\text{---}}$$

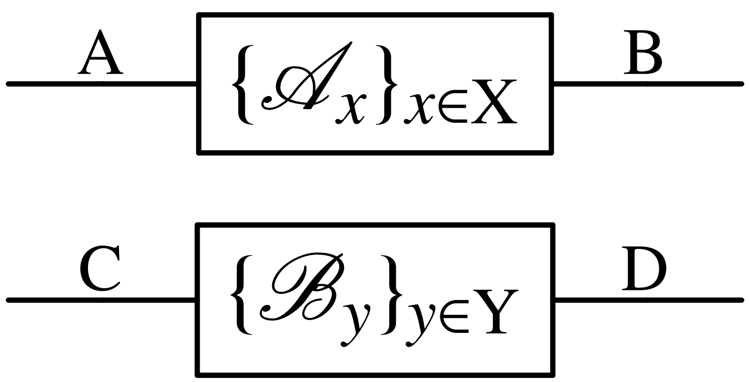
Identity test

$$\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{I}_{\mathrm{A}}}\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{---}} \quad = \quad \overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{---}}$$

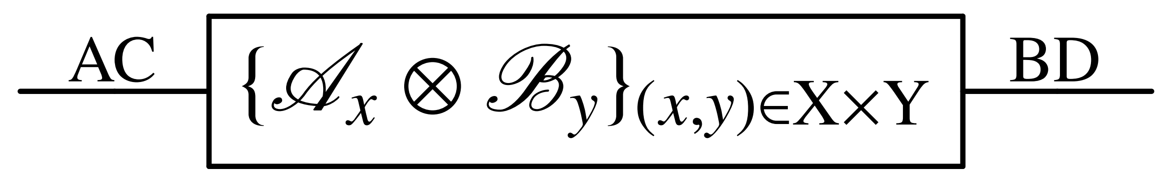
$$\overset{\mathrm{B}}{\text{---}}\boxed{\mathscr{D}}\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{I}_{\mathrm{A}}}\overset{\mathrm{A}}{\text{---}} \quad = \quad \overset{\mathrm{B}}{\text{---}}\boxed{\mathscr{D}}\overset{\mathrm{A}}{\text{---}}$$

Operational Probabilistic Theory

Parallel composition (associative)



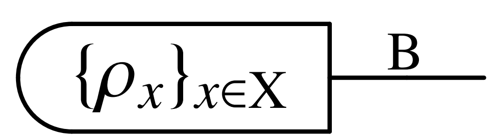
$=:$



$$\mathrm{AB} = \mathrm{BA}$$

$$\mathrm{AI} = \mathrm{IA} = \mathrm{A}$$

$$\mathrm{A(BC)} = \mathrm{(AB)C}$$



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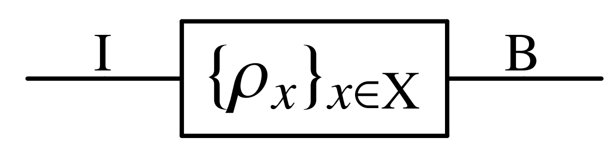

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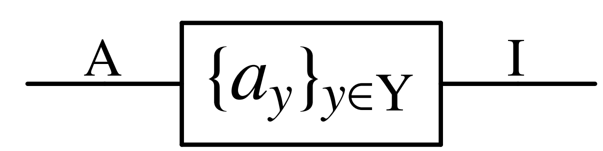
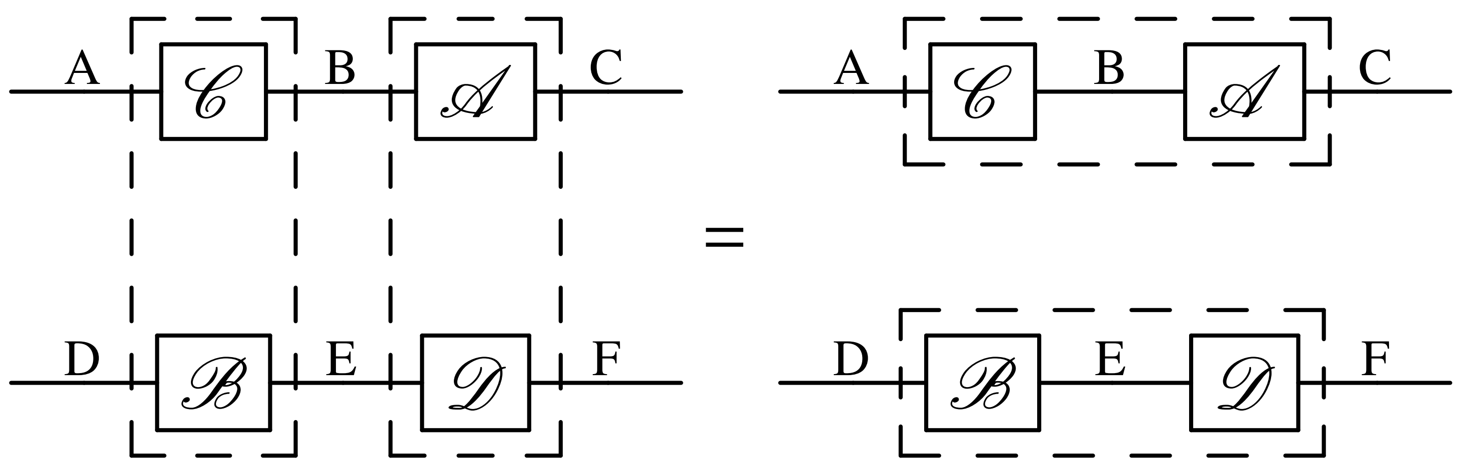
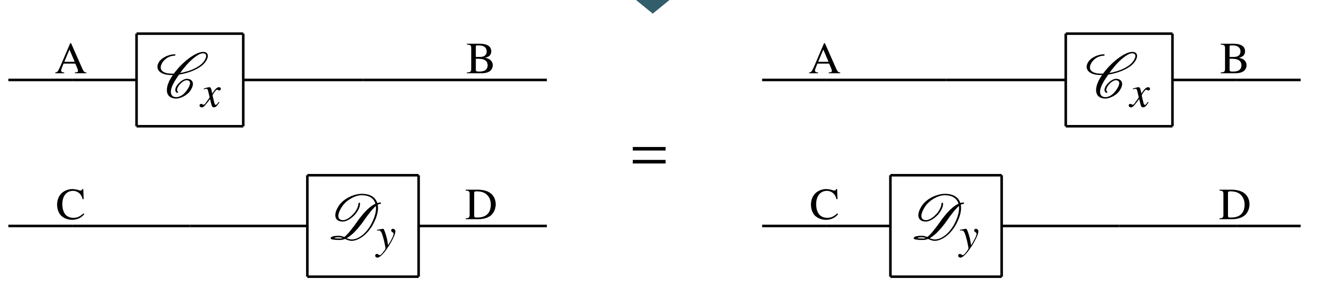


Operational Probabilistic Theory

Sequential and parallel compositions commute



$$(\mathscr{A} \otimes \mathscr{D}) \circ (\mathscr{C} \otimes \mathscr{B}) = (\mathscr{A} \circ \mathscr{C}) \otimes (\mathscr{D} \circ \mathscr{B})$$



wire-stretching

(foliations)

Operational Probabilistic Theory

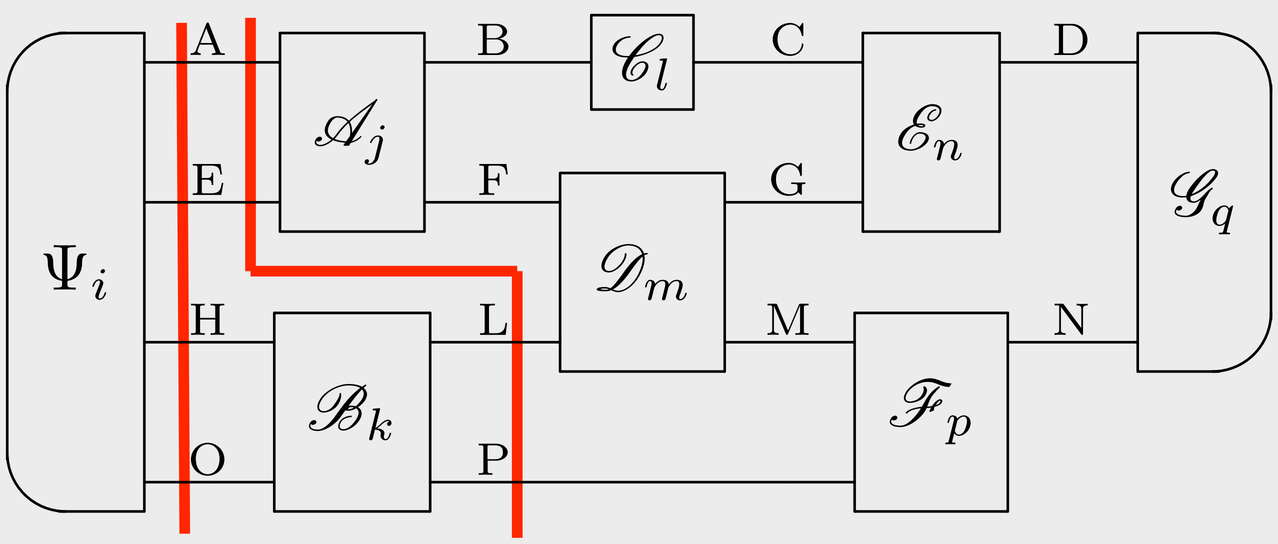
The framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Maximal set of
independent systems
= *"leaf"*

Operational Probabilistic Theory

The framework

Logic ⊂ Probability ⊂ OPT

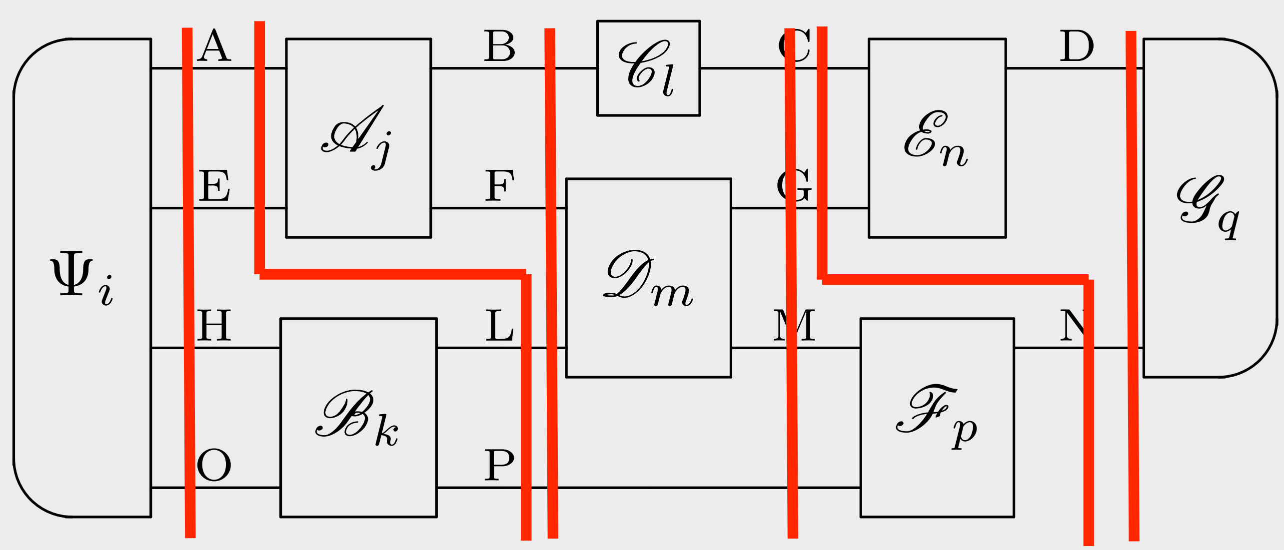
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Maximal set of independent systems = *"leaf"*

Foliation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

States are functionals for effects

States are separating for effects

Effects are functionals on states

Effects are separating for states

Embedding in real vector spaces

$\mathsf{St}(\mathrm{A}),\ \mathsf{St}_1(\mathrm{A}),\ \mathsf{St}_{\mathbb{R}}(\mathrm{A})$

$\mathsf{Eff}(\mathrm{A}),\ \mathsf{Eff}_1(\mathrm{A}),\ \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

Dimension D_{A}

$\mathsf{Eff}_{\mathbb{R}}(\mathrm{A}) = \mathsf{St}_{\mathbb{R}}(\mathrm{A})^{\vee}$

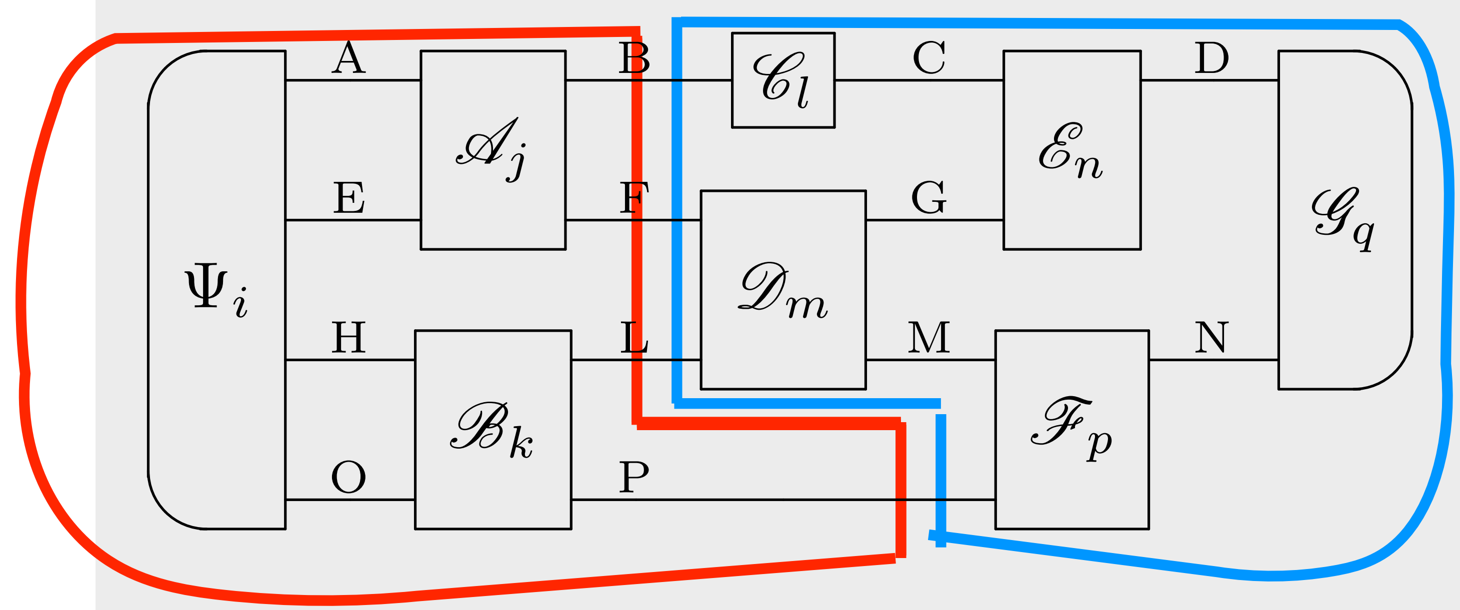
$\mathsf{St}_{\mathbb{R}}(\mathrm{A}) = \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})^{\vee}$

Paring notation:

$\rho \in \mathsf{St}(\mathrm{A}), a \in \mathsf{Eff}(\mathrm{A}),\quad (\rho| \xrightarrow{\mathrm{A}} |a) = (a|\rho)$

$(\Psi_i, \mathscr{A}_j, \mathscr{B}_k| \xrightarrow{\mathrm{BFLP}} |\mathscr{D}_m, \mathscr{F}_p, \mathscr{C}_l, \mathscr{E}_n, \mathscr{G}_q)$

$p(i, j, k, l, m, n, p, q|\text{circuit})$



Operational Probabilistic Theory

$$\{\mathscr{T}_i\}_{i\in\{\underbrace{i_1,\,i_2,\,\dots,\,i_n}_{j_1},\underbrace{i_{n+1},\,i_{n+2},\,\dots}_{j_2},\underbrace{\;\dots\;}_{\dots}\}}$$

Coarse-graining $\downarrow$ $\uparrow$ Refinement

$$\{\hat{\mathscr{T}}_j\}_{j\in\{j_1,j_2,\dots\}}$$

$$\hat{\mathscr{T}}_S = \sum_{i\in S} \mathscr{T}_i$$

Partial ordering

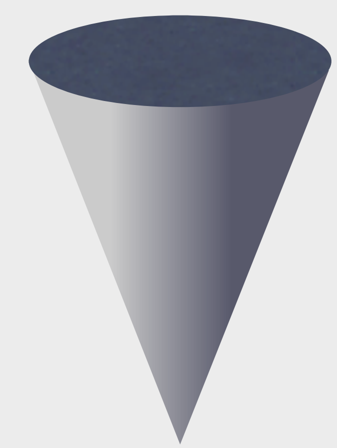
Conditioned test (needs causality)

$$\text{A}\ \boxed{\mathscr{C}_i}\ \text{B}\ \boxed{\mathscr{D}^{(i)}_{j_i}}\ \text{C} \;:=\; \text{A}\ \boxed{\mathscr{D}^{(i)}_{j_i}\circ\mathscr{C}_i}\ \text{C}$$

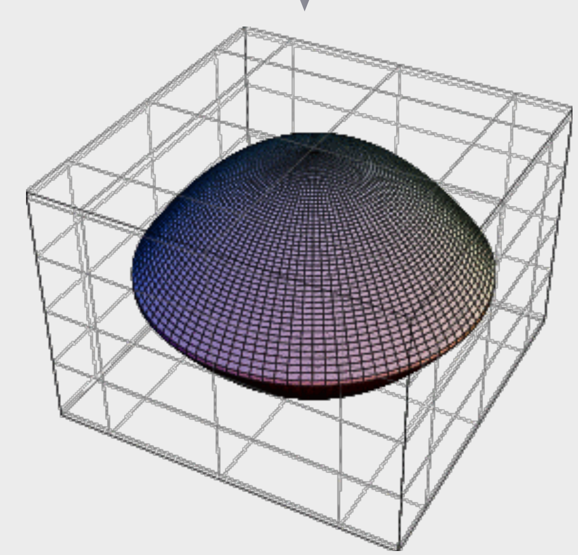
Circuit multiplication: randomize tests

$$p_i\ \text{A}\ \boxed{\mathscr{C}^{(i)}_{j_i}}\ \text{B} \;:=\; \begin{array}{l} \text{A}\ \boxed{\mathscr{C}^{(i)}_{j_i}}\ \text{B} \\ \text{I}\ \boxed{p_i}\ \text{I} \end{array} .$$

Cone structure



Convex structure



Operational
Probabilistic Theory

State tomography

$\{l_i\}_{i\in\mathrm{X}} \subseteq \mathsf{Eff}(\mathrm{A})$ separating for states $\Rightarrow$ span $\mathsf{Eff}(\mathrm{A})$

$$\forall a \in \mathsf{Eff}(\mathrm{A}),\ a = \sum_{i\in\mathrm{X}} c_i(a) l_i \qquad c_i \in \mathsf{St}_{\mathbb{R}}(\mathrm{A}).$$

$\{c_i\}_{i\in\mathrm{X}}$ is a *dual set* for $\{l_i\}_{i\in\mathrm{X}}$

$\rho \in \mathsf{St}_1(\mathrm{A})$ deterministic

$$\forall a \in \mathsf{Eff}_{\mathbb{R}}(\mathrm{A}),\ (a|\rho) = \sum_{i\in\mathrm{X}} c_i(a)(l_i|\rho)$$

state-tomography

$\{l_i\}_{i\in\mathrm{X}}$ *informationally complete* for states

Principles for Quantum Theory

$\{\rho_0, \rho_1\} \subseteq \mathsf{St}(\mathrm{A})$ preparation test

$\{a_0, a_1\}$ observation test

success probability of discrimination

$$p_{\mathrm{succ}} = (a_0|\rho_0) + (a_1|\rho_1)$$
$$= (a|\rho_0) + (a_1|\rho_1 - \rho_0)$$
$$= (a|\rho_1) + (a_0|\rho_0 - \rho_1)$$
$$= \frac{1}{2}[1 + (a_1 - a_0|\rho_1 - \rho_0)]$$

$$a := a_0 + a_1$$

Metric

$$p^{(\mathrm{opt})}_{\mathrm{succ}} = \frac{1}{2}[1 + ||\rho_1 - \rho_0||]$$

$$||\delta|| := \sup_{\{a_0, a_1\}} (a_0 - a_1|\delta)$$

$$||\delta|| = \sup_{a_0 \in \mathsf{Eff}(\mathrm{A})} (a_0|\delta) - \inf_{a_1 \in \mathsf{Eff}(\mathrm{A})} (a_1|\delta)$$

monotonicity

$$\mathscr{C} \in \mathsf{Transf}_1(\mathrm{A}, \mathrm{B})$$

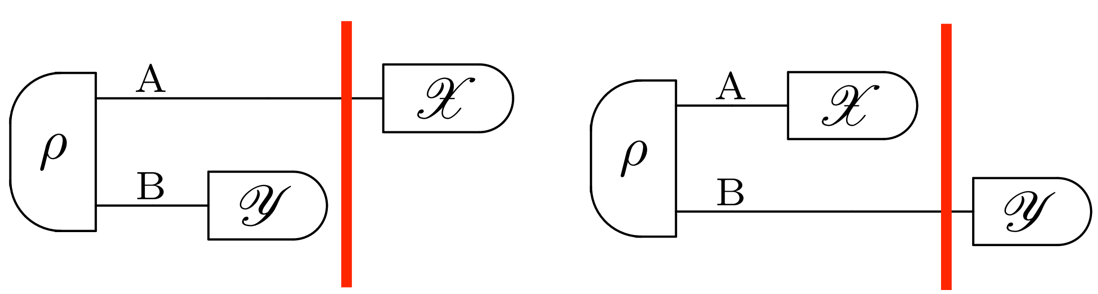
$$||\mathscr{C}\delta||_{\mathrm{B}} \leq ||\delta||_{\mathrm{A}}$$

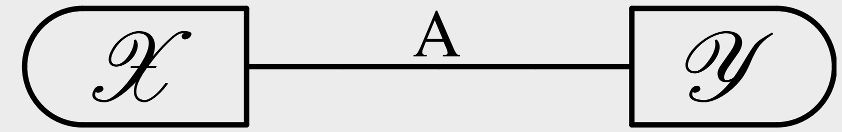
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

no signaling without interaction

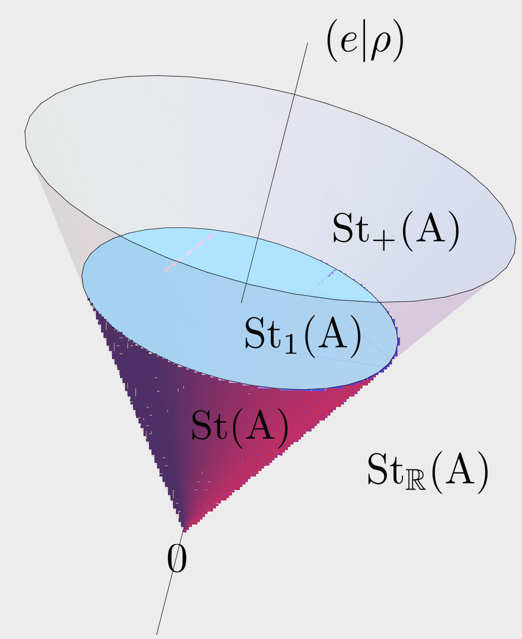


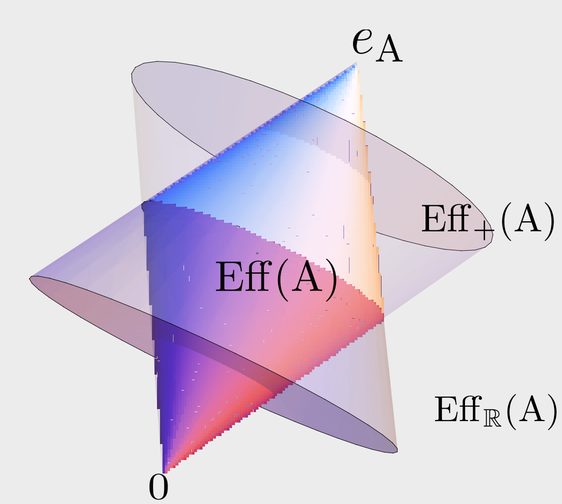


$$p(i, j|\mathscr{X}, \mathscr{Y}) := (a_j|\rho_i)$$

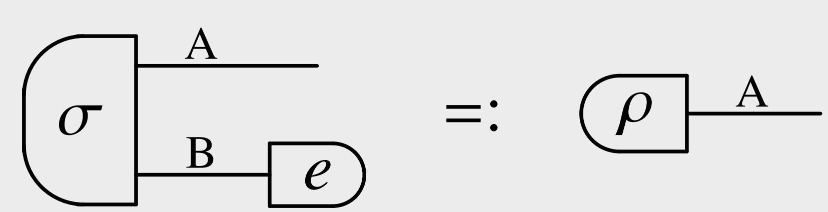
$$p(i|\mathscr{X}, \mathscr{Y}) = p(i|\mathscr{X}, \mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique; b) states are “normalizable”





marginal state



Principles for Quantum Theory

P1. Causality
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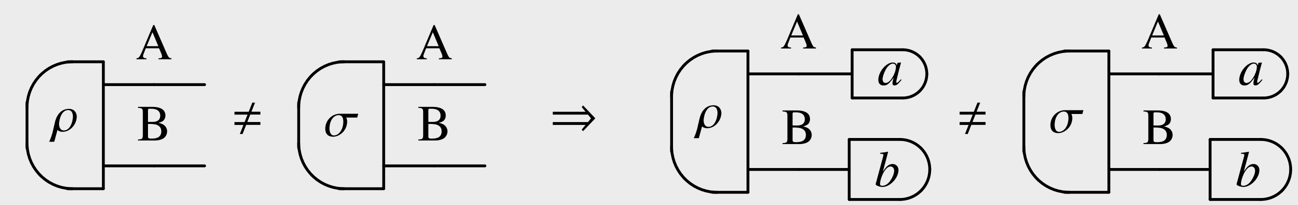
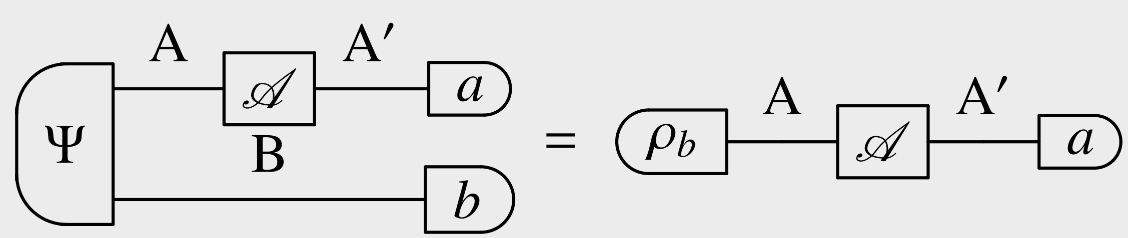
It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product

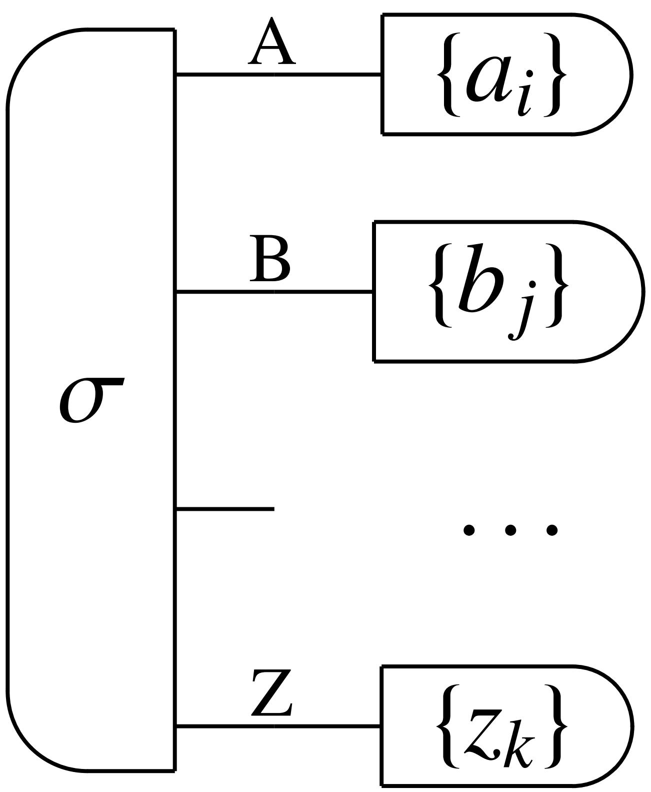


Local characterization of transformations

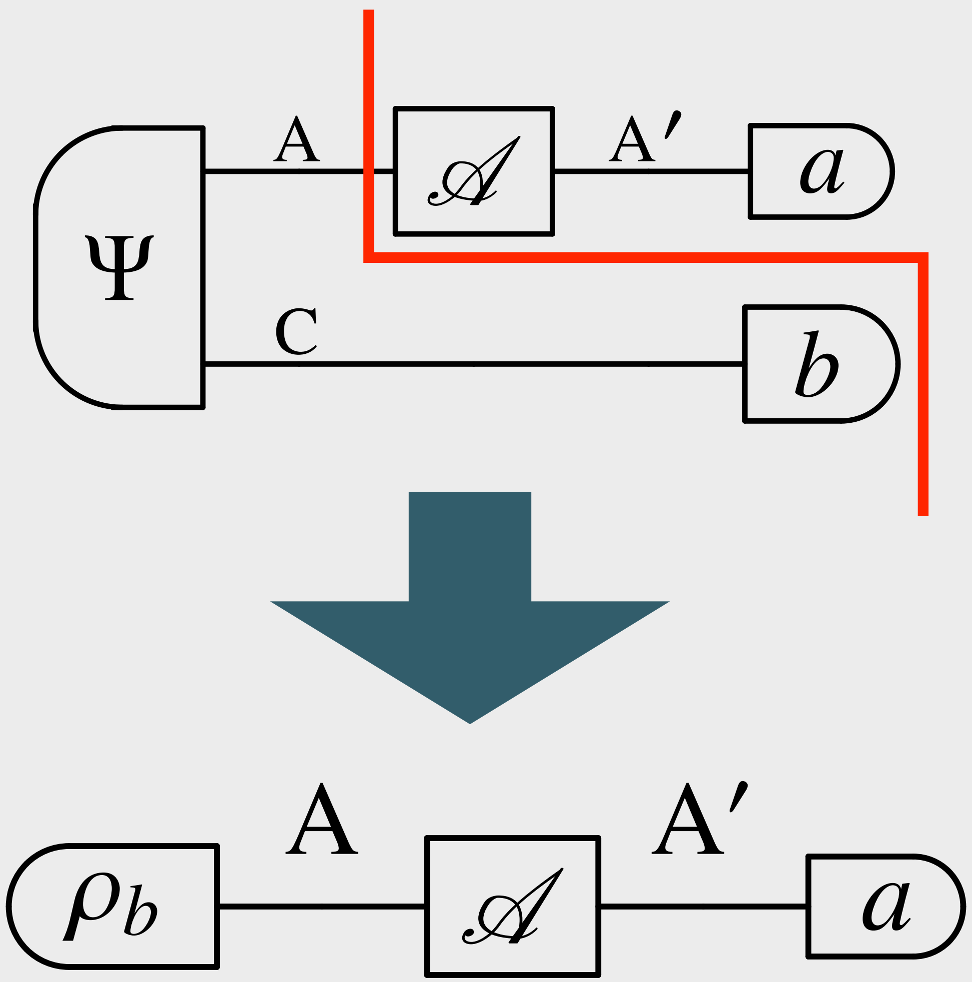


Principles for Quantum Theory

Local effects are separating for joint states



Tomography



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

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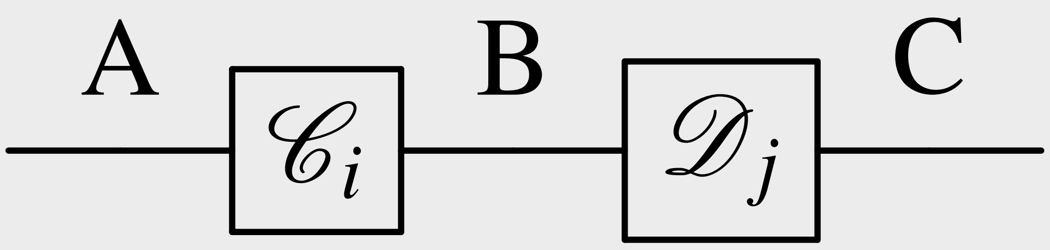
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis



Principles for Quantum Theory

P1. Causality
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P4. Atomicity of composition
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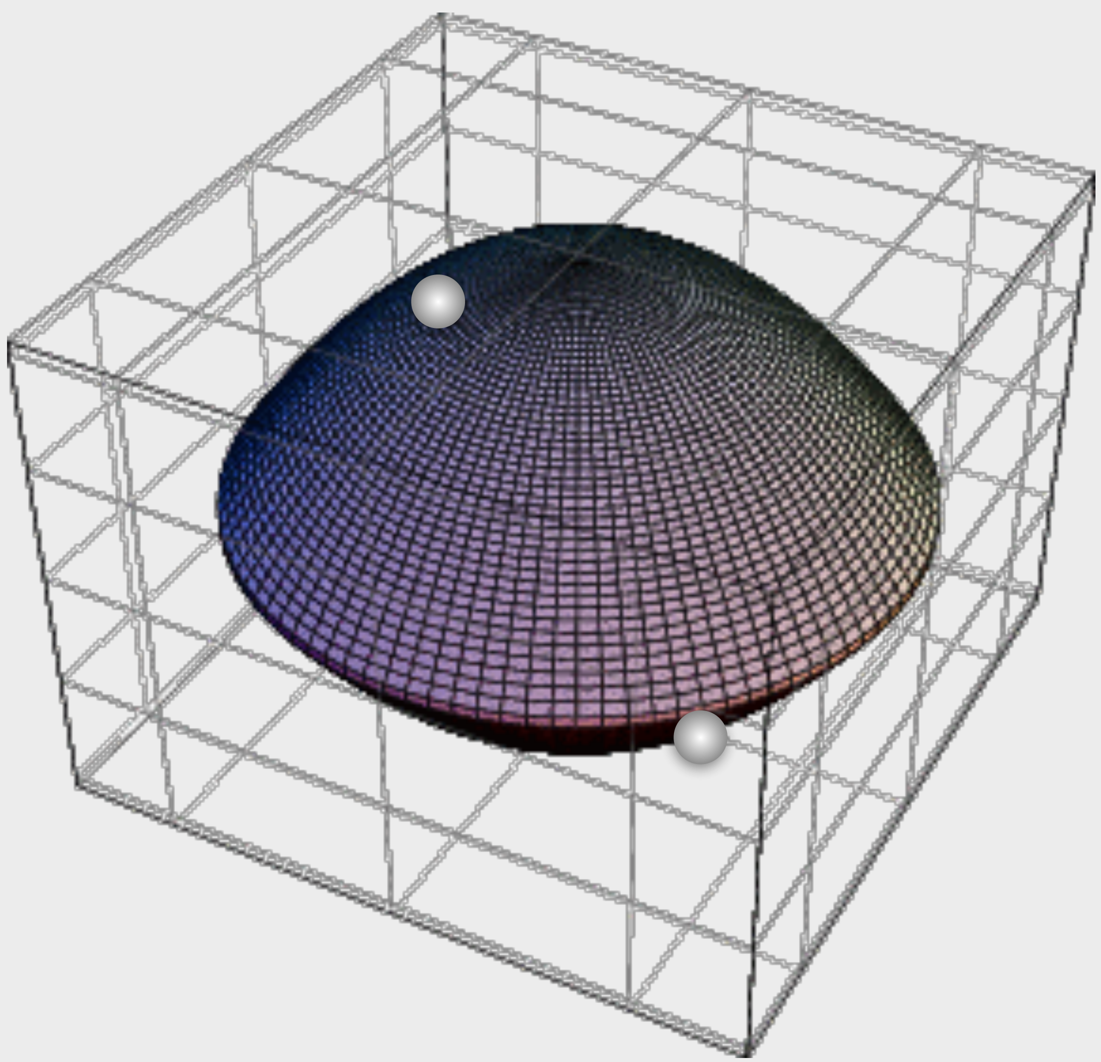
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory

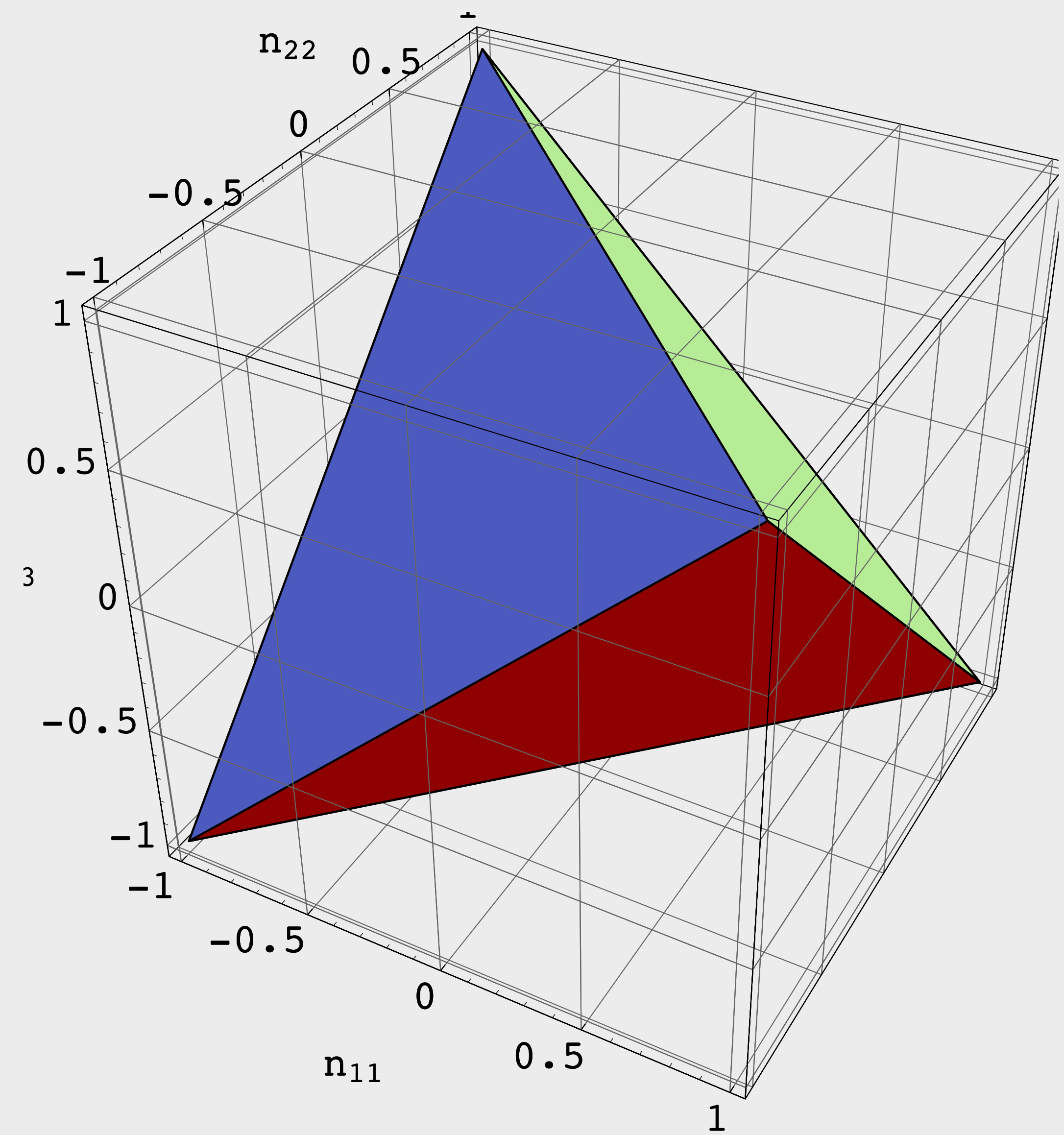


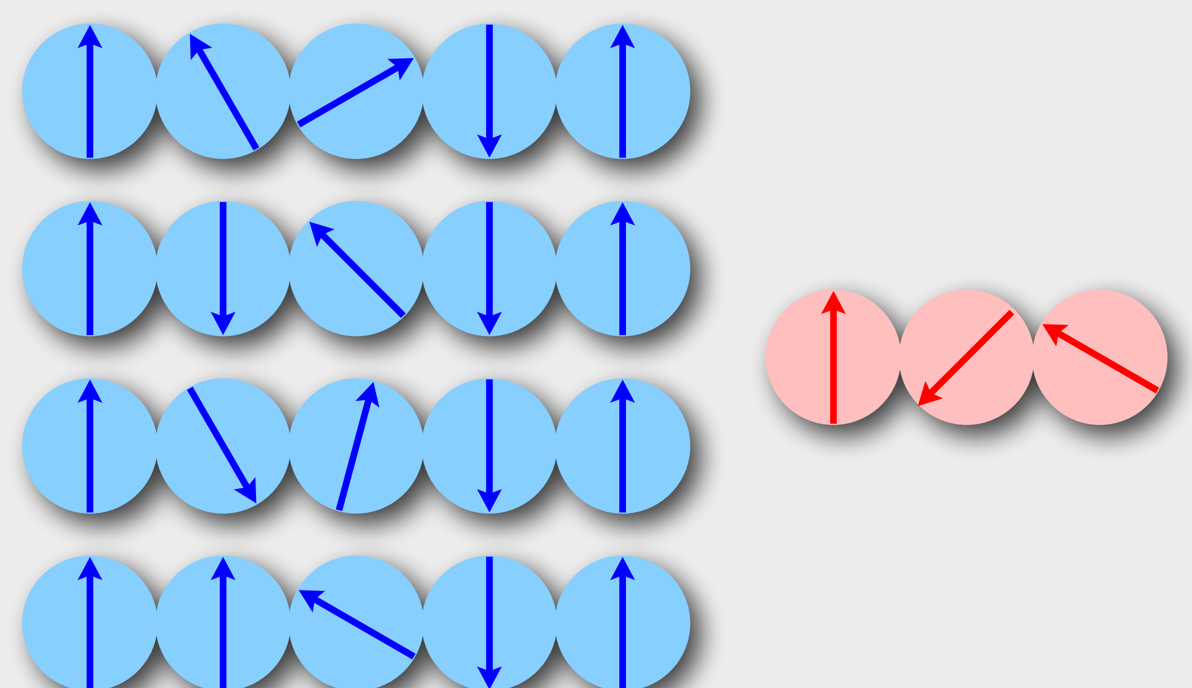
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. $\boxed{\text{Lossless Compressibility}}$

For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system





Principles for Quantum Theory

P1. Causality

P2. Local discriminability

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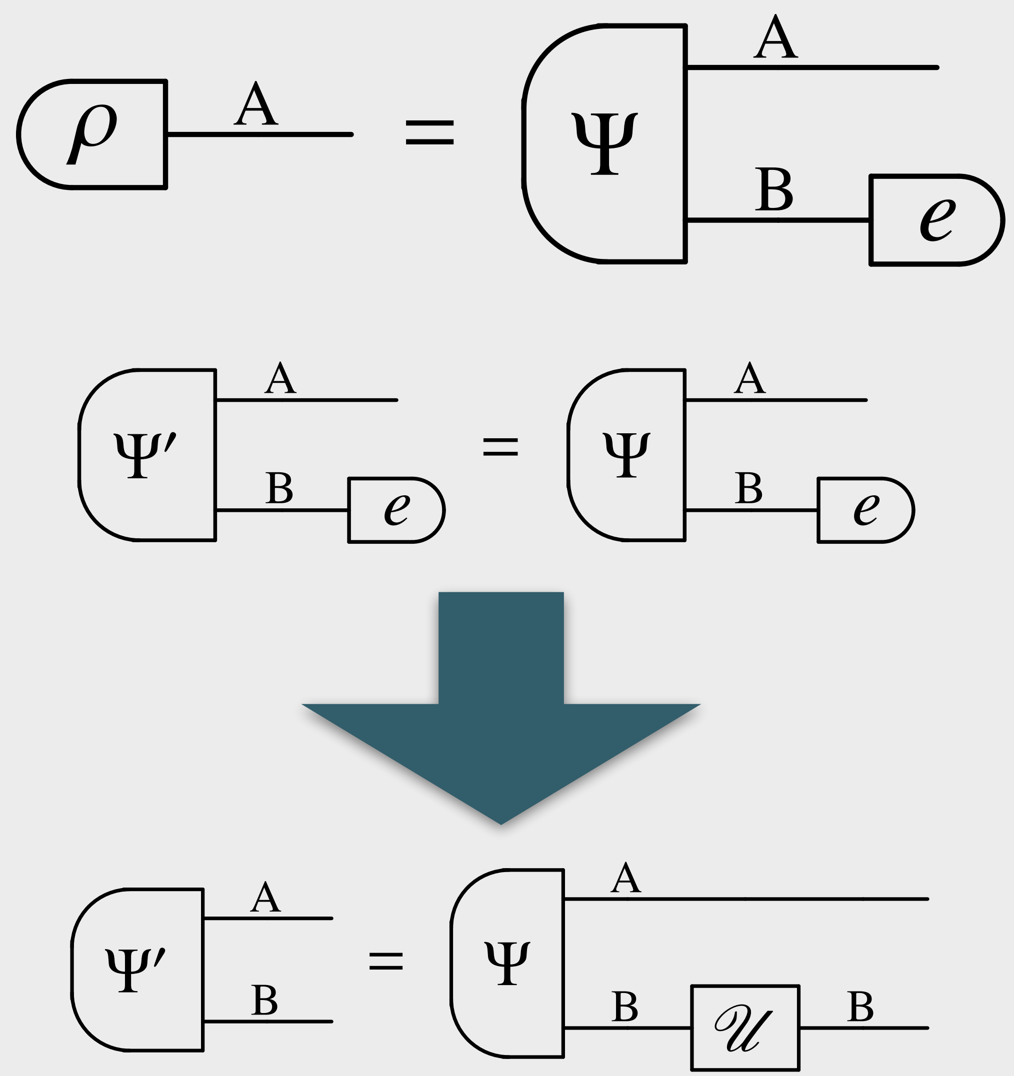
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system



Principles for Quantum Theory

P1. Causality

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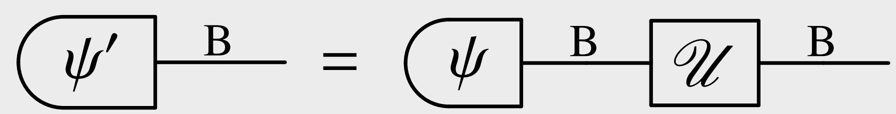
Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

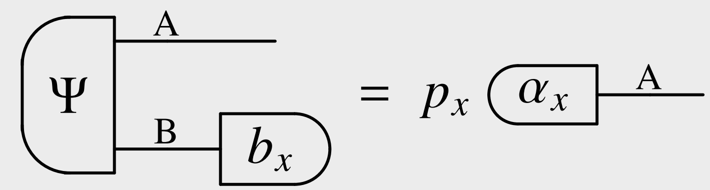
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

2. *Every two normalized pure states of the same system are connected by a reversible transformation*

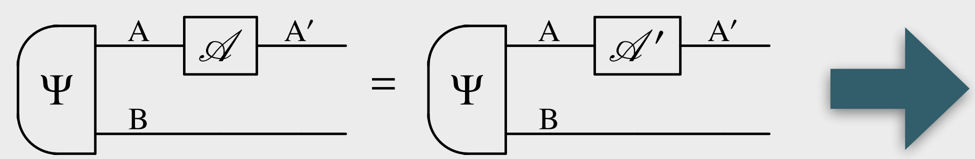


3. ***Steering:*** Let **Ψ** purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



$\forall x \in X$

4. ***Process tomography (faithful state):***



$\mathscr{A}\rho = \mathscr{A}'\rho$
$\forall \rho$

5. ***No information without disturbance***

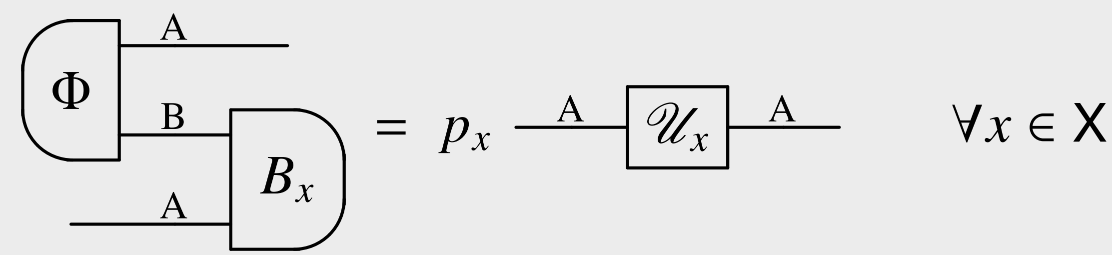
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

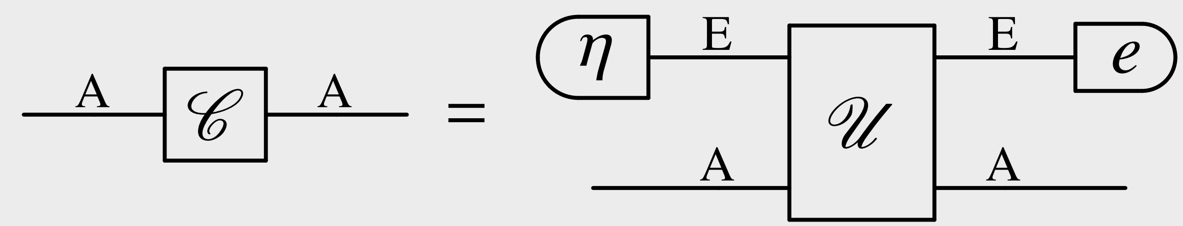
Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

Consequences

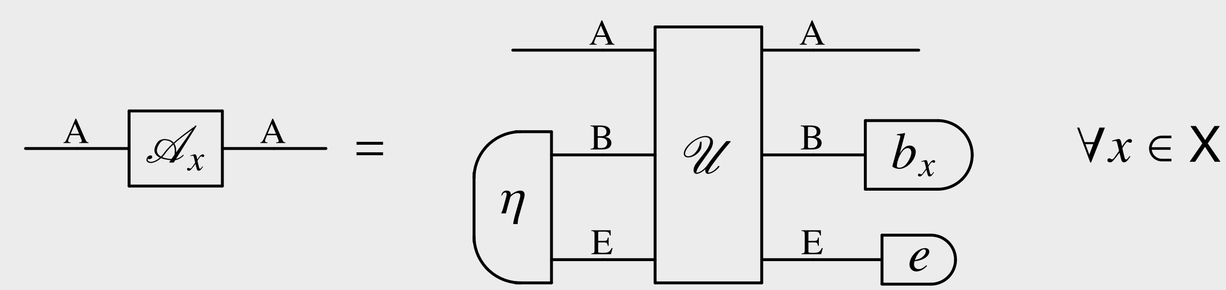
6. ***Teleportation***



7. ***Reversible dilation of "channels"***



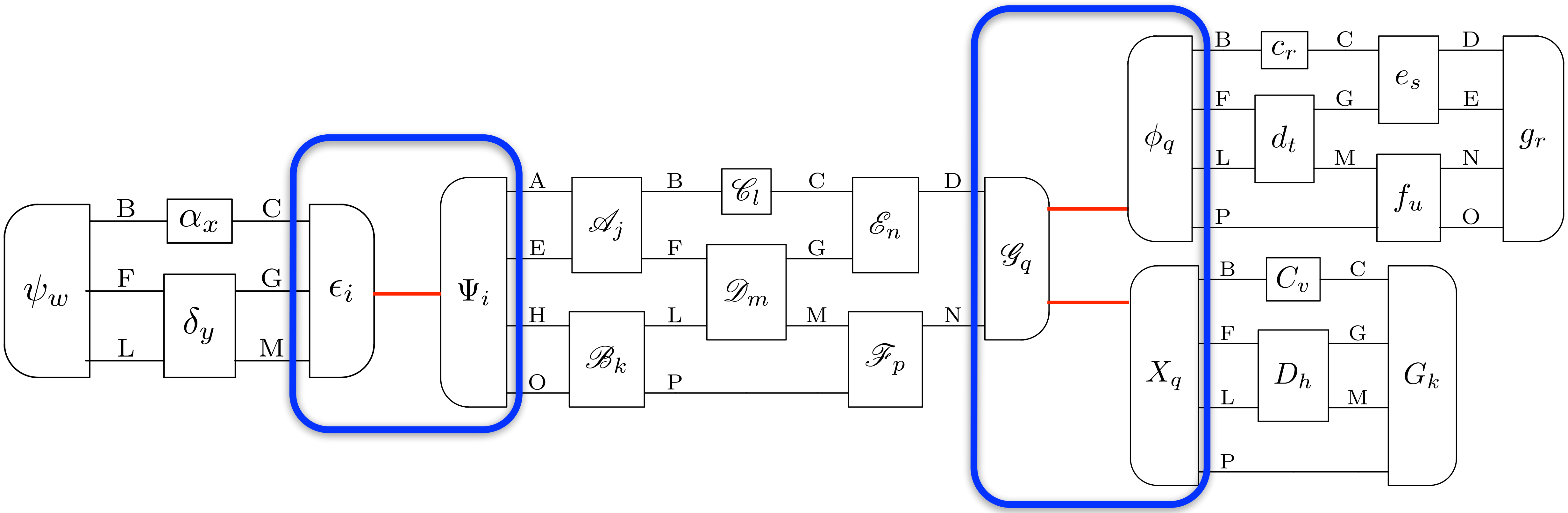
8. ***Reversible dilation of "instruments"***



9. ***State-transformation cone isomorphism***

10. ***Rev. transform. for a system make a compact Lie group***

On the von Neumann postulate



Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
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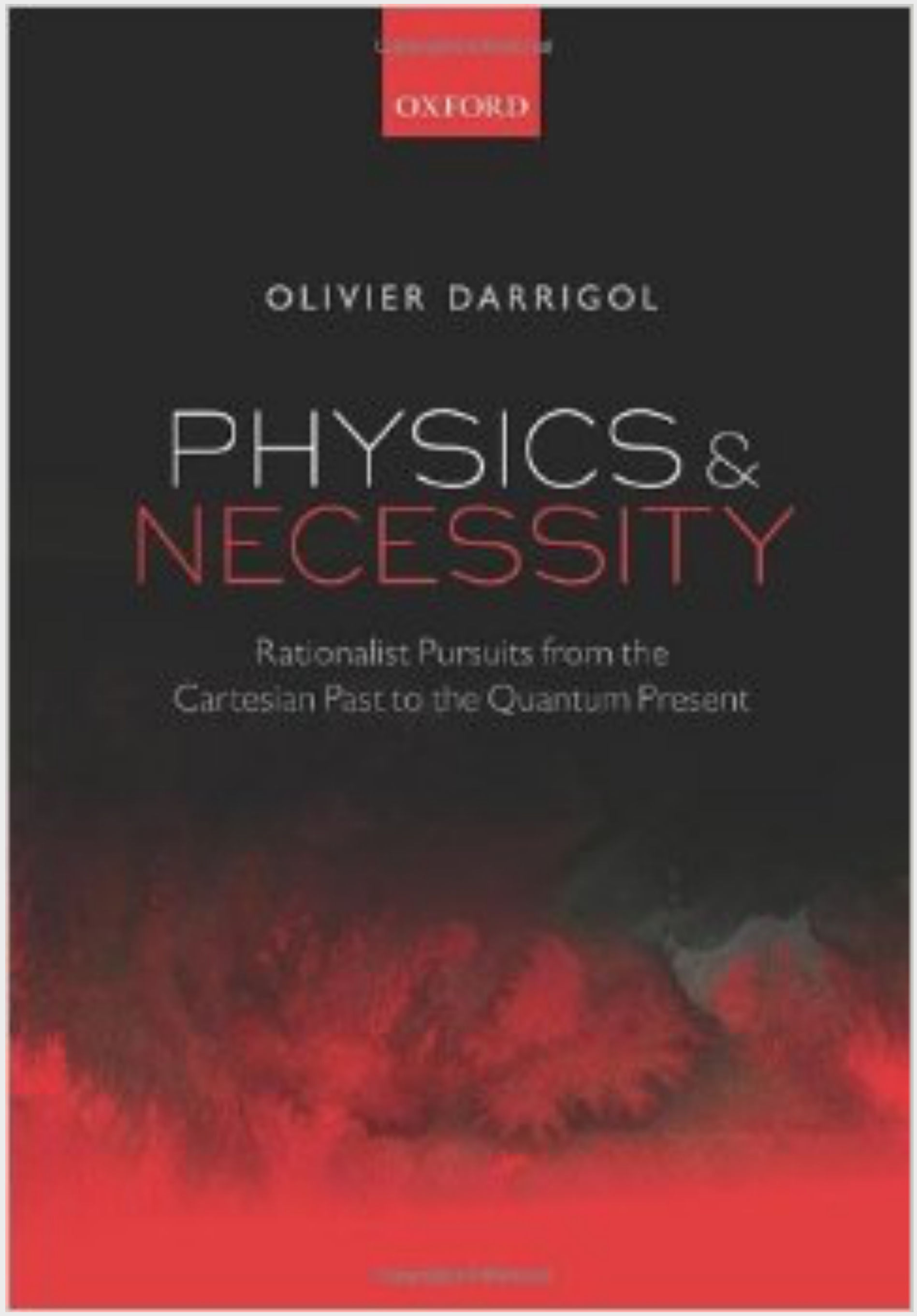
Epistemological principles

Are they *necessary*?

Fermionic quantum theory?



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Thank you!