Free quantum field theory from general principles

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Reality and Measurement in Algebraic Quantum Theory

QUit quantum information theory group

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D'Ariano and Perinotti, *Derivation of the Dirac Equation from Principles of Information processing*, Phys. Rev. A **90** 062106 (2014)
Bisio, D'Ariano,Tosini, *Quantum Field as a Quantum Cellular Automaton: the Dirac free evolution in 1d*, Annals of Physics **354** 244 (2015)
D'Ariano, Mosco, Perinotti, Tosini, *Path-integral solution of the one-dimensional Dirac quantum cellular automaton,* PLA **378** 3165 (2014)
D'Ariano, Mosco, Perinotti, Tosini, *Discrete Feynman propagator for the Weyl quantum walk in 2 + 1 dimensions,* EPL **109** 40012 (2015)
D'Ariano, Manessi, Perinotti, Tosini, *The Feynman problem and Fermionic entanglement …,* Int. J. Mod. Phys. **A17** 1430025 (2014)
Bibeau-Delisle, Bisio, D'Ariano, Perinotti, Tosini, *Doubly-Special Relativity from Quantum Cellular Automata,* EPL (in press)
Bisio, D'Ariano, Perinotti, *Quantum Cellular Automaton Theory of Light,* arXiv:1407.6928
Bisio, D'Ariano, Perinotti, *Lorentz symmetry for 3d Quantum Cellular Automata,* arXiv:1503.01017
D'Ariano, *A Quantum Digital Universe,* Il Nuovo Saggiatore **28** 13 (2012)
D'Ariano, *The Quantum Field as a Quantum Computer,* Phys. Lett. A **376** 697 (2012)
D'Ariano, *Physics as Information Processing*, AIP CP1327 7 (2011)
D'Ariano, *On the "principle of the quantumness", the quantumness of Relativity, and the computational grand-unification*, in AIP CP1232 (2010)

How far we can go with general principles?

Much farer than what one can imagine!

Index

- How a physical theory should be
- Principles for physics
- Localization issue in QFT and the particle notion
- QCA field theory
- Nonlinear Lorentz and group-theoretical notion of particle

How a physical theory should be

- Axioms must be mathematical

    axioms contain no physical notion, e.g. mass, Lagrangian, …,

        variables are adimensional, …

- Axioms and theorems must have physical interpretation

    physics emergent (e.g. mechanics, …)

- Units of measure must be provided in terms of special values of the adimensional variables

Principles for Physics

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03 67 Ac, 03 65 Ta

$\boxed{\text{Principles for Quantum Theory}}$

P1. Causality

P2. Local discriminability

P3. Purification*

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Book from CUP (summer 2015)

Principles for Physics

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- linearity
- unitarity
- locality
- homogeneity
- isotropy
- minimal-dimension
- qi-embedding in Euclidean space

- Quantum Cellular Automata (QCA) theory

- *Relativistic limit ($k \ll 1$):* free QFT (Weyl, Dirac, and Maxwell)
- *Ultra-relativistic limit ($k \sim 1$)* [Planck scale]: nonlinear Lorentz (Camelia/Smolin *Doubly Special Relativity)*
- QFT derived:
  - without assuming Special Relativity
  - quantum ab-initio (mechanics emergent)
- QCA is a *discrete* theory.

Motivations to keep it discrete:

1. Existence of continuum is metaphysical (only mathematical convenience)
2. Continuum is special case of discrete
3. Testing mechanisms in simulations
4. Falsifiable Planck-scale hypothesis
5. Natural scenario for holographic principle
6. Solves all issues in QFT originating from continuum:

   i) uv divergencies
   ii) localization issue
   iii) Computability and path-integral

Localization issue in QFT

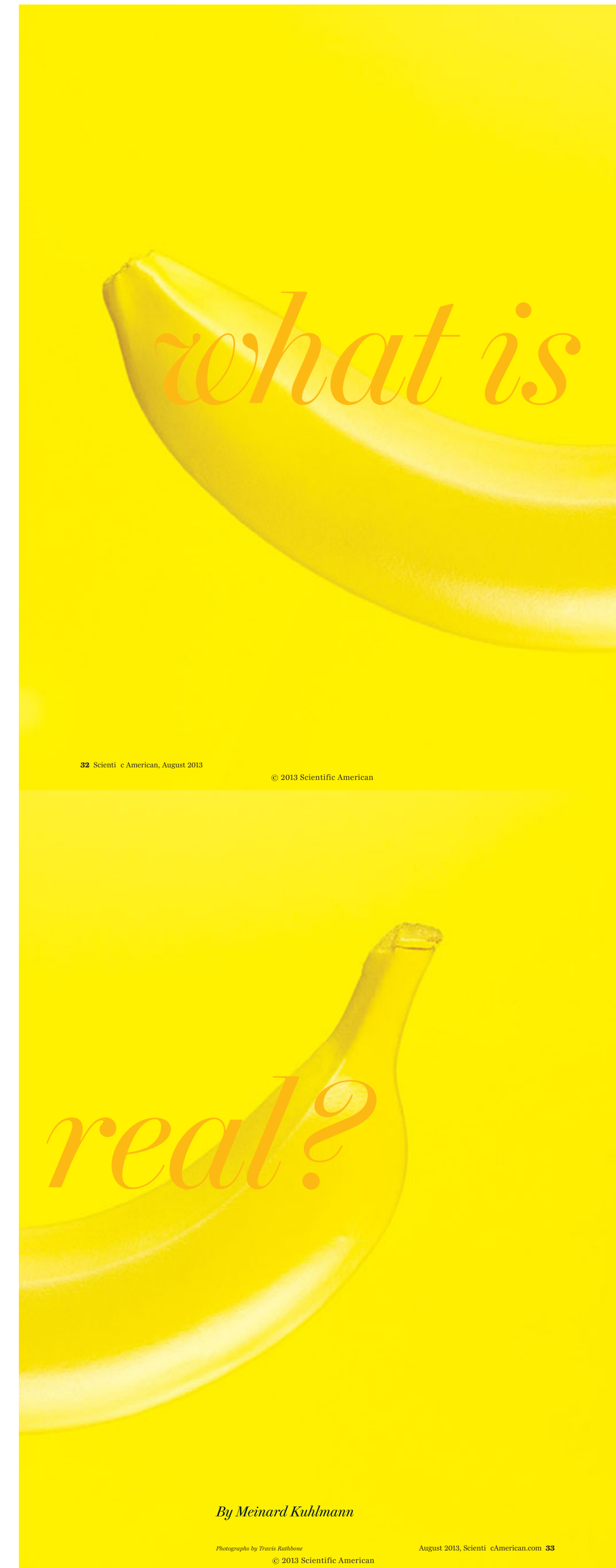
Physicists routinely describe the universe as being made of tiny subatomic particles that push and pull on one another by means of force elds. They call their subject particle physics and their instruments p article accel erators. They hew to a Legoli ke model of the world. But this view sweeps a littleknown fact under the rug: the particle interpretation of quantum physics, as well as the eld interpretation, stretches our conventional notions of p article and eld to such an extent that ever more people think the world might be made of something else entirely.

The problem is not that physicists lack a valid theory of the subatomic realm. They do have one: it is called quantum eld the ory. Theorists developed it between the late 1920s and early 1950s by merging the earlier theory of quantum mechanics with Ein stein s special theory of relativity. Quantum eld theory provides the conceptual underpinnings of the Standard Model of particle physics, which describes the fundamental building blocks of mat ter and their interactions in one common framework. In terms of empirical precision, it is the most successful theory in the history of science. Physicists use it every day to calculate the aftermath of particle collisions, the synthesis of matter in the big bang, the ex treme conditions inside atomic nuclei, and much besides.

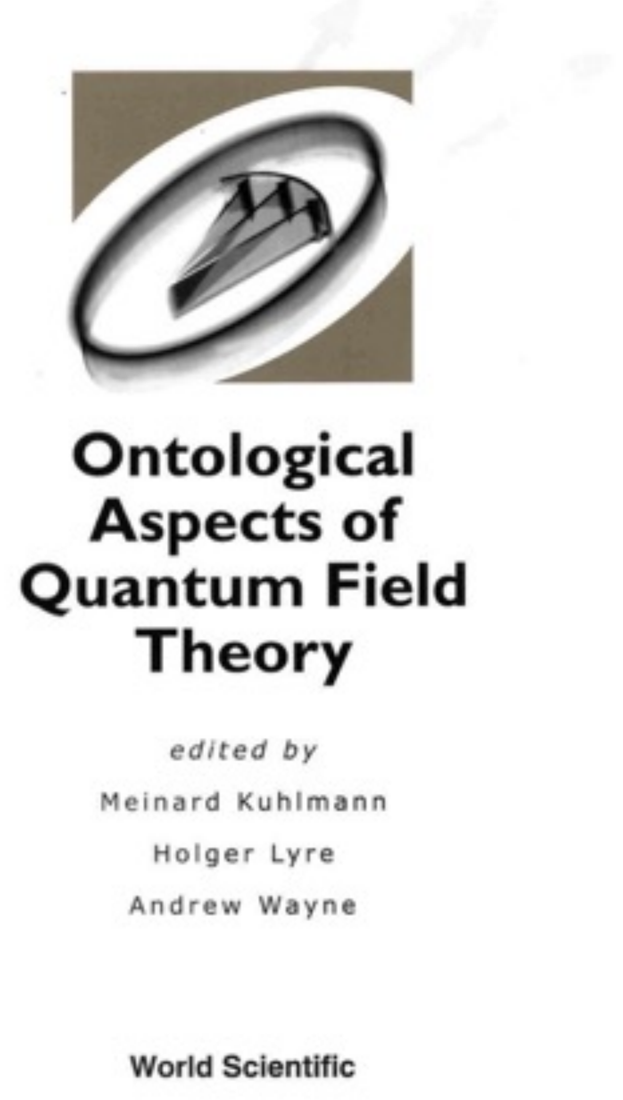
ican articles. However compelling it might appear, it is not at all satisfactory.

For starters, the two categories blur together. Quantum eld theory assigns a eld to each type of elementary particle, so there is an electron eld as surely as there is an electron. At the same time, the force elds are quantized rather than continu ous, which gives rise to particles such as the photon. So the dis tinction between particles and elds appears to be arti cial, and physicists often speak as if one or the other is more fundamen tal. Debate has swirled over this point over whether quantum eld theory is ultimately about particles or about elds. It start ed as a battle of titans, with eminent physicists and philoso phers on both sides. Even today both concepts are still in use for illustrative purposes, although most physicists would admit that the classical conceptions do not match what the theory says. If the mental images conjured up by the words particle and eld do not match what the theory says, physicists and philosophers must gure out what to put in their place.

With the two standard, classical options gridlocked, some phi losophers of physics have been formulating more radical alterna tives. They suggest that the most basic constituents of the materi al world are intangible entities such as relations or properties. One particularly radical idea is that everything can be reduced to intangibles alone, without any reference to individual things. It is



Localization issue in QFT



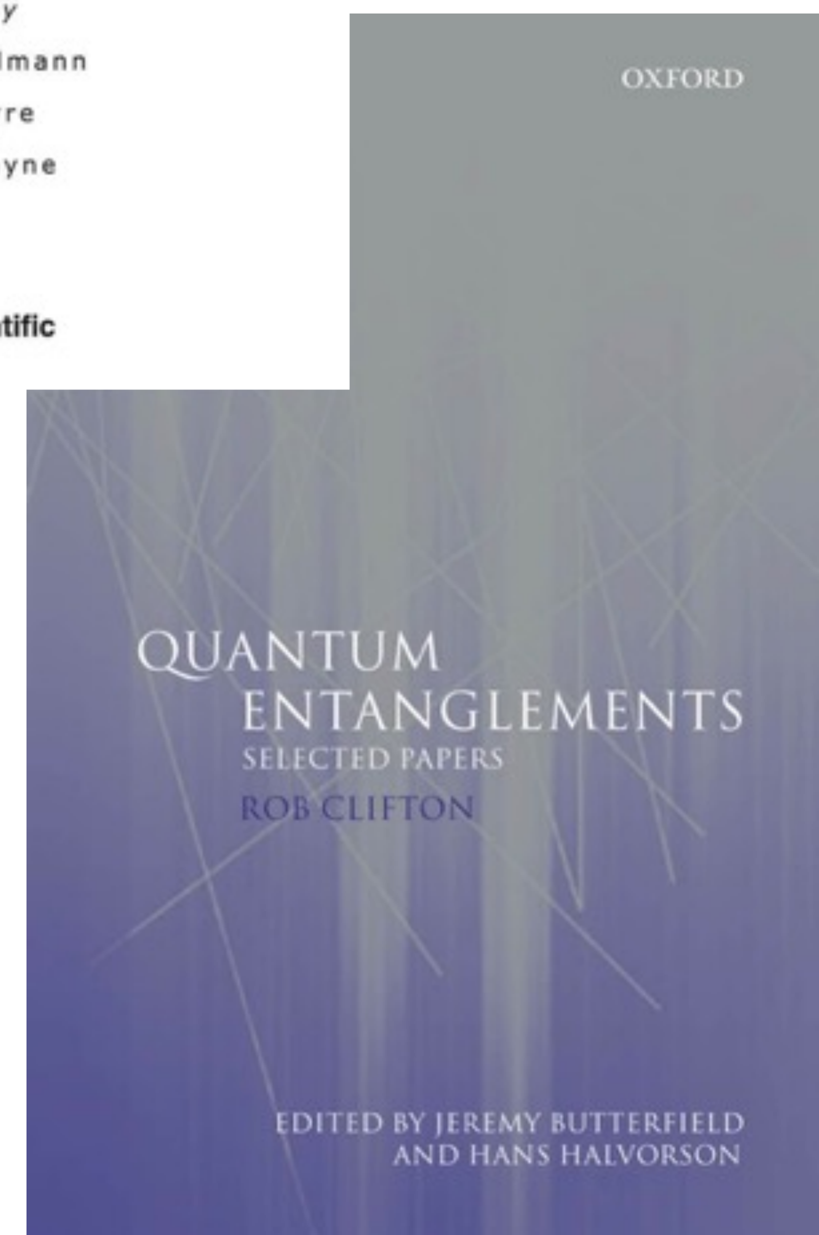
Chapter 10

No Place for Particles in Relativistic Quantum Theories?

Hans Halvorson
Princeton University

Rob Clifton
University of Pittsburgh

Abstract. *David Malament (1996) has recently argued that there can be no relativistic quantum theory of (localizable) particles. We consider and rebut several objections that have been made against the soundness of Malament's argument. We then consider some further objections that might be made against the generality of Malament's conclusion, and we supply three no-go theorems to counter these objections. Finally, we dispel potential worries about the counterintuitive nature of these results by showing that relativistic quantum field theory itself explains the appearance of "particle detections."*



No Place for Particles in Relativistic Quantum Theories?

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Localization issue in QFT

Malament (1996)

Theorem 1 ***(Malament).*** *Let* $(\mathcal{H}, \Delta \mapsto E_\Delta, \mathbf{a} \mapsto U(\mathbf{a}))$ *be a localization system over Minkowski spacetime that satisfies:*

(1) Localizability
(2) Translation covariance
(3) Energy bounded below
(4) Microcausality

Then $E_\Delta = 0$ *for all* Δ.

Theorem 5 *Suppose that the unsharp localization system* $(\mathcal{H}, \Delta \mapsto A_\Delta, \mathbf{a} \mapsto U(\mathbf{a}))$ *satisfies:*

(1) Additivity
(2) Translation covariance
(3) Energy bounded below
(4) Microcausality
(5) No absolute velocity

Then $A_\Delta = 0$ *for all* Δ.

Localization issue in QFT

No Place for Particles in Relativistic Quantum Theories?

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Additivity: If Δ and Δ' are disjoint subsets of a single hyperplane, then $N_\Delta + N_{\Delta'} = N_{\Delta \cup \Delta'}$.

Number conservation: If $\{\Delta_n : n \in \mathbb{N}\}$ is a disjoint covering of Σ, then the sum $\sum_n N_{\Delta_n}$ converges to a densely defined, self-adjoint operator N on $\mathcal{H}$ (independent of the chosen covering), and $U(\mathbf{a})NU(\mathbf{a})^* = N$ for any timelike translation $\mathbf{a}$ of M.

Theorem 6 *Suppose that the system* $(\mathcal{H}, \Delta \mapsto N_\Delta, \mathbf{a} \mapsto U(\mathbf{a}))$ *of local number operators satisfies:*

(1) Additivity
(2) Translation covariance
(3) Energy bounded below
(4) Number conservation
(5) Microcausality
(6) No absolute velocity

Then $N_\Delta = 0$ *for all* Δ.

Localization issue in QFT

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10.8 Conclusion

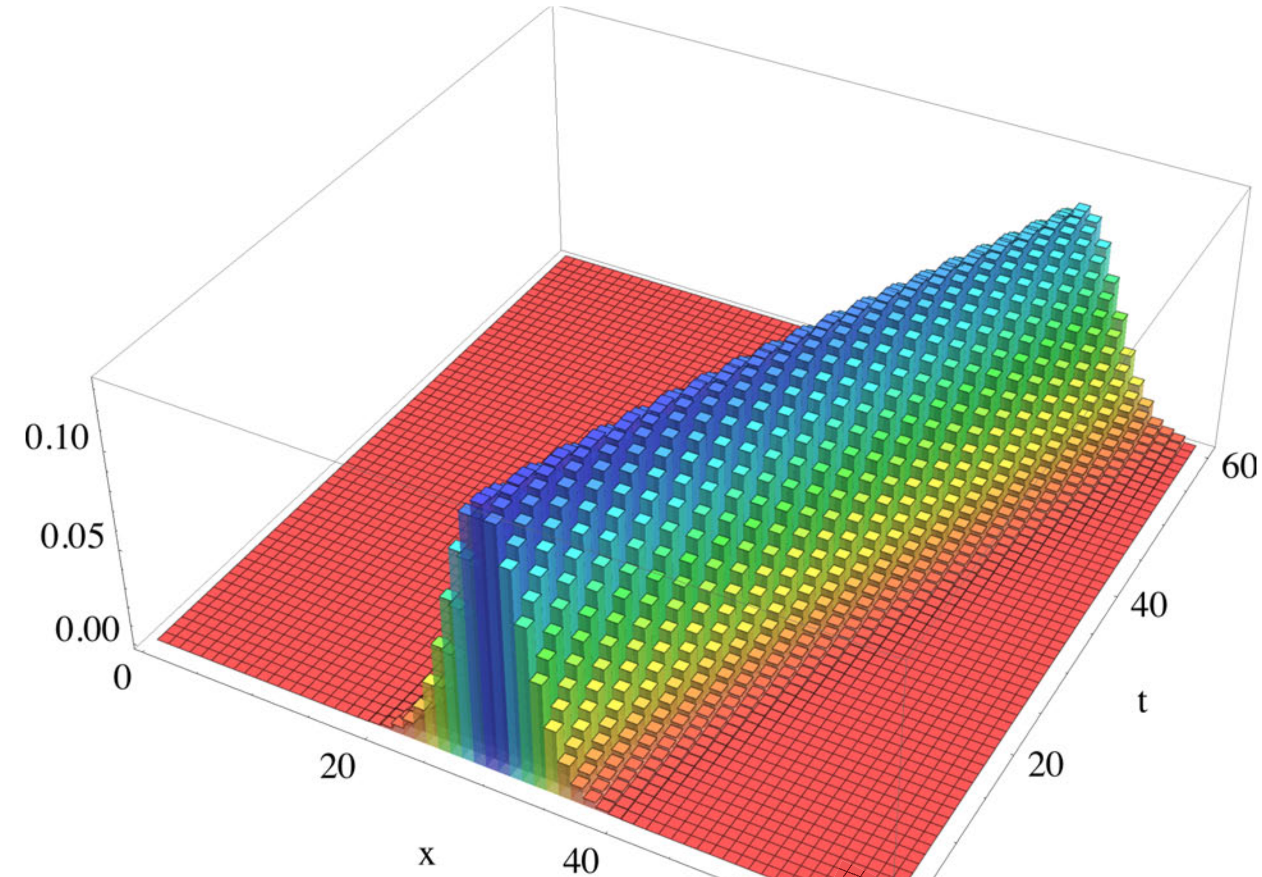
Malament claims that his theorem justifies the belief that,

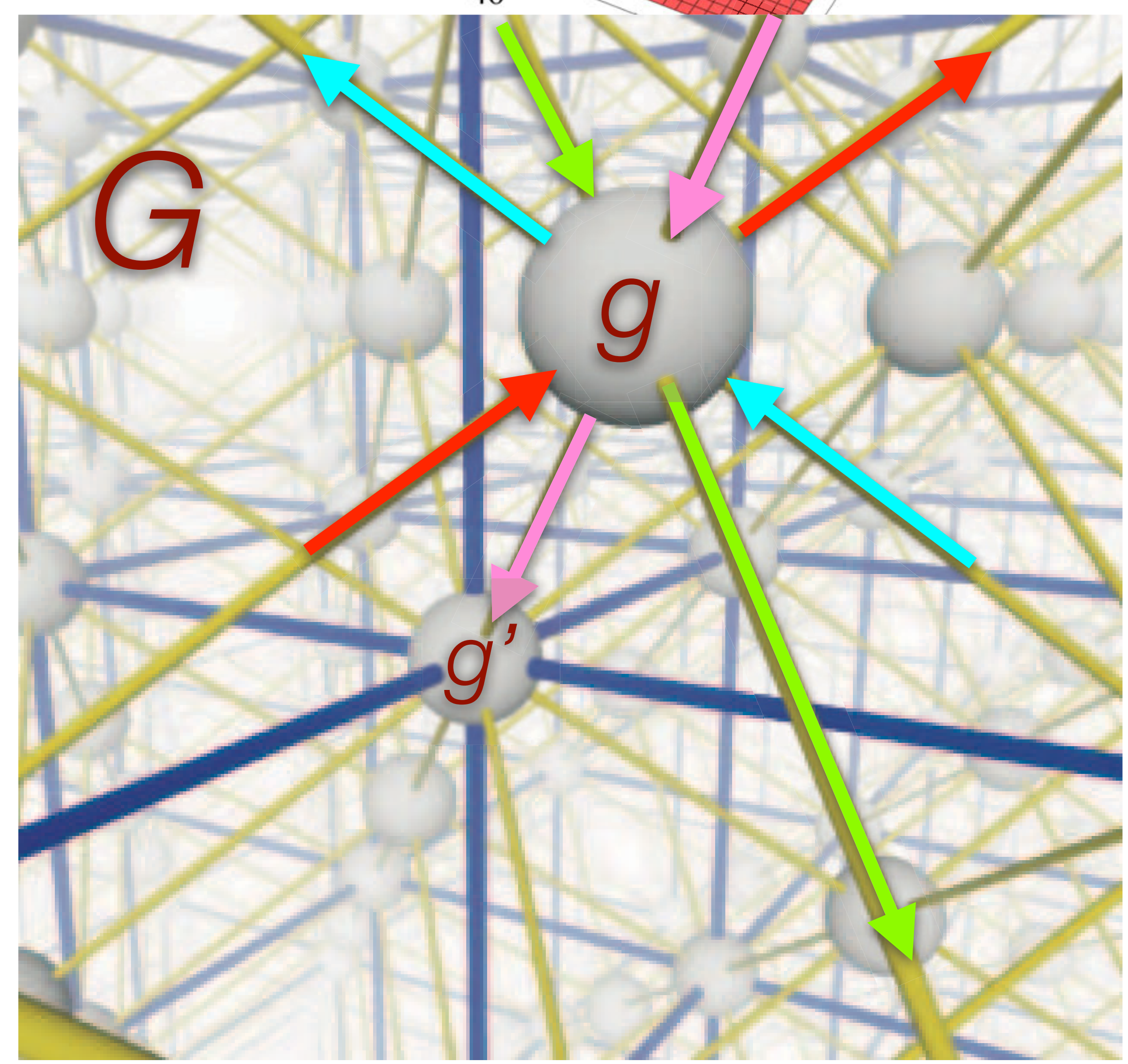
> ...in the attempt to reconcile quantum mechanics with relativity theory...one is driven to a field theory; all talk about "particles" has to be understood, at least in principle, as talk about the properties of, and interactions among, quantized fields. (Malament 1996, 1)

In order to buttress Malament's argument for this claim, we provided two further results (Theorems 3 and 5) which show that the conclusion continues to hold for generic spacetimes, as well as for unsharp localization observables. We then went on to show that RQFT does not permit an ontology of localizable particles; and so, strictly speaking, our talk about localizable particles is a fiction. Nonetheless, RQFT does permit *talk* about particles — albeit, if we understand this talk as really being about the properties of, and interactions among, quantized fields. Indeed, modulo the standard quantum measurement problem, RQFT has no trouble explaining the appearance of macroscopically well-localized objects, and shows that our talk of particles, though a *façon de parler*, has a legitimate role to play in empirically testing the theory.

QCA on Cayley graph

- The notion of quantum particle is emergent.
- Free theory (Fock space): Quantum walk on the Cayley graph of a group
- Interacting theory (von Neumann algebra) : QCA.





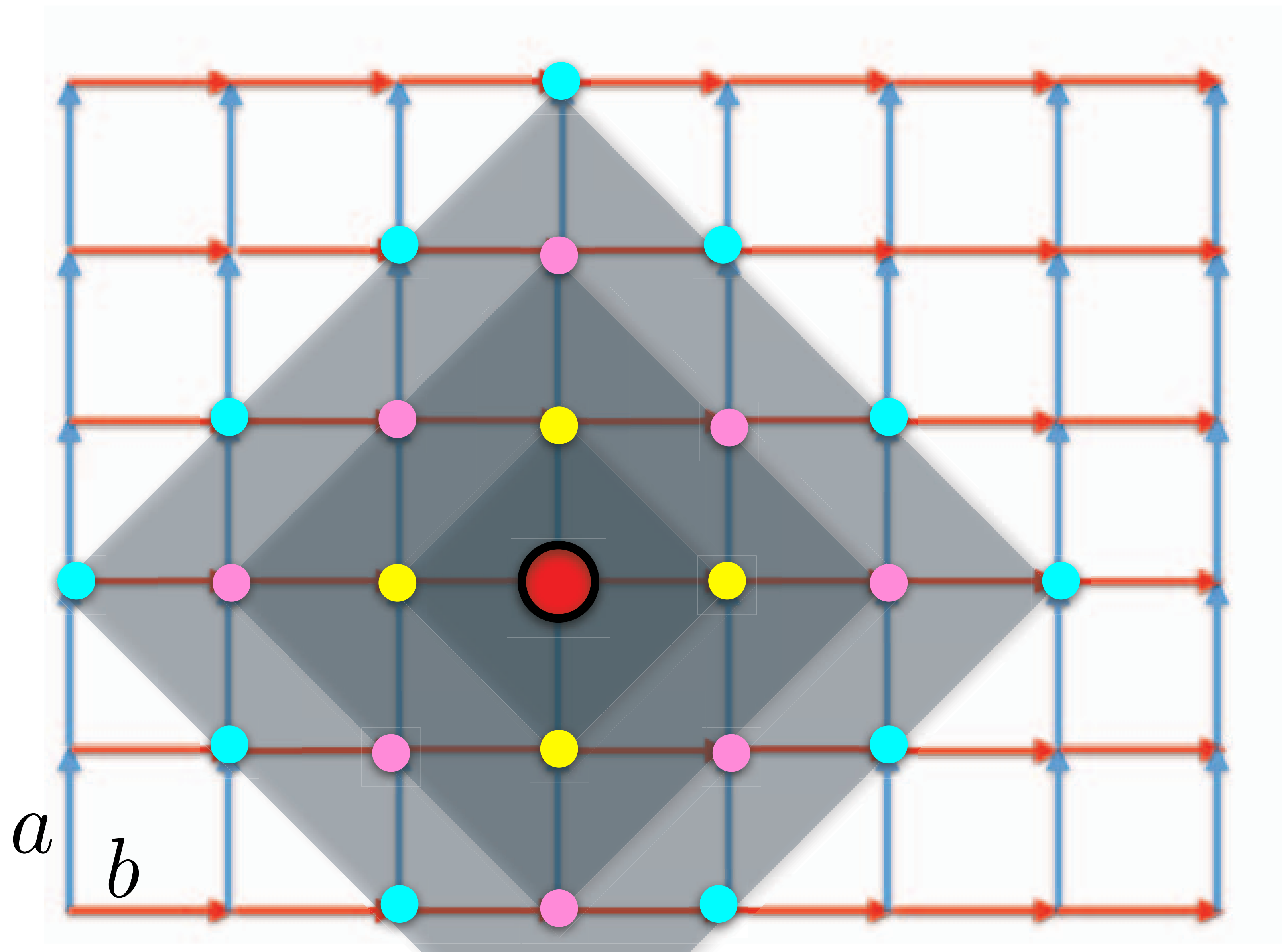
Quantum walk on Cayley graph

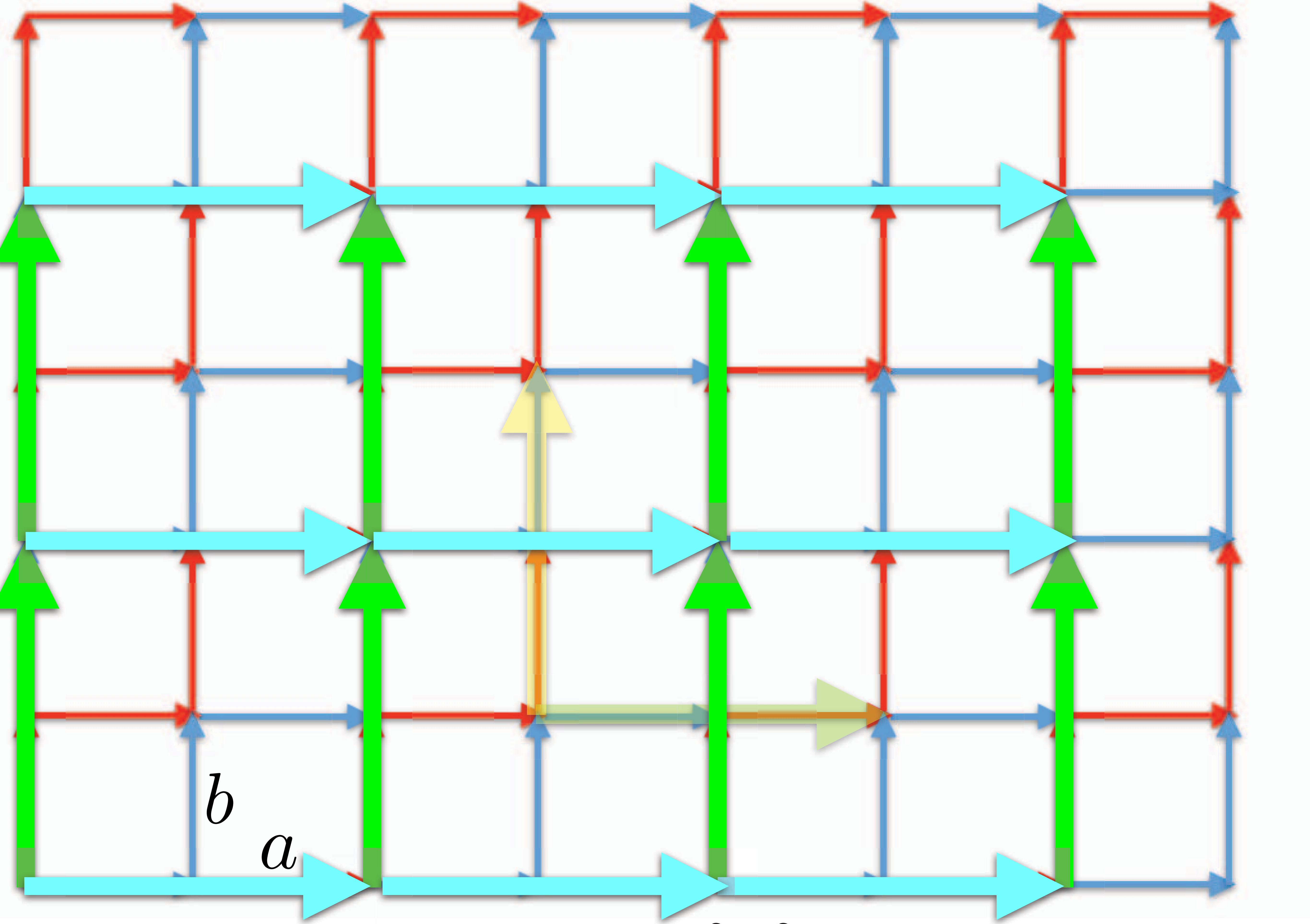
Theorem (Gromov): A group is quasi-isometrically embeddable in R^d iff it is <u>virtually Abelian</u>

Virtually Abelian groups have polynomial growth

points $\sim r^d$

$$G = \langle a, b | aba^{-1}b^{-1} \rangle$$





$$G = \langle a, b | a^2 b^{-2} \rangle$$

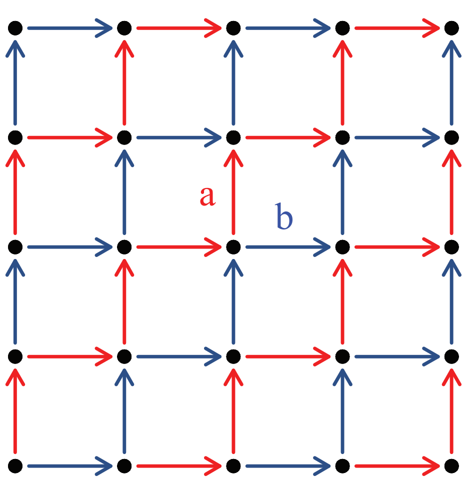
Quantum walk on Cayley graph

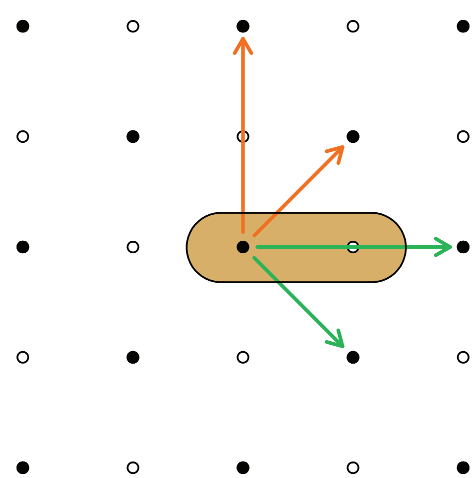
Remark 2. *One can prove that for QWCG* $Q = (G, S_+, s, \{A_h\}_{h\in S})$ *with* G *virtually Abelian there exists a quantum walk* $Q' = (H, S_+^H, s \cdot i_H, \{B_h\}_{h\in S^H})$ *with Abelian* $H \subset G$*, with finite index* i_H*, such that*

$$A_{Q'} = V A_Q V^\dagger, \quad \text{with } V : u_{g_i a} \otimes \psi \mapsto V u_{g_i a} \otimes \psi = v_a \otimes e_i \otimes \psi, \qquad (13)$$

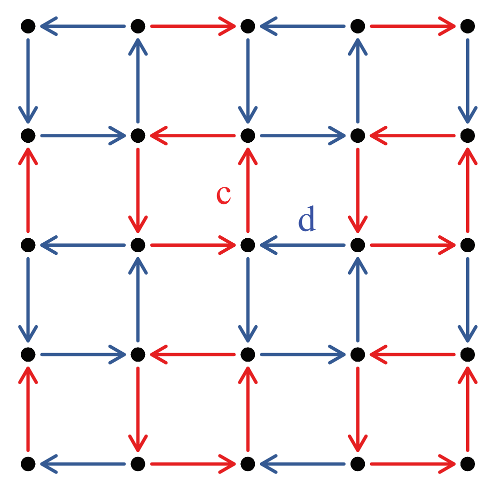
with $\{g_i\}_{i=1,\dots,i_H}$ *being coset representatives,* v_a *with* $a \in H$ *canonical orthonormal basis of* $\ell^2(H)$*,* $\{e_i\}_{i=1,\dots,i_H}$ *canonical basis in* $\mathbb{C}^{i_H}$*,* $\psi \in \mathbb{C}^s$*, and* V *isomorphism between* $\ell^2(G) \otimes \mathbb{C}^s$ *and* $\ell^2(H) \otimes \mathbb{C}^{s\cdot i_H}$*.*

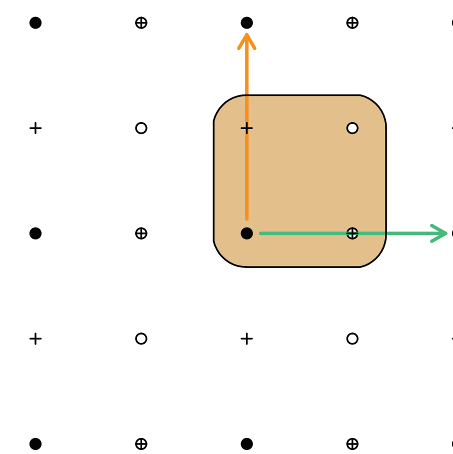
$\langle\, a, b \mid a^2 b^{-2} \,\rangle$



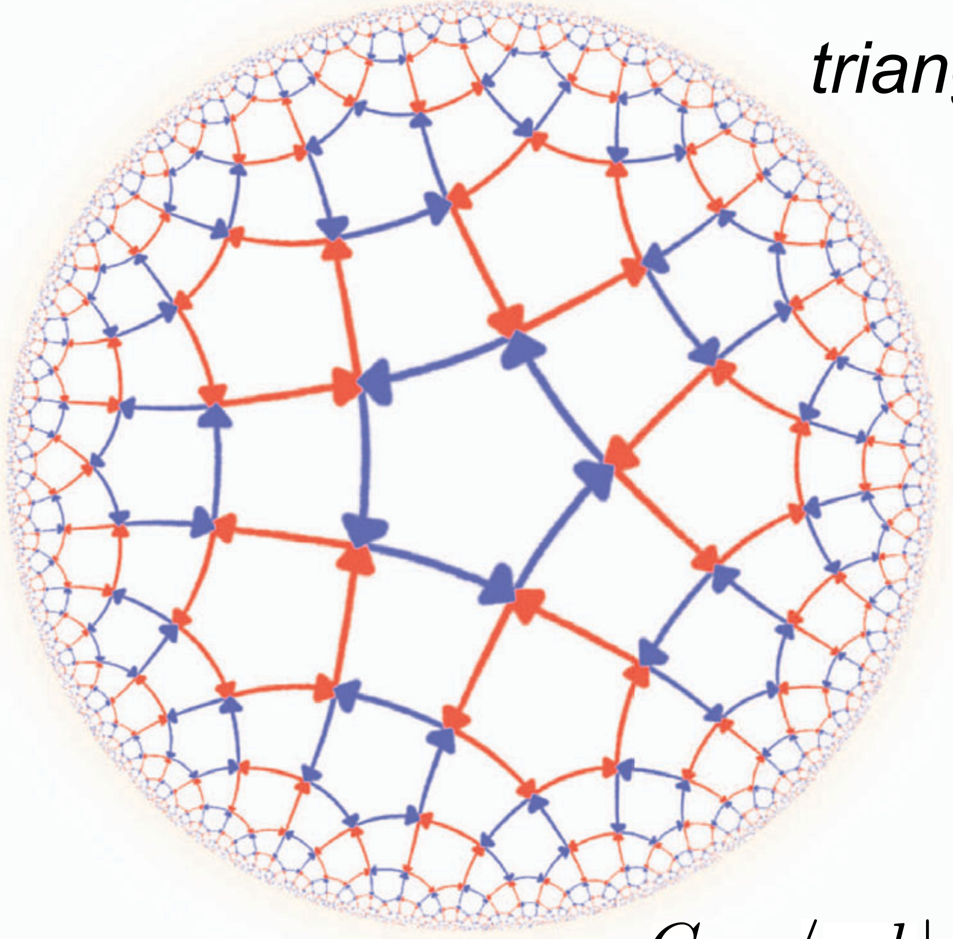


$\langle\, c, d \mid c^4, d^4, (cd)^2 \,\rangle$

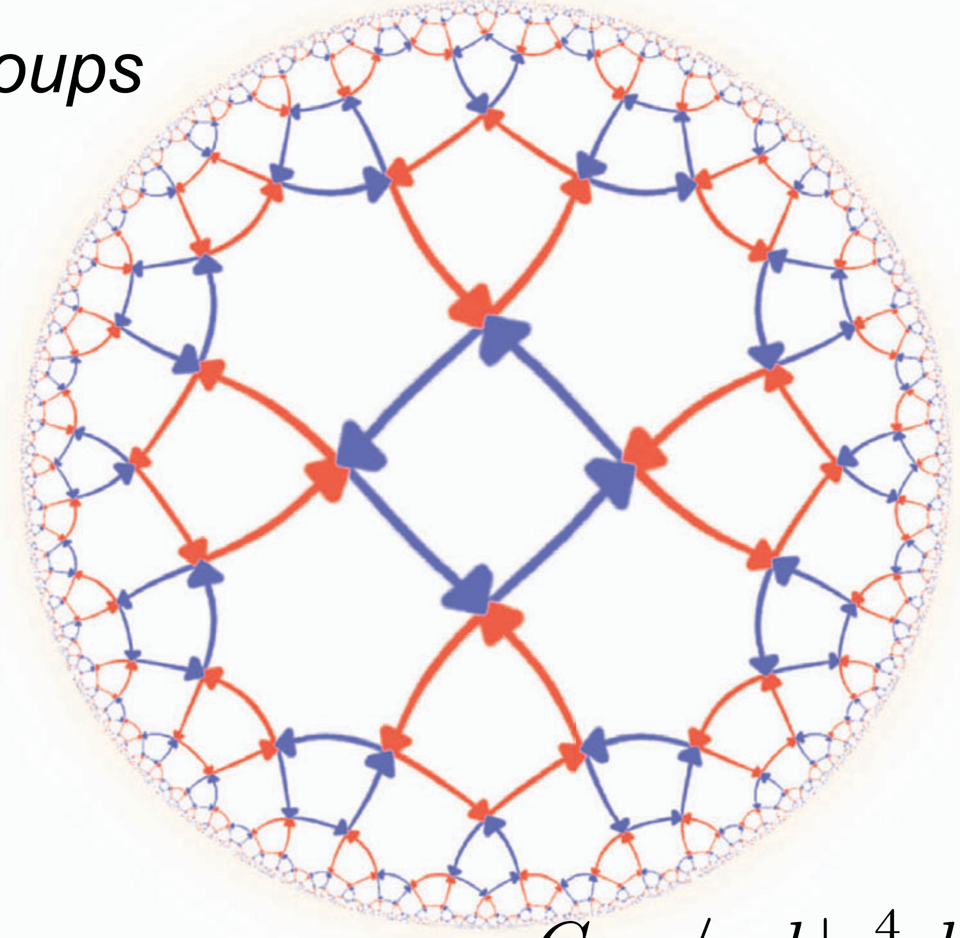




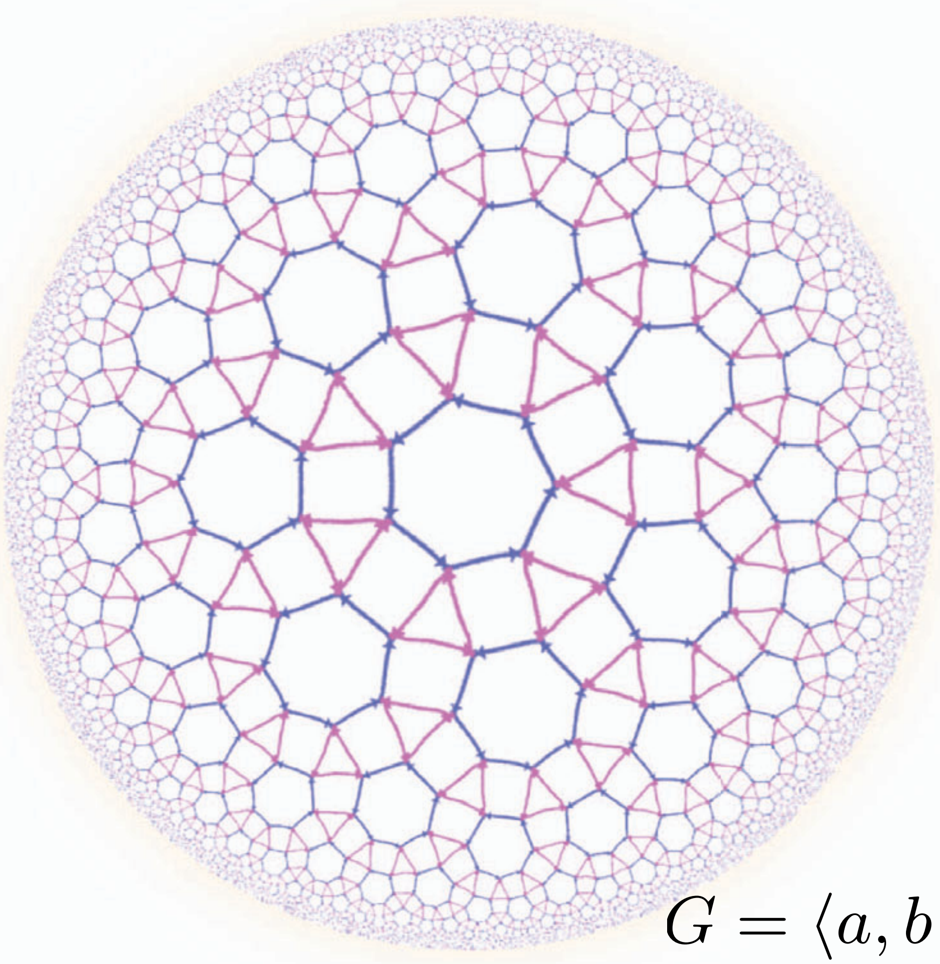
triangle orbifold groups



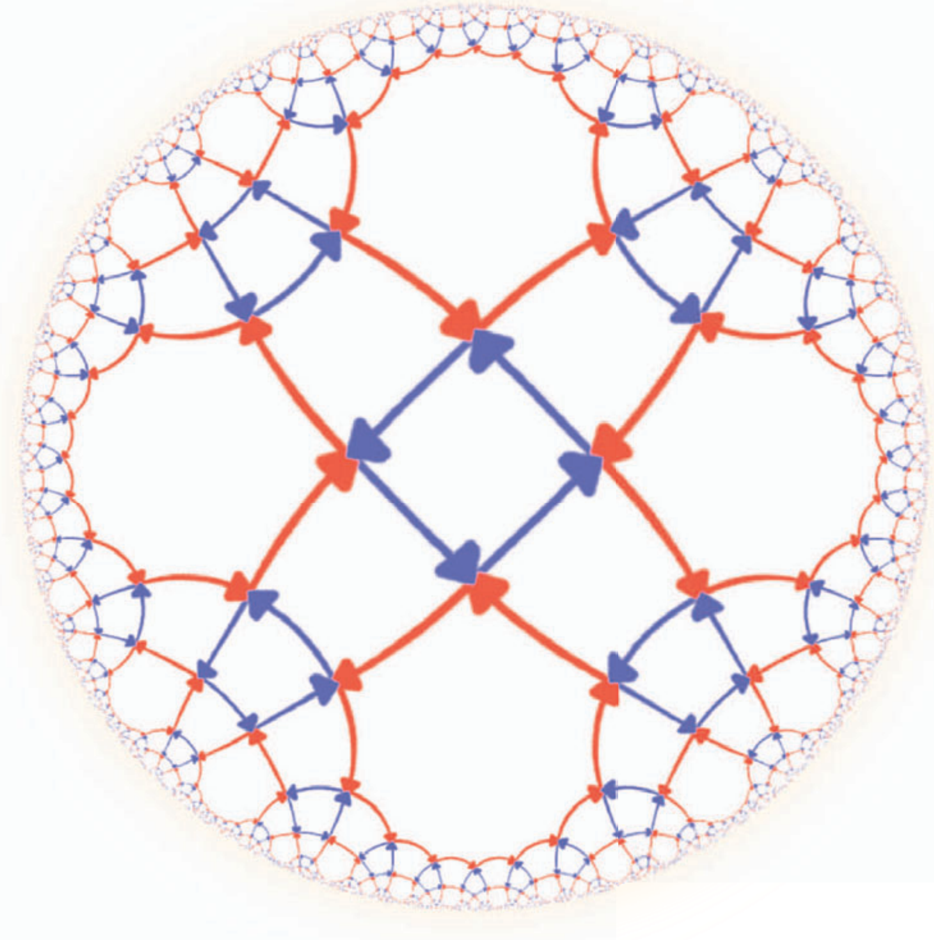
$G = \langle a, b | a^5, b^5, (ab)^2 \rangle$



$G = \langle a, b | a^4, b^4, (ab)^3 \rangle$

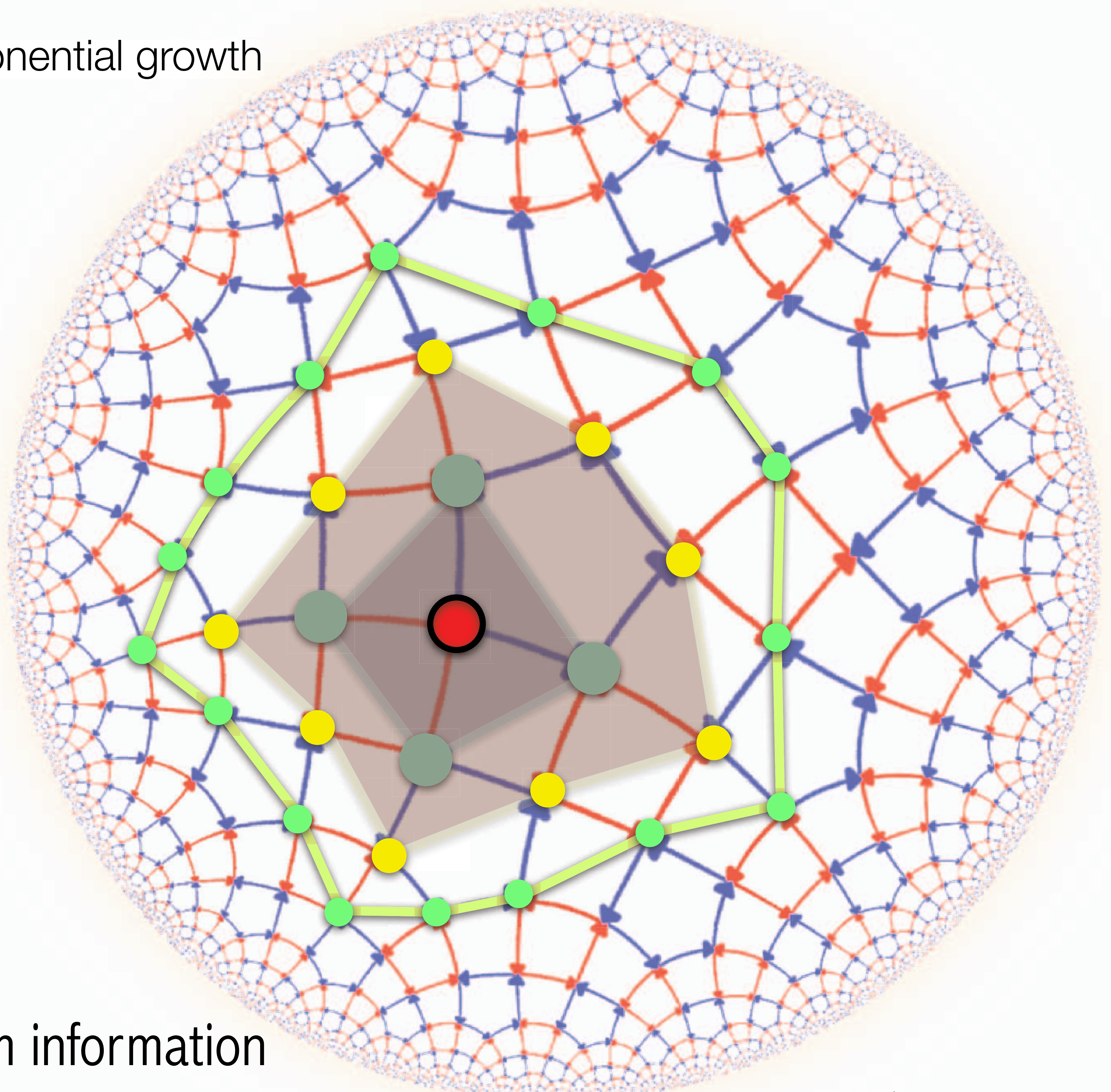


$G = \langle a, b | a^7, b^3, (ab)^2 \rangle$



$G = \langle a, b | a^4, b^{10}, (ab)^2 \rangle$

- G hyperbolic → exponential growth



points ~exp(r)

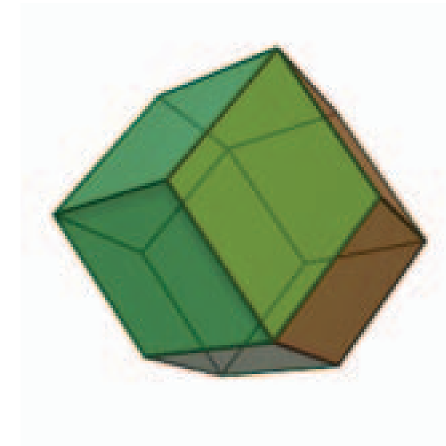
transmitted quantum information decrease as exp(-r)

$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

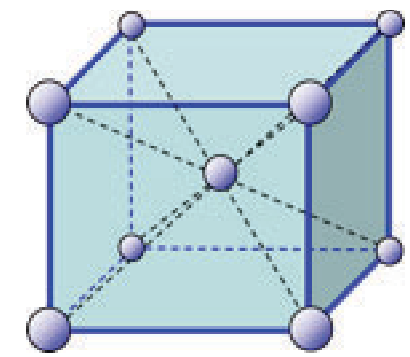
The Weyl QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

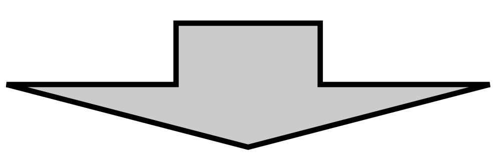
☞ Minimal dimension for nontrivial unitary *A:* s=2



- Unitarity ⇒ for d=3 the only possible *G* is the BCC!!
- Isotropy ⇒ Fermionic ψ (d=3)



Unitary operator: $$A = \int_{\mathsf{B}} \mathrm{d}^3\mathbf{k}\, |\mathbf{k}\rangle\langle\mathbf{k}| \otimes A_{\mathbf{k}}$$



Two QCAs connected by CPT

$$\boxed{\begin{aligned} A_{\mathbf{k}}^{\pm} = & - i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ & \mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ & - i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ & + I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

The Weyl QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

$$i\partial_t \psi(t) \simeq \tfrac{i}{2}[\psi(t+1) - \psi(t-1)] = \tfrac{i}{2}(A - A \ )\psi(t)$$

$$\begin{aligned}\tfrac{i}{2}(A^{\pm}_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+ \sigma_x(s_x c_y c_z \pm c_x s_y s_z)\\ &\pm \sigma_y(c_x s_y c_z \mp s_x c_y s_z)\\ &+ \sigma_z(c_x c_y s_z \pm s_x s_y c_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$

$$\boxed{i\partial_t \psi = \frac{1}{\sqrt{3}}\boldsymbol{\sigma}^{\pm} \cdot \mathbf{k}\,\psi}$$ ☜ Weyl equation!

$$\boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$$

Two QCAs connected by CPT

$$\boxed{\begin{aligned}A^{\pm}_{\mathbf{k}} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z)\\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z)\\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z)\\ &+ I(c_x c_y c_z \mp s_x s_y s_z)\end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QCA

$\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$E_{\mathbf{k}}^{\blacksquare} = \begin{pmatrix} nA_{\mathbf{k}}^{\blacksquare} & imI \\ imI & nA_{\mathbf{k}}^{\blacksquare\blacksquare} \end{pmatrix}$$

$$n^2 + m^2 = 1$$

$E_{\mathbf{k}}$ CPT-connected!

$$\omega_{\pm}^{E}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

m≤1: mass

n^{-1}: refraction index

Bisio, D'Ariano, Perinotti, arXiv:1407.6928

Maxwell QCA

$\otimes$

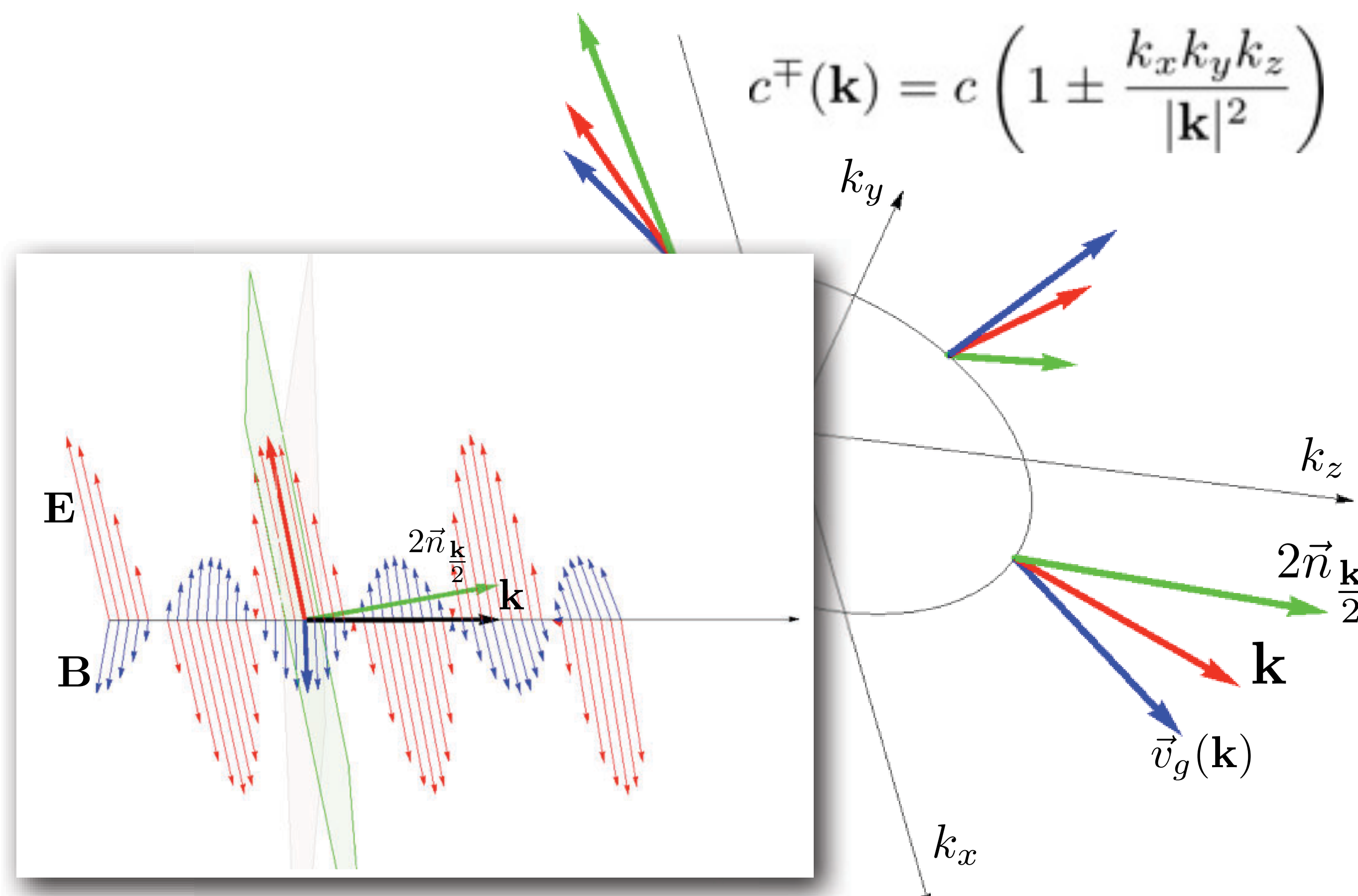
$$M_{\mathbf{k}} = A_{\mathbf{k}} \otimes A_{\mathbf{k}}^{*}$$

$$F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)



Determining dimensional units [L][T][M]

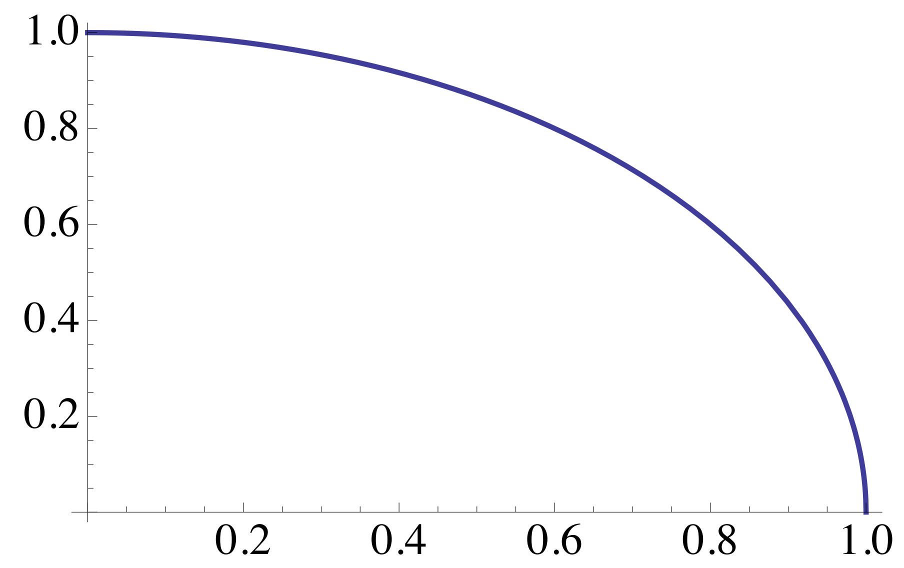
Dimensionless variables

$$x = \frac{x_m}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_s}{\mathfrak{t}} \in \mathbb{N}, \quad m = \frac{m_g}{\mathfrak{m}} \in [0,1]$$

$$c = \mathfrak{a}/\mathfrak{t}$$

Measure $\mathfrak{m}$ from mass-refraction-index

$$n(m_g) = \sqrt{1 - \left(\frac{m_g}{\mathfrak{m}}\right)^2}$$



Measure $\mathfrak{a}$ from light-refraction-index

$$c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$

Conversion to dimensional units

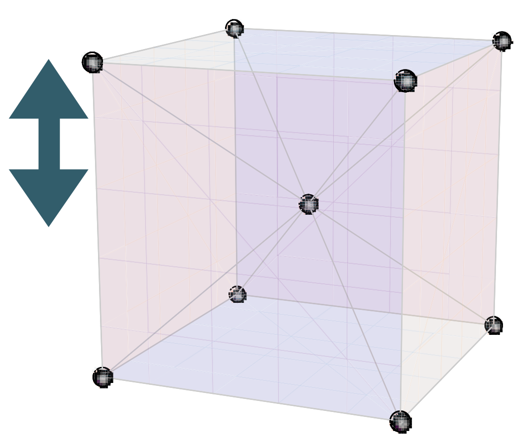
Dimensionless variables

$$x = \frac{x_m}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_s}{\mathfrak{t}} \in \mathbb{N}, \quad m = \frac{m_g}{\mathfrak{m}} \in [0,1]$$

Relativistic limit: $\quad \boldsymbol{c} = \mathfrak{a}/\mathfrak{t} \qquad \boldsymbol{\hbar} = \mathfrak{m}\mathfrak{a}\boldsymbol{c}$

Mini black-hole: $\quad G = \mathfrak{a}\mathfrak{t}^{-2}/(\mathfrak{m}\mathfrak{a}^{-2})$

l_P



$$\mathfrak{a} = l_P \qquad \mathfrak{t} = t_P \qquad \mathfrak{m} = m_P$$

[L] [T] [M]

fundamental system (Wilczek)

Dirac emerging from the QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

$$F = |\langle \exp[-iN\boldsymbol{\Delta}(\mathbf{k})]\rangle|$$

$$\Delta(\mathbf{k}) := (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k})$$

$$= \frac{\sqrt{3}k_x k_y k_z}{(m^2+\frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2+\frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2+\tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \;\; \Rightarrow \; t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \;\Rightarrow N \simeq k^{-2} = 10^{16} \;\Rightarrow\; 5 * 10^{-28}$ s

A. Bisio, G. M. D'Ariano, A. Tosini,
Ann. Phys. **354** 244 (2015)

The general dispersive Schrödinger equation

$$i\partial_t e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k}\ \ t) = s[\omega(\mathbf{k}) - \omega_0]e^{-i\mathbf{k}_0\ \ \mathbf{x}+i\omega_0 t}\psi(\mathbf{k}\ \ t)$$

$$i\partial_t \psi(\mathbf{k}\ \ t) = s[\omega(\mathbf{k}) - \omega_0]\psi(\mathbf{k}\ \ t)$$

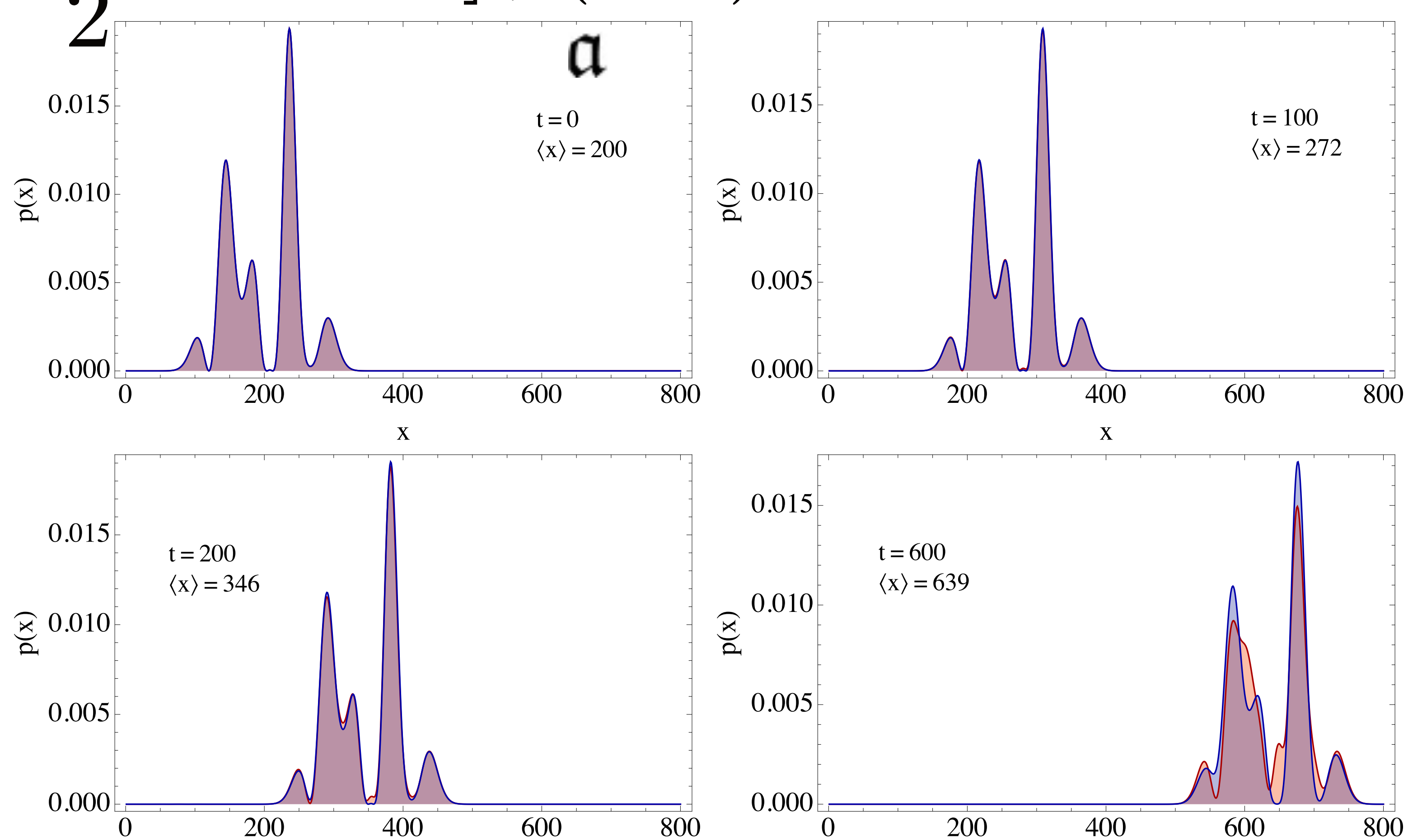
$$s = \pm$$

$$i\partial_t \psi(\mathbf{x}\ \ t) = s[\mathbf{v}\ \ \ \boldsymbol{\nabla} + \frac{1}{2}\mathbf{D}\ \ \ \boldsymbol{\nabla}\boldsymbol{\nabla}]\psi(\mathbf{x}\ \ t)$$

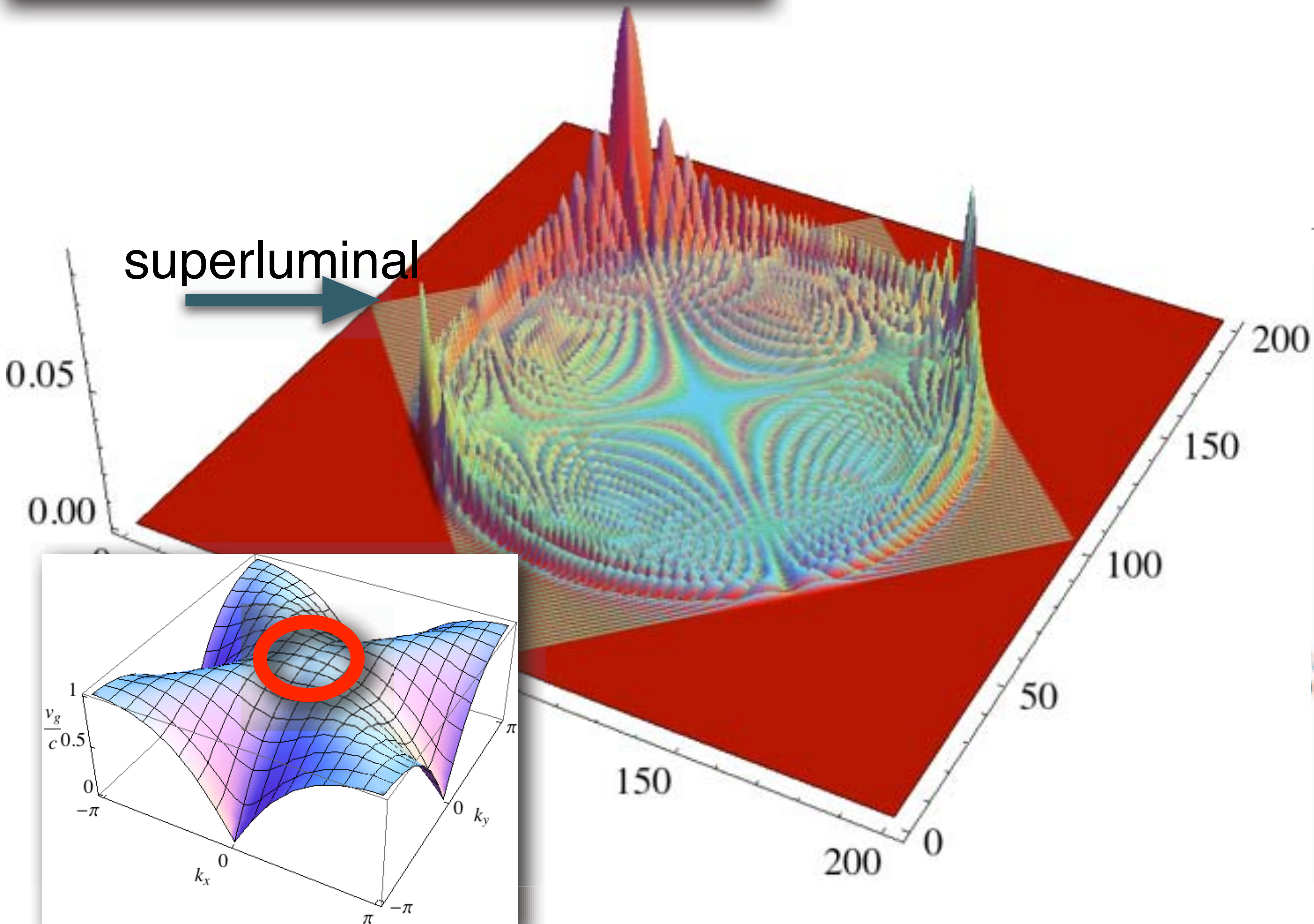
k_0=3π/10. m=.6

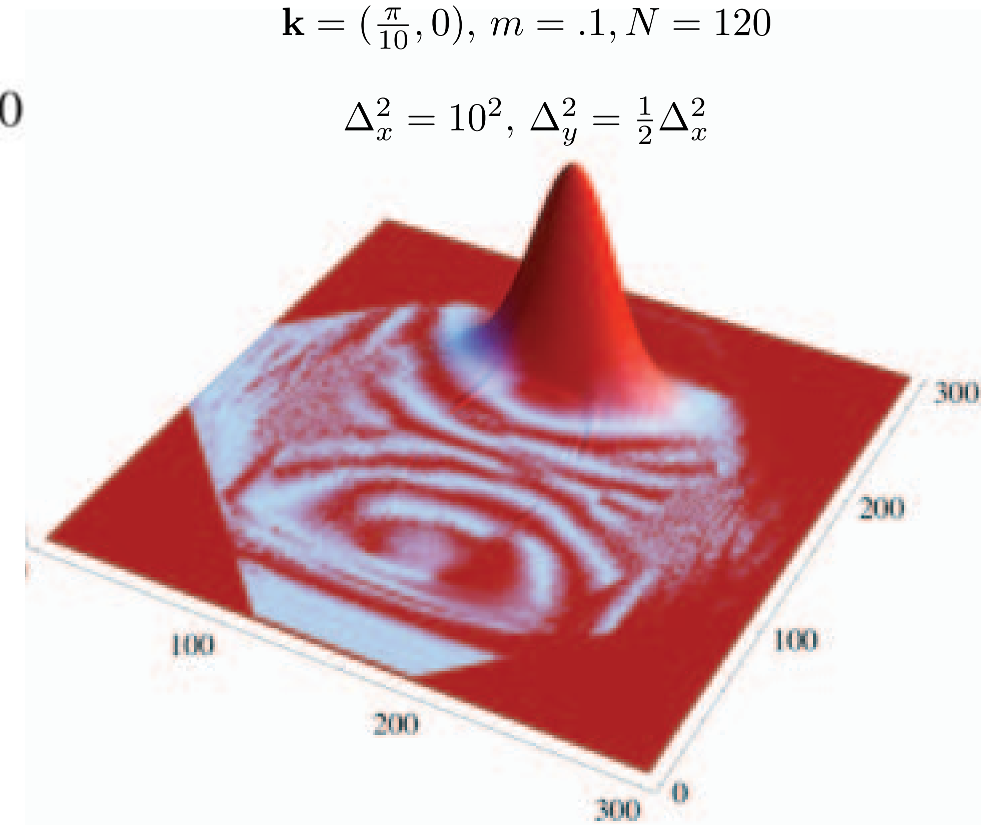
$$\mathbf{v} = (\boldsymbol{\nabla}_{\mathbf{k}}\omega)\,(\mathbf{k}_0)$$

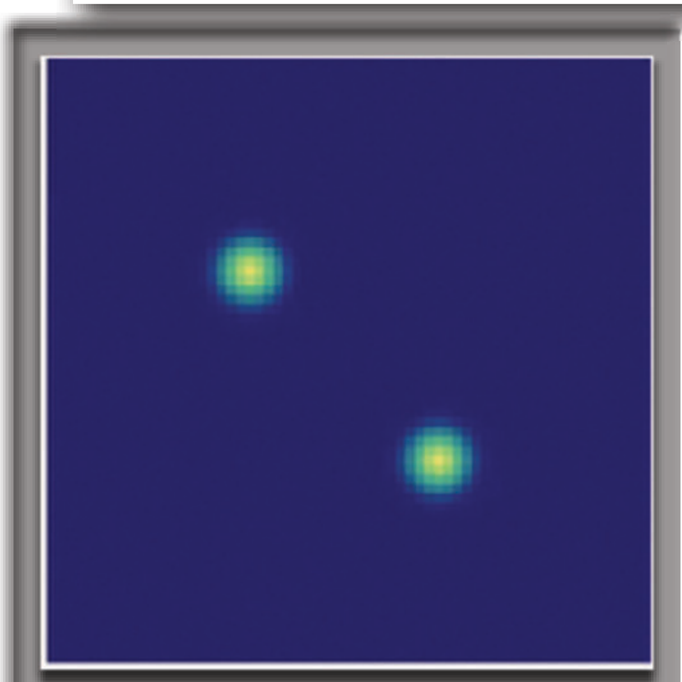
$$\mathbf{D} = (\boldsymbol{\nabla}_{\mathbf{k}}\boldsymbol{\nabla}_{\mathbf{k}}\omega)\,(\mathbf{k}_0)$$

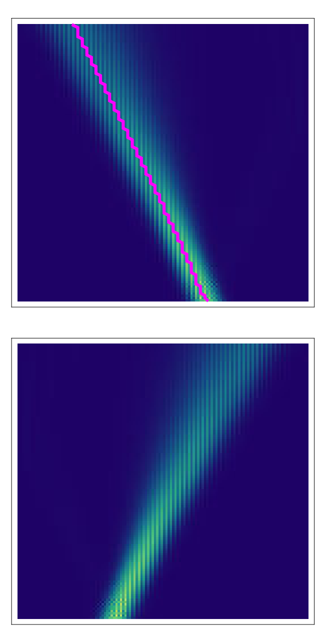


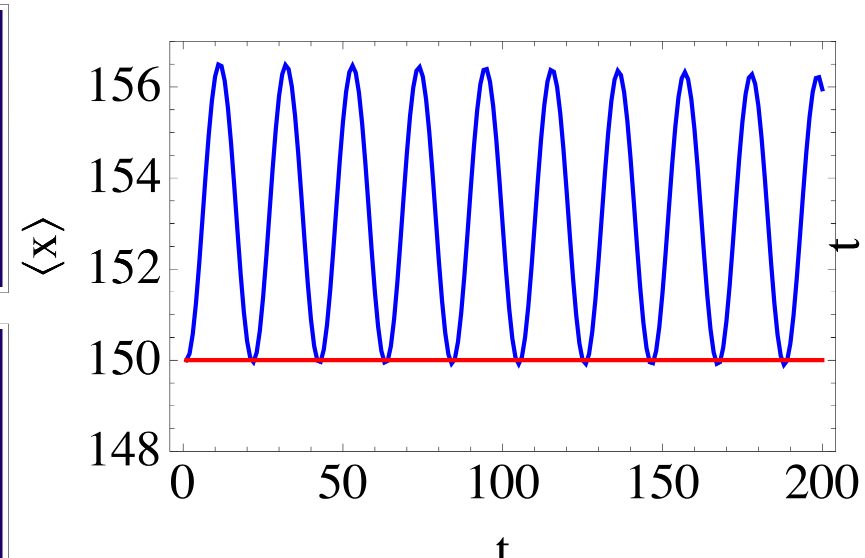
Dirac QCA

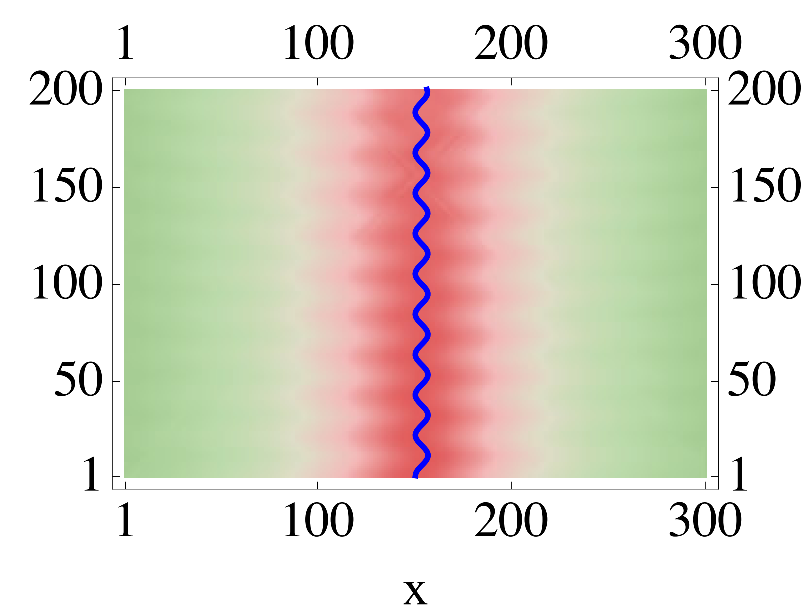










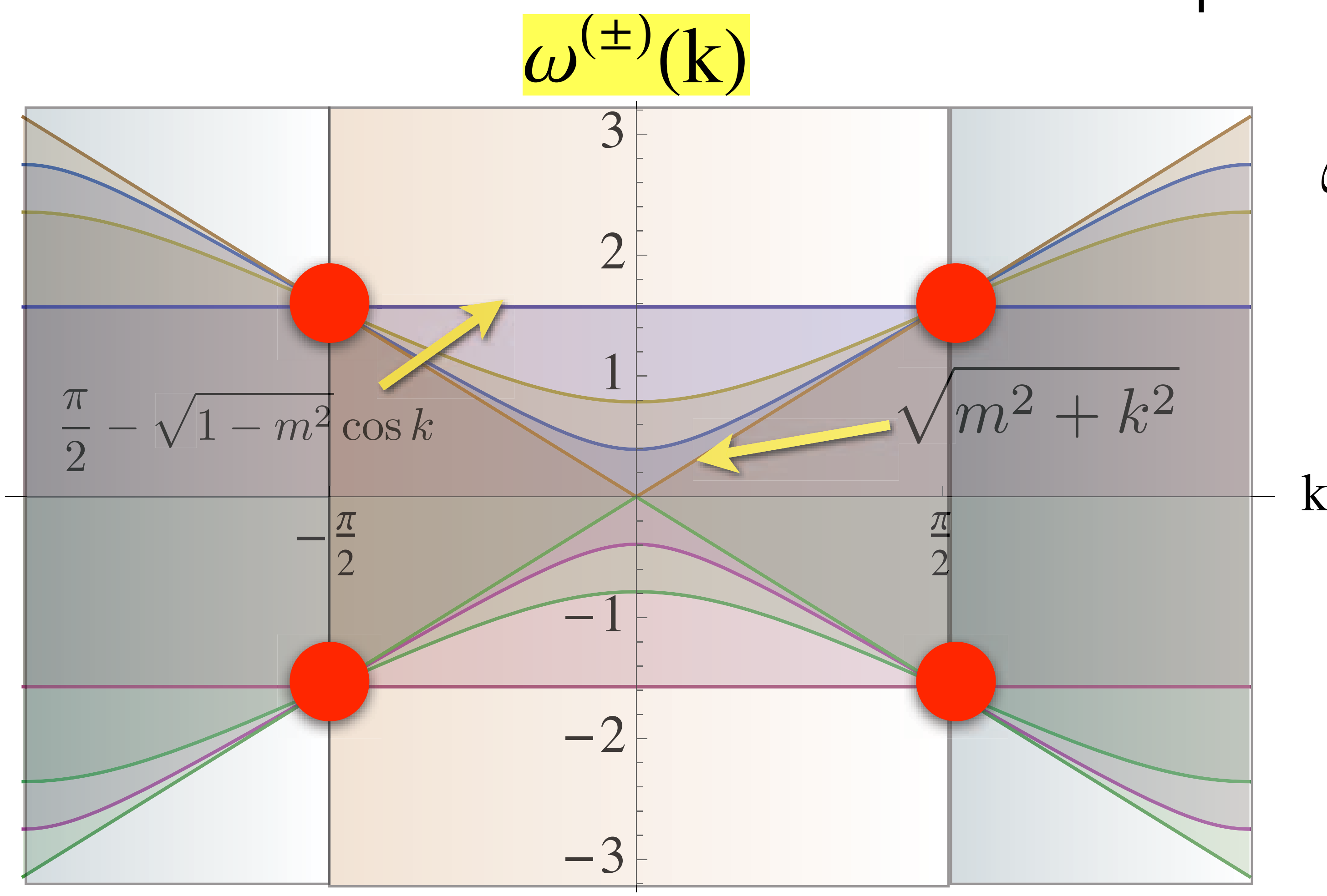


Particle state: $k_0 = 0$, $m = 0.15$, $\sigma = 40$. Oscillation frequency $\nu = 0.048$

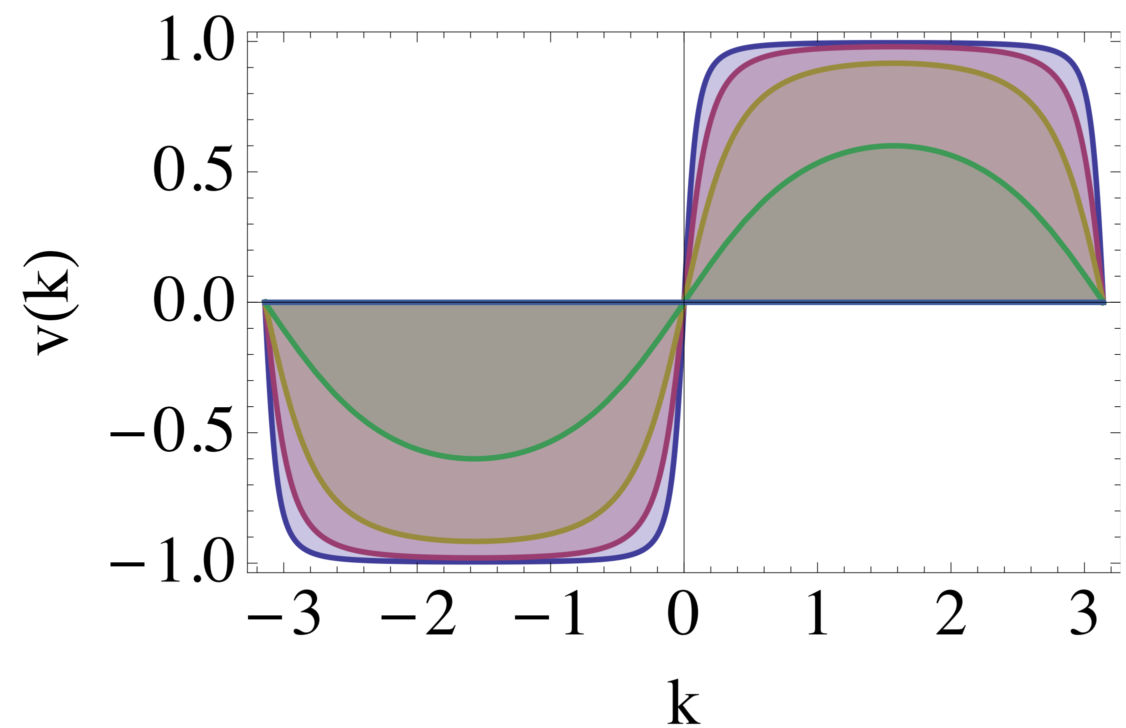
Bibeau-Delisle, Bisio, D'Ariano,
Perinotti, Tosini, EPL (in press)

Planck-scale effects: Lorentz covariance distortion

Transformations that leave the dispersion relation invariant



$$\omega_E(k) := \quad \cos^{-1}(\sqrt{1-m^2}\cos k)$$



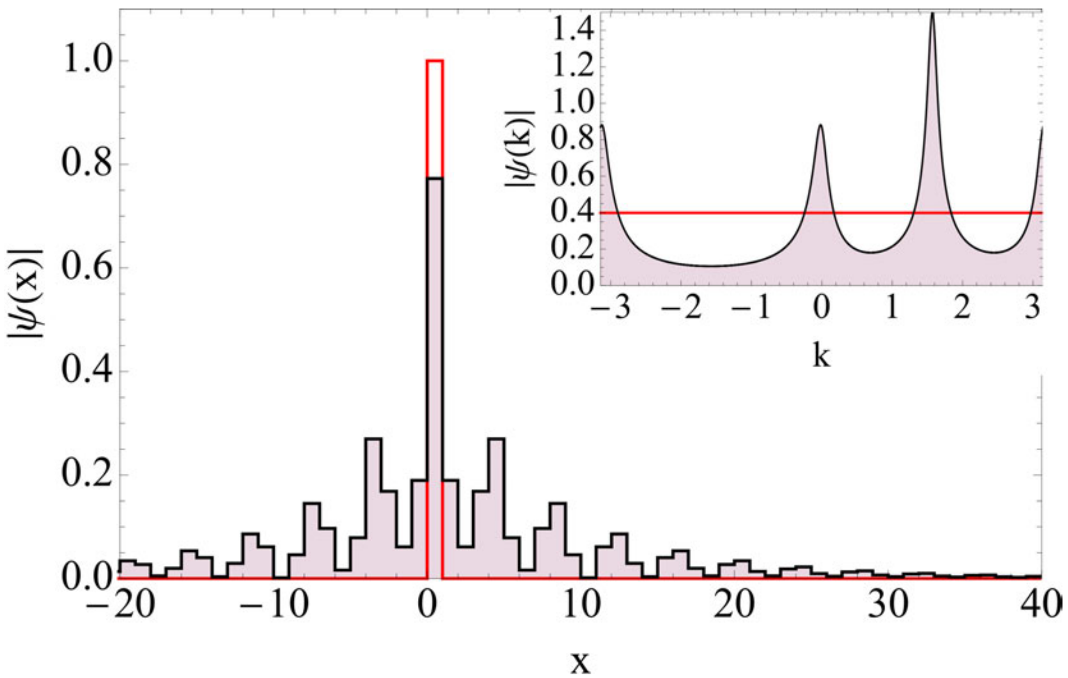
$$\omega' = \arcsin\left[\gamma\left(\sin\omega/\cos k - \beta\tan k\right)\cos k'\right]$$

$$k' = \arctan\left[\gamma\left(\tan k - \beta\sin\omega/\cos k\right)\right]$$

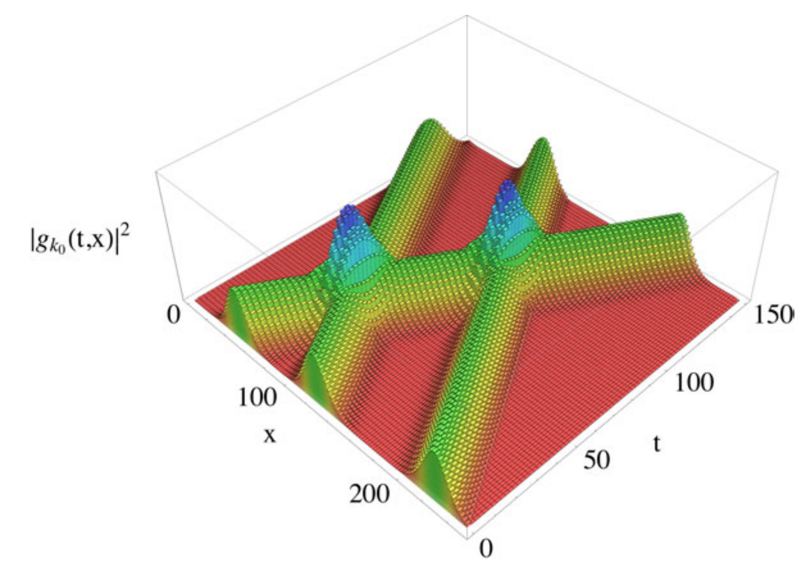
$$\gamma := (1-\beta^2)^{-1/2}$$

Bibeau-Delisle, Bisio, D'Ariano, Perinotti, Tosini, arXiv:1310.6760

Planck-scale effects: Lorentz covariance distortion

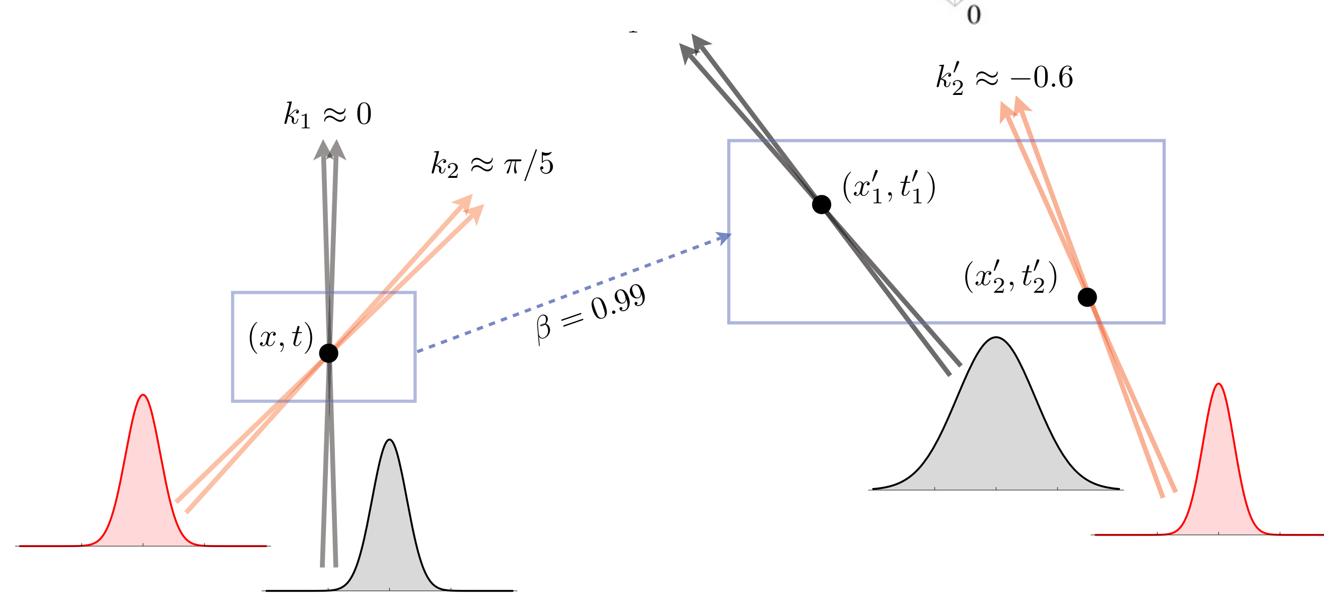


For narrow-band states we can linearize Lorentz transformations around *k*=*k₀* and we get *k*-dependent Lorentz transformations



Delocalization under boost

$$|\psi\rangle = \int dk \mu(k) \hat{g}(k) |k\rangle \xrightarrow{L^D_\beta} \int dk \mu(k)\, \hat{g}(k) |k'\rangle =$$
$$= \int dk \mu(k')\, \hat{g}(k(k')) |k'\rangle$$



Relative locality

R. Schützhold and W. G. Unruh, J. Exp. Theor. Phys. Lett. **78** 431 (2003)
G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, and L. Smolin, arXiv:1106.0313 (2011)

Lorentz covariance: Weyl automaton

$$A^{\pm}_{\mathbf{k}} := \lambda^{\pm}(\mathbf{k})I - i\mathbf{n}^{\pm}(\mathbf{k}) \cdot \boldsymbol{\sigma}^{\pm}$$

$$\mathbf{n}^{\pm}(\mathbf{k}) := \begin{pmatrix} s_x c_y c_z \pm c_x s_y s_z \\ c_x s_y c_z \mp s_x c_y s_z \\ c_x c_y s_z \pm s_x s_y c_z \end{pmatrix}$$

$$A = \int_{\mathsf{B}} \mathrm{d}^3\mathbf{k}\, |\mathbf{k}\rangle\langle\mathbf{k}| \otimes A_{\mathbf{k}}$$

$$\lambda^{\pm}(\mathbf{k}) := (c_x c_y c_z \mp s_x s_y s_z)$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}} \qquad s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$A_{\mathbf{k}}\psi(\mathbf{k},\omega) = e^{i\omega}\psi(\mathbf{k},\omega)$$

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma})\psi(\mathbf{k},\omega) = 0$$

$$\sin^2\omega - |\mathbf{n}(\mathbf{k})|^2 = 0$$

Inertial frame: decomposition into irreducible representations

Change of frame: $\mathbf{k} \to \mathbf{k}'(\mathbf{k},\omega), \qquad \omega \to \omega'(\mathbf{k},\omega)$

Lorentz covariance: Weyl automaton

Change of frame: $\mathbf{k} \to \mathbf{k}'(\mathbf{k}, \omega), \qquad \omega \to \omega'(\mathbf{k}, \omega)$

Requirement that the change of frame leaves the dynamics invariant:

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma}) = \Lambda^\dagger (\sin\omega' I - \mathbf{n}(\mathbf{k'}) \cdot \boldsymbol{\sigma})\Lambda \qquad \Lambda = \Lambda(\mathbf{k}, \omega) \in \mathrm{SL}(s, \mathbb{C})$$

Assume linearity: Λ independent on $\mathbf{k}$ and ω $\Rightarrow$ $\Lambda^\dagger \sigma \Lambda = L_\beta^{-1} \sigma$

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma})\psi(\mathbf{k}, \omega) = 0$$

$$f(\omega, \mathbf{k})(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma})\psi(\mathbf{k}, \omega) = 0 \quad (\omega, \mathbf{k}) \in \mathcal{X}.$$

$f(\omega, \mathbf{k})$ continuous non vanishing on $\mathcal{X}$

Define the 4-vector: $p := f(\omega, \mathbf{k})\,(\sin(\omega), \mathbf{n}(\mathbf{k}))$

Lorentz covariance: Weyl automaton

$$f(\omega, \mathbf{k})(\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma})\psi(\mathbf{k}, \omega) = 0$$

$$\boxed{(p_\mu \sigma^\mu)\psi(\mathbf{k}, \omega) = 0}$$

action on $(\mathbf{k}, \omega)$ given by the non-linear representation of the Lorentz group

$$\mathcal{L}_\beta := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D} \qquad \mathcal{D}(\omega, \mathbf{k}) := f(\omega, \mathbf{k})(\sin \omega, \mathbf{n}(\mathbf{k}))$$

$$\psi(\mathbf{k}, \omega) \mapsto \Lambda \psi(\mathbf{k}', \omega')$$

Lorentz covariance: Weyl automaton

$f(\omega, \mathbf{k})(\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma})\psi(\mathbf{k}, \omega) = 0$ $\rightarrow$ $\boxed{(p_\mu \sigma^\mu)\psi(\mathbf{k}, \omega) = 0}$

action on $(\mathbf{k}, \omega)$ given by the non-linear representation of the Lorentz group

$$\mathcal{L}_\beta := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D} \qquad \mathcal{D}(\omega, \mathbf{k}) := f(\omega, \mathbf{k})(\sin \omega, \mathbf{n}(\mathbf{k}))$$

$$\psi(\mathbf{k}, \omega) \mapsto \Lambda \psi(\mathbf{k}', \omega')$$

take f monotonic: you can study $J_{\mathbf{n}}(\mathbf{k})$ instead of $J_{\mathcal{D}}(\mathbf{k})$ Also: $J_{\mathcal{D}}(\mathbf{0}) = I$

The Brillouin zone separates into four regions $\mathsf{B} = \left(\cup_{i=0}^3 \mathsf{B}_i\right) \cup \mathsf{F}$

F: zero-measure set where the Jacobian $J_{\mathbf{n}}(\mathbf{k})$ of the map $\mathbf{n}(\mathbf{k})$ vanishes

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Jacobian must go to the identity

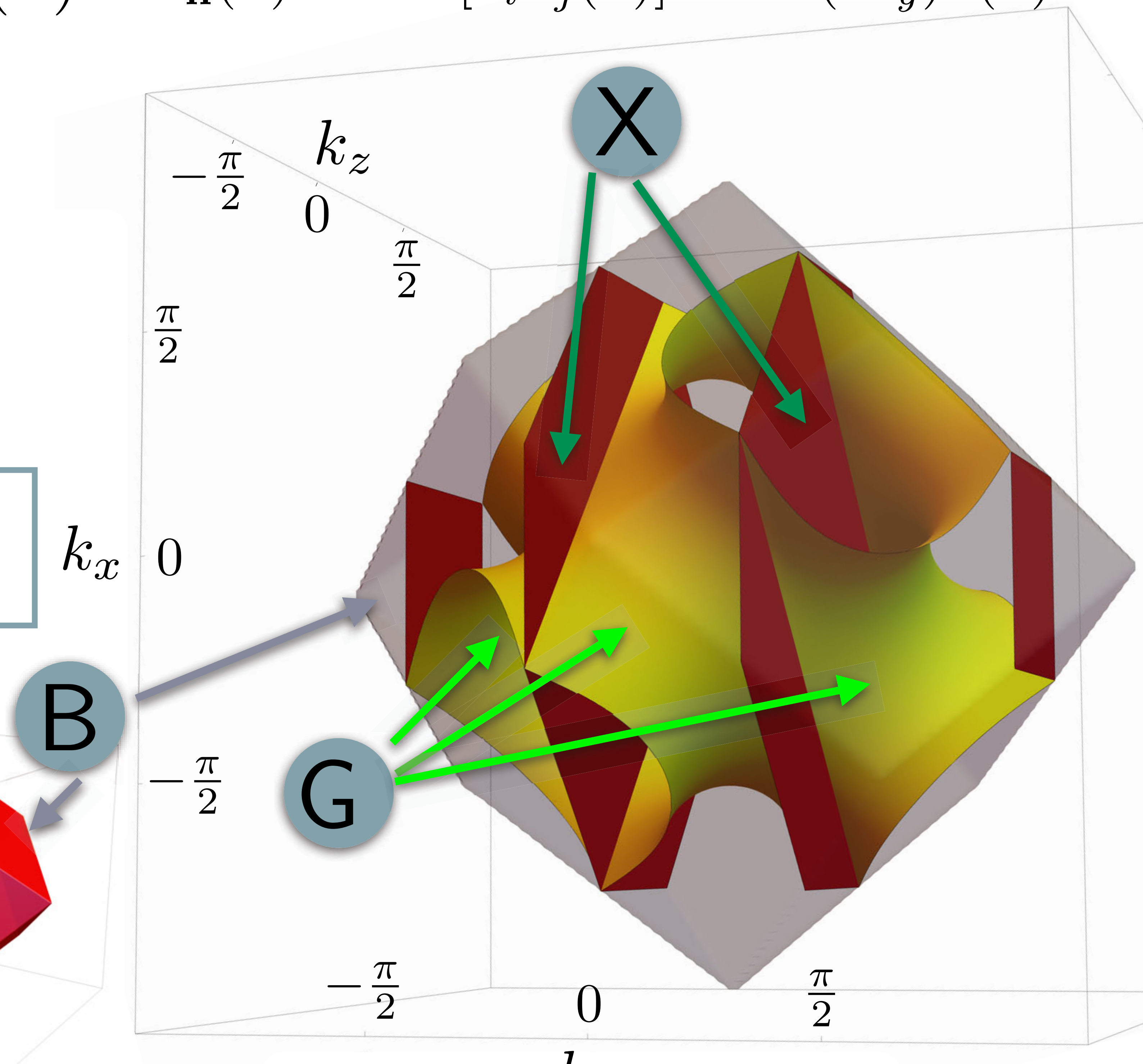
Lorentz covariance: ... aton

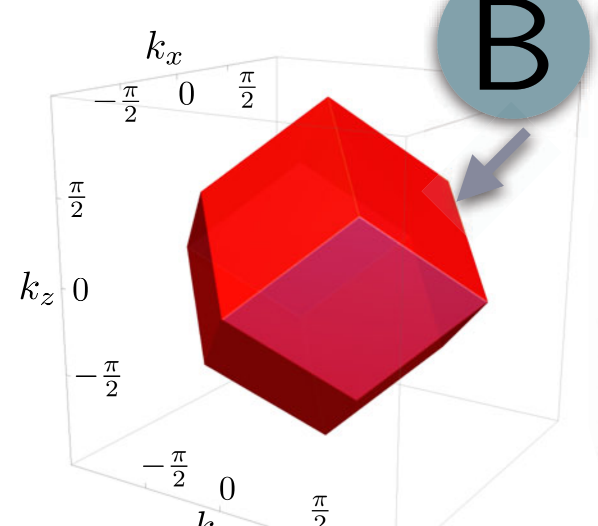
Jacobian $J_{\mathbf{n}}(\mathbf{k})$ of the map $\mathbf{n}(\mathbf{k})$: $\quad J_{\mathbf{n}}(\mathbf{k}) := \det[\partial_i n_j(\mathbf{k})] = \cos(2k_y)\lambda(\mathbf{k})$

vanishes on the set:

$$\mathsf{F} = \mathsf{G} \cup \mathsf{X},$$
$$\mathsf{X} := \{\mathbf{k} \in \mathsf{B} | \cos(2k_y) = 0\}$$
$$\mathsf{G} := \{\mathbf{k} \in \mathsf{B} | \lambda(\mathbf{k}) = 0\}$$

$$\boxed{\mathsf{B} \setminus \mathsf{F} = \bigcup_i \mathsf{B}_i, \quad \mathsf{B}_i \cap \mathsf{B}_j = \emptyset \text{ for } i \neq j}$$





Lorentz covariance: Weyl automaton

Each region B_i is diffeomorphic to the unit ball U pierced by with two semi-ellipses T_i

$$\mathbf{n}^{(0)}(\mathsf{B}_0) = \mathbf{n}^{(2)}(\mathsf{B}_2) = \mathsf{Q}_a$$

$$\mathbf{n}^{(1)}(\mathsf{B}_1) = \mathbf{n}^{(3)}(\mathsf{B}_3) = \mathsf{Q}_b$$

$\mathbf{n}^{(i)}$ restriction of $\mathbf{n}$ to B_i

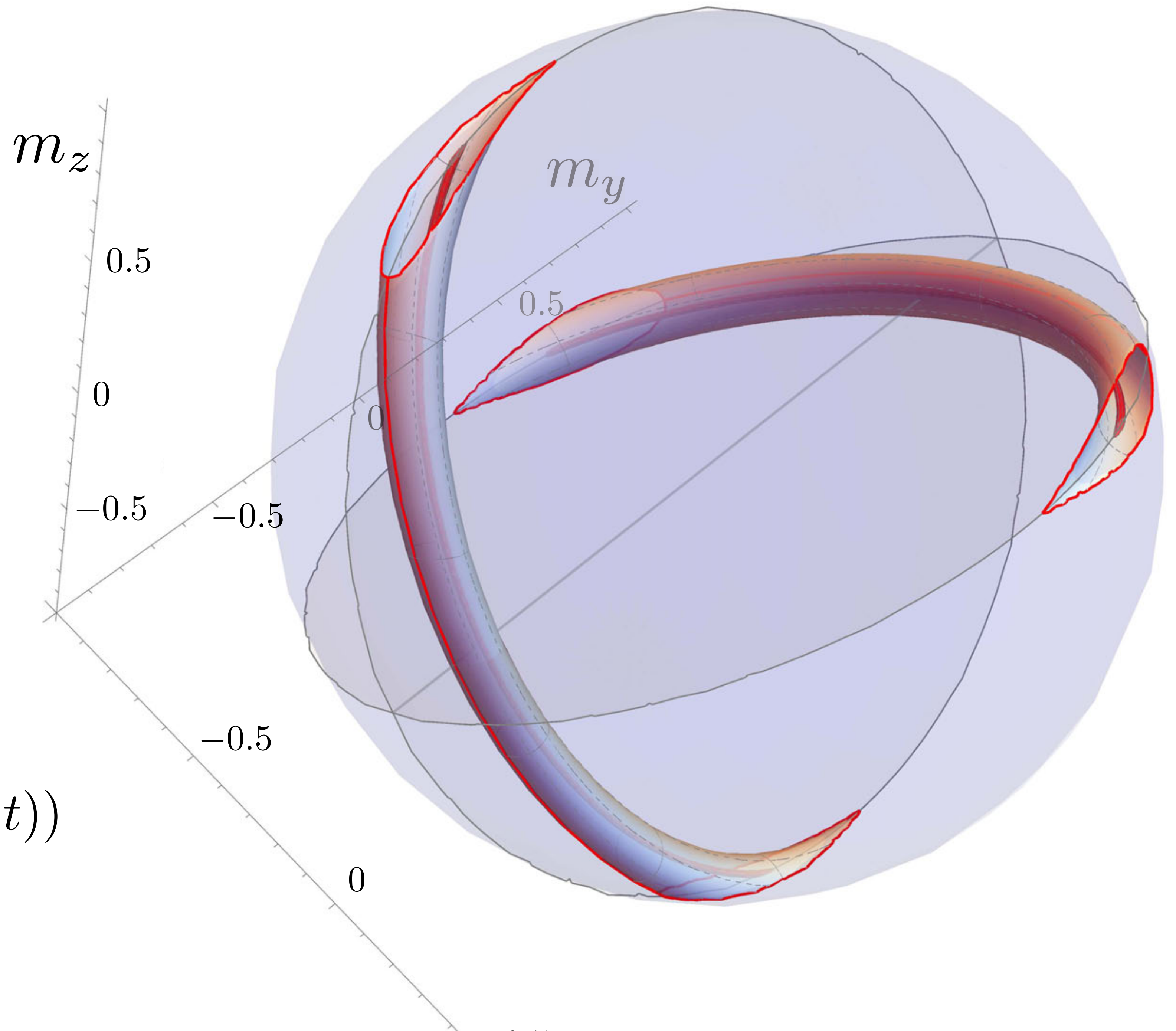
$$\mathsf{Q}_a := \mathsf{U} \setminus (\mathbf{e}_+(\mathsf{T}_1) \cup \mathbf{e}_-(\mathsf{T}_2))$$

$$\mathsf{Q}_b := \mathsf{U} \setminus (\mathbf{e}_+(\mathsf{T}_2) \cup \mathbf{e}_-(\mathsf{T}_1))$$

$$\mathsf{T}_1 := (-\tfrac{\pi}{2}, \tfrac{\pi}{2})$$

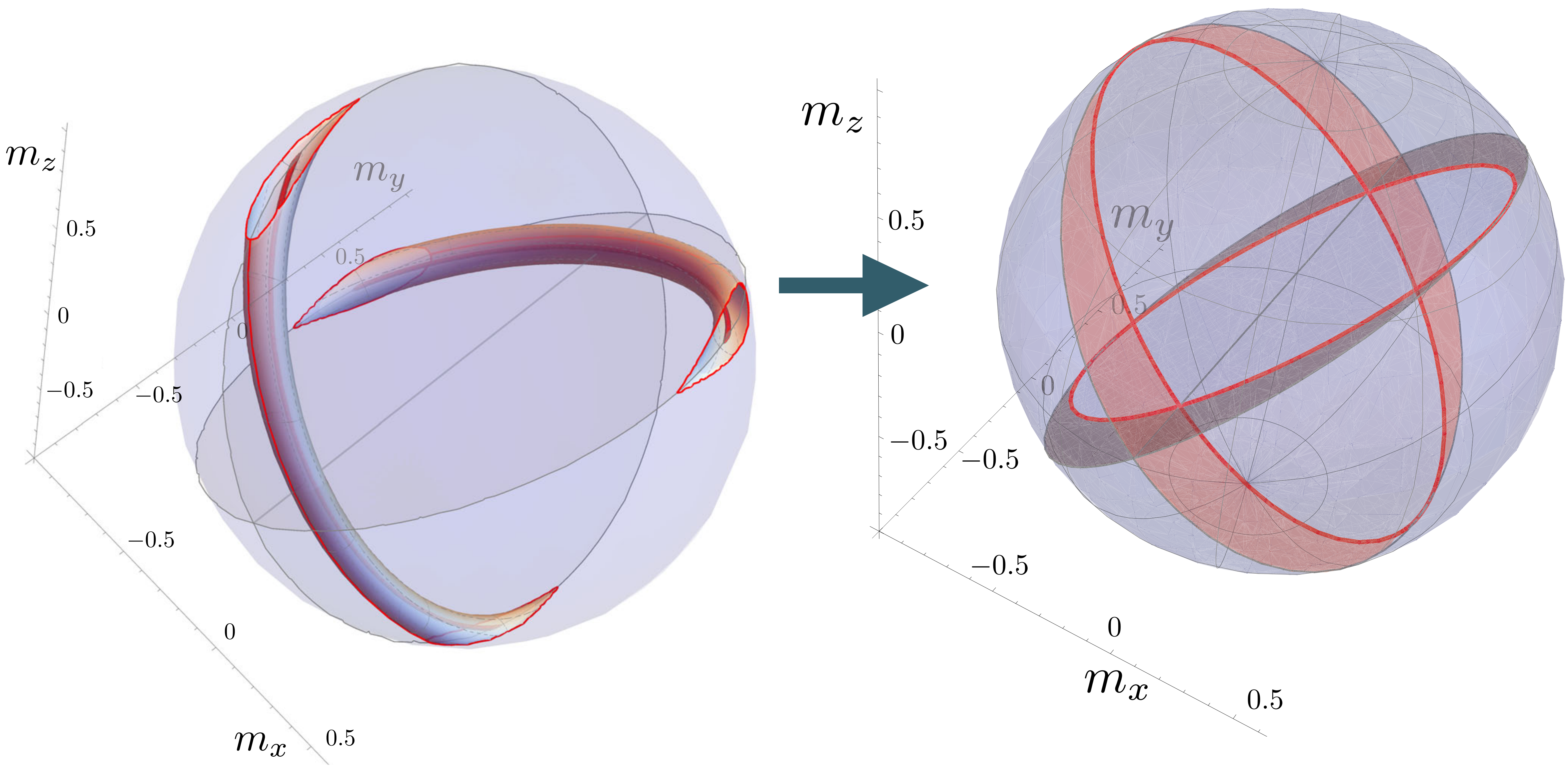
$$\mathsf{T}_2 := (-\pi, -\tfrac{\pi}{2}) \cup (\tfrac{\pi}{2}, \pi].$$

$$\mathbf{e}_\pm(t) := \frac{1}{\sqrt{2}}(\sin(t), \cos(t), \pm\sin(t))$$



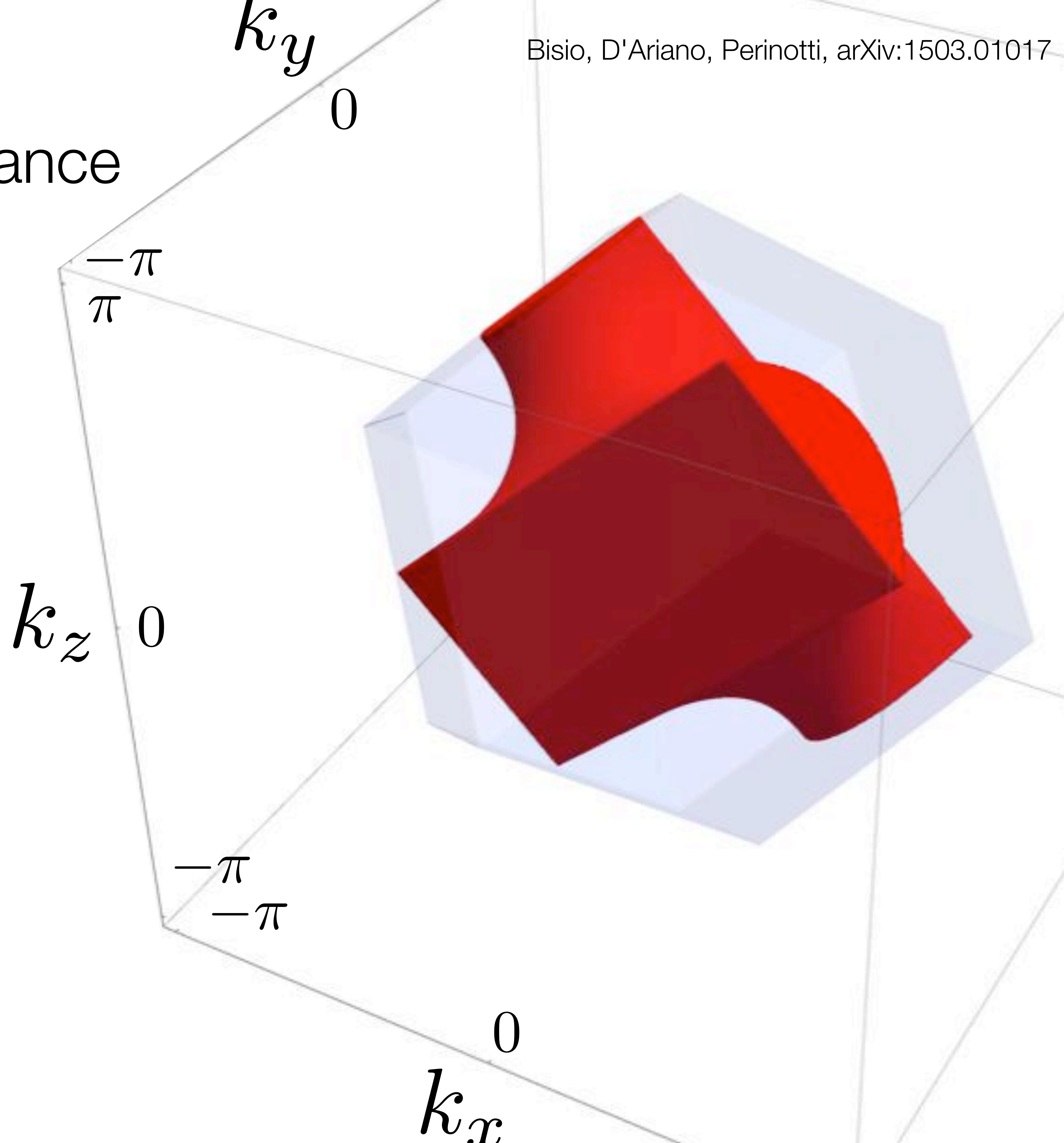
Lorentz covariance: Weyl automaton

By cutting out a zero-measure region, we get the stellate set, which is diffeomorphic to a ball



Bisio, D'Ariano, Perinotti, arXiv:1503.01017

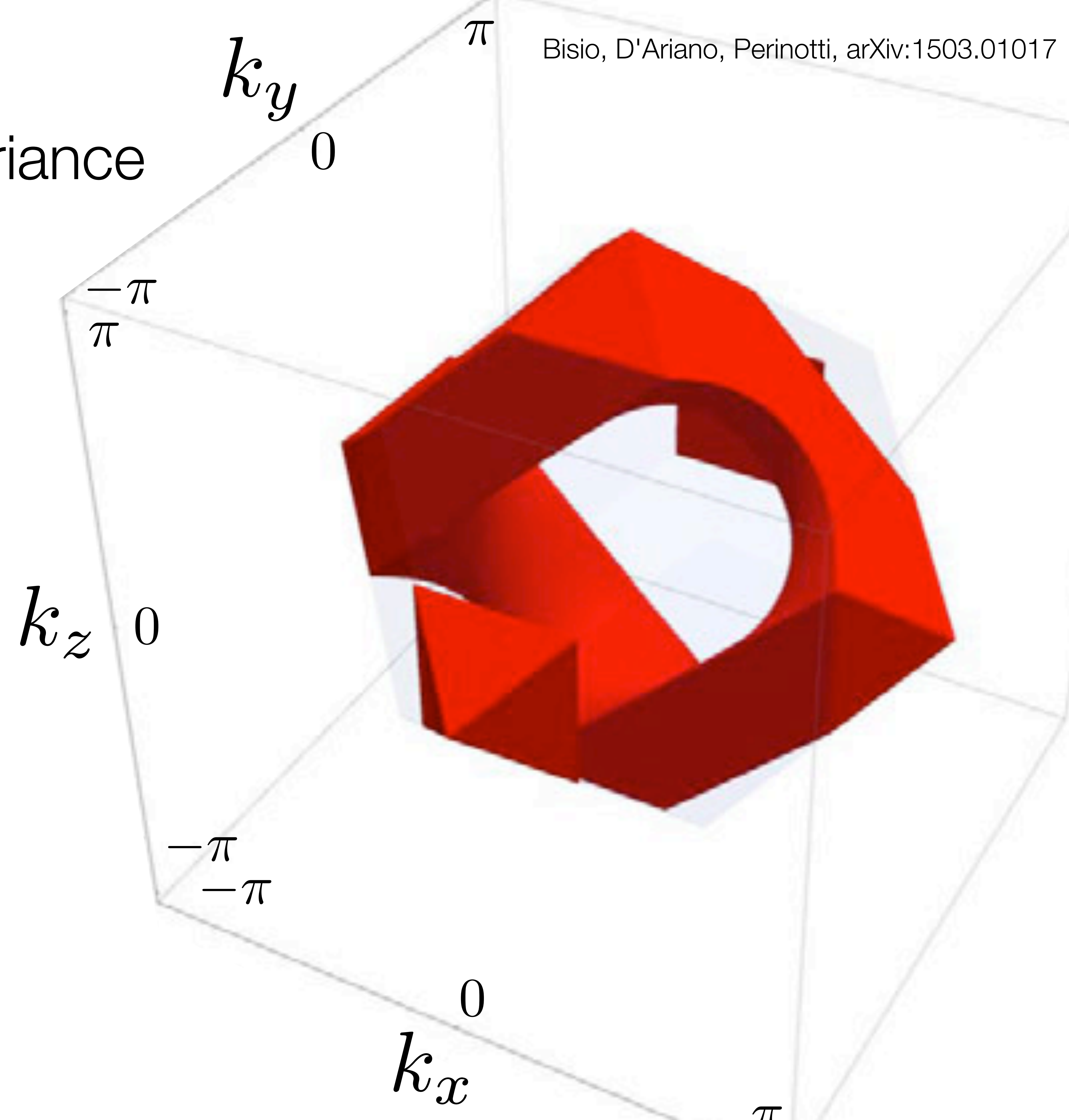
Lorentz covariance



B$_0$ zone

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

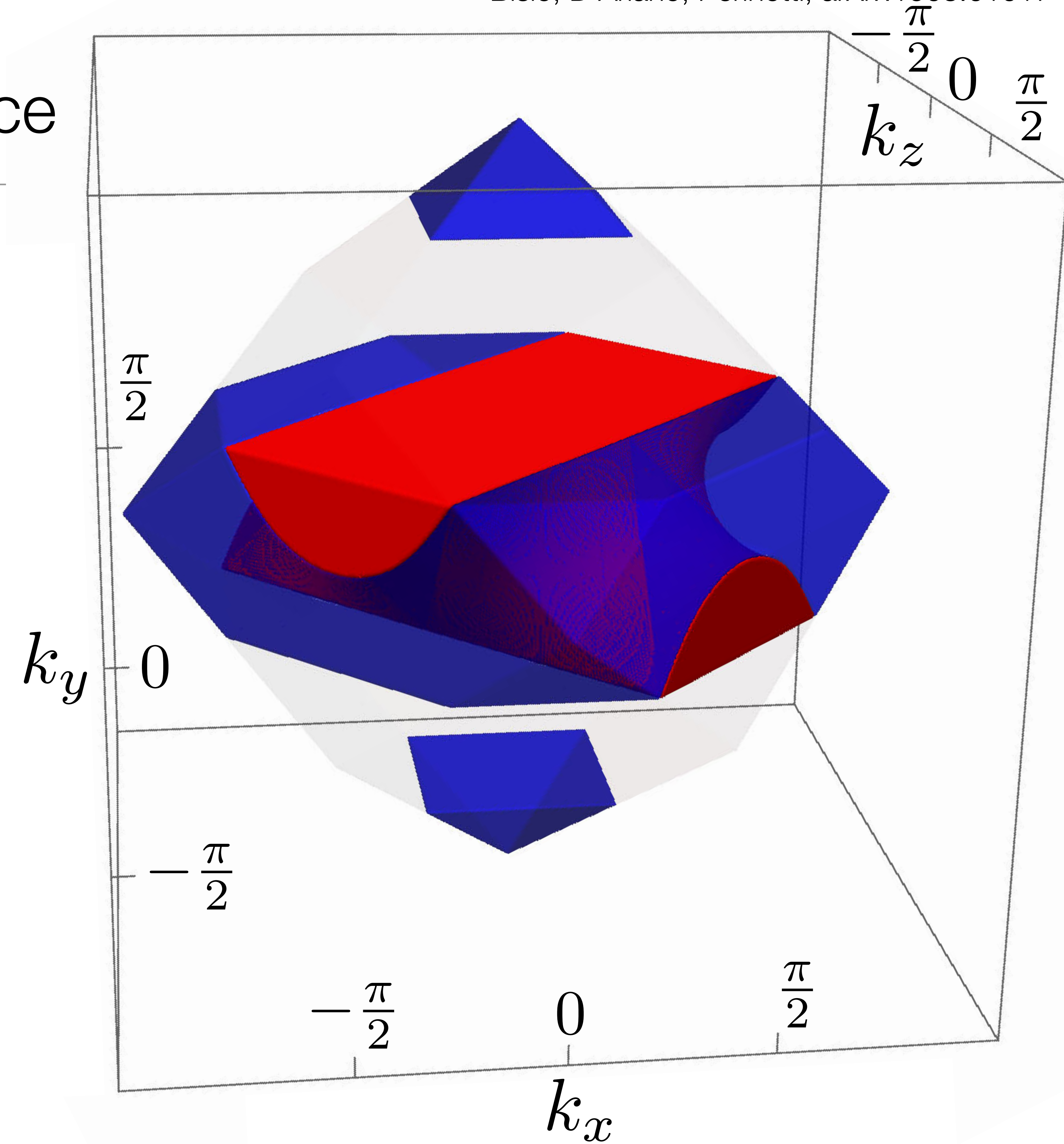
Lorentz covariance



B$_1$ zone

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

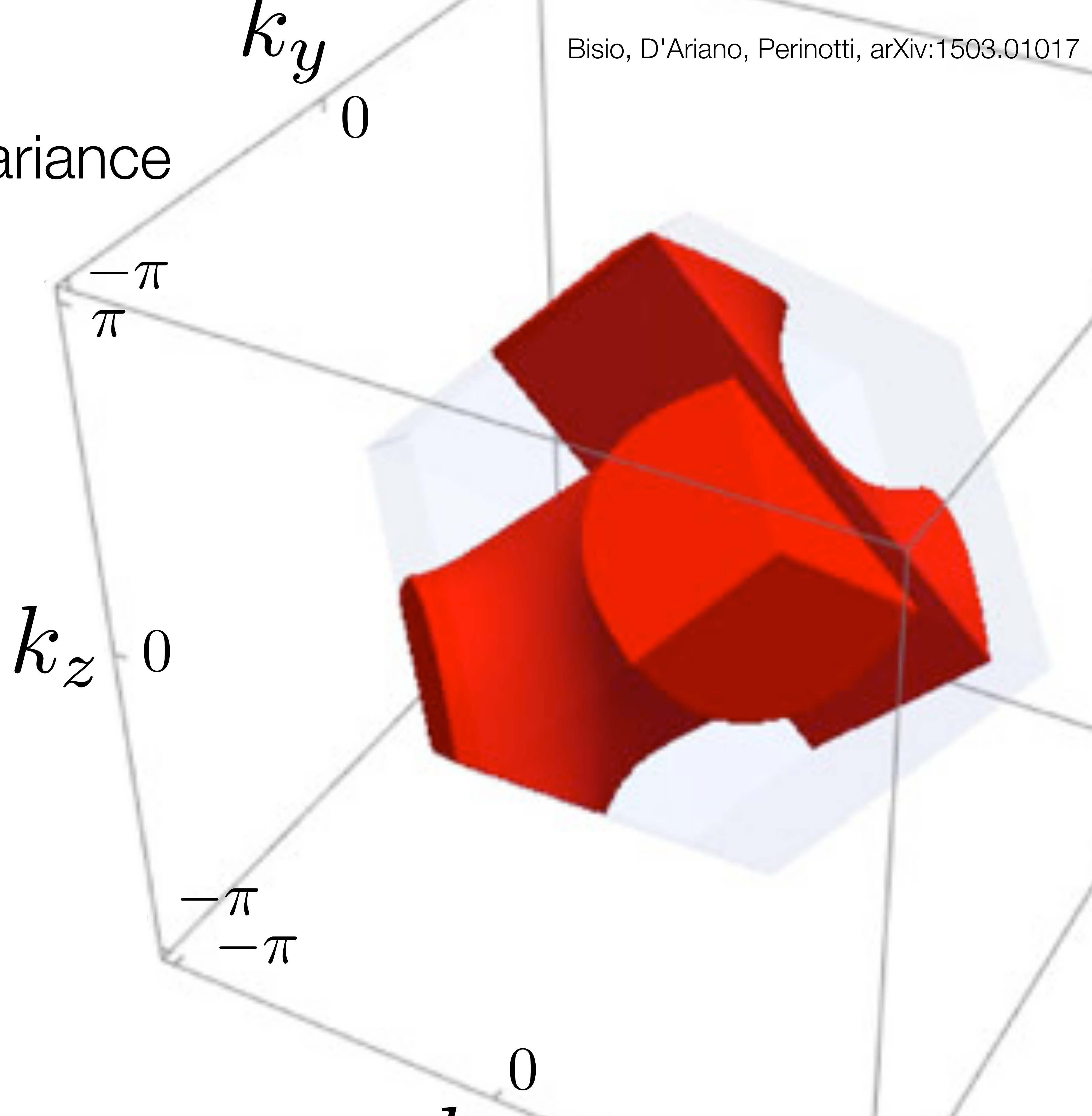
Lorentz covariance



$\mathbf{B}_0$ & $\mathbf{B}_1$ zones

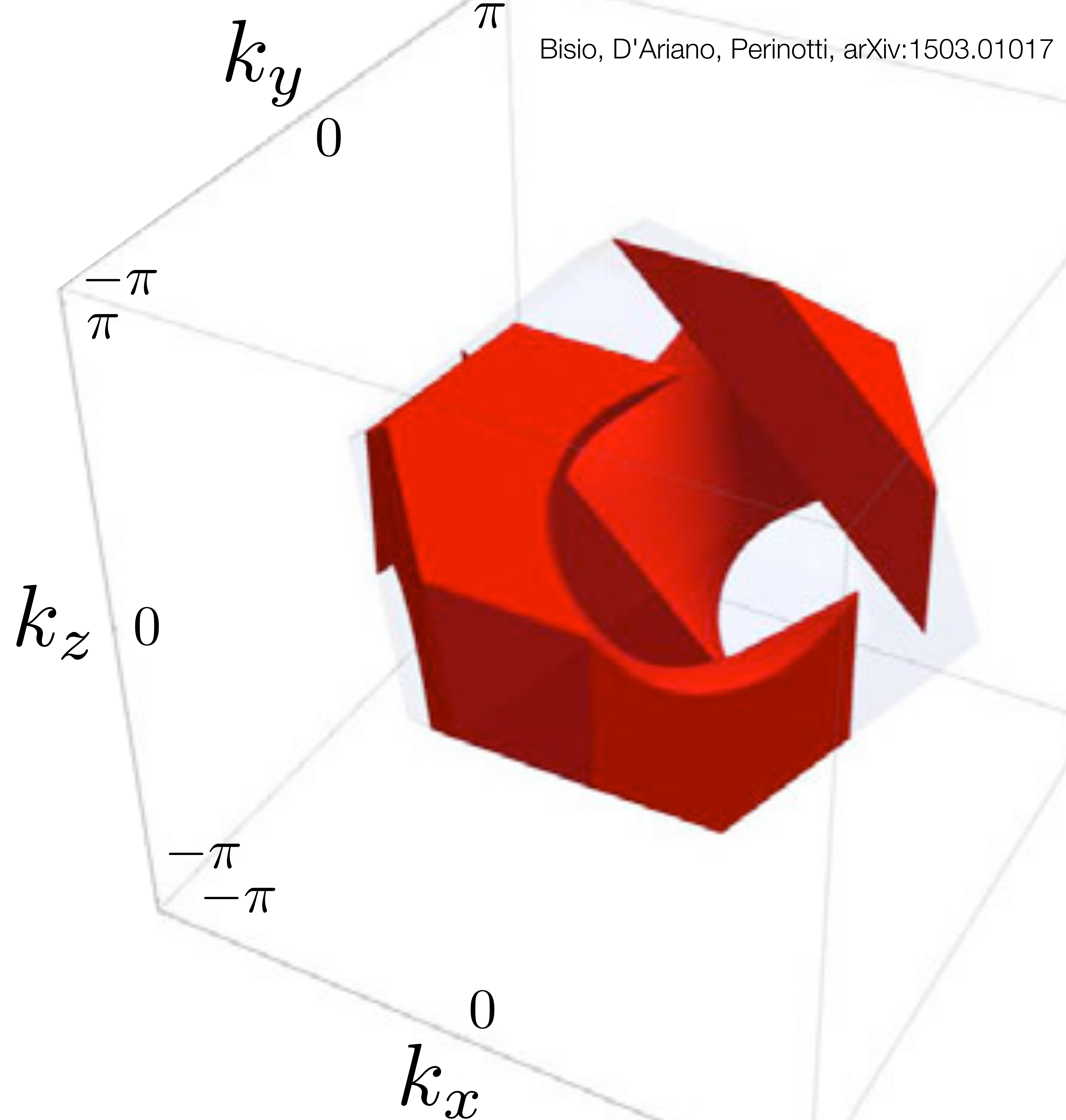
Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Lorentz covariance



B$_2$ zone

Lorentz covar

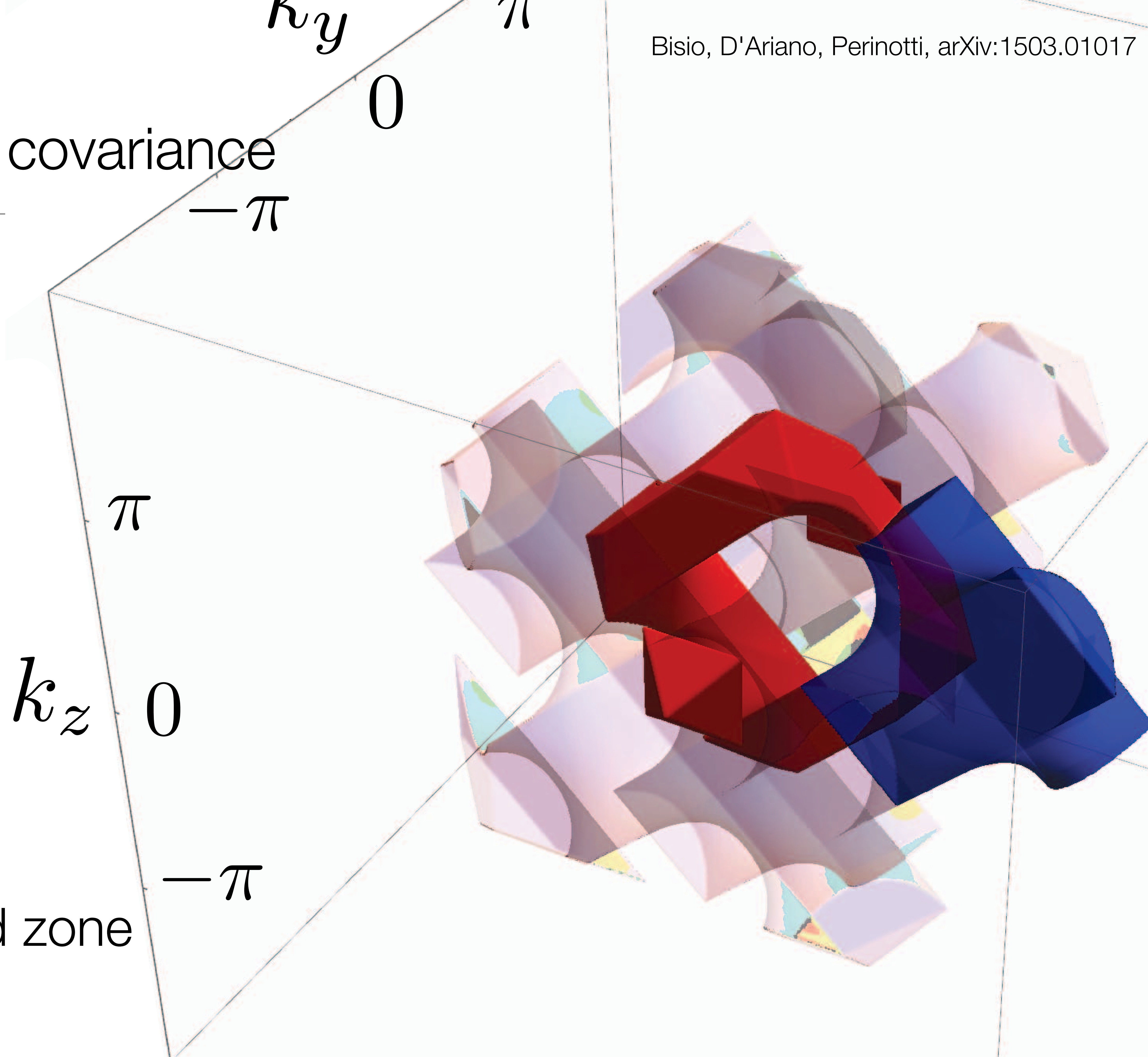


Bisio, D'Ariano, Perinotti, arXiv:1503.01017

B$_3$ zone

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Lorentz covariance

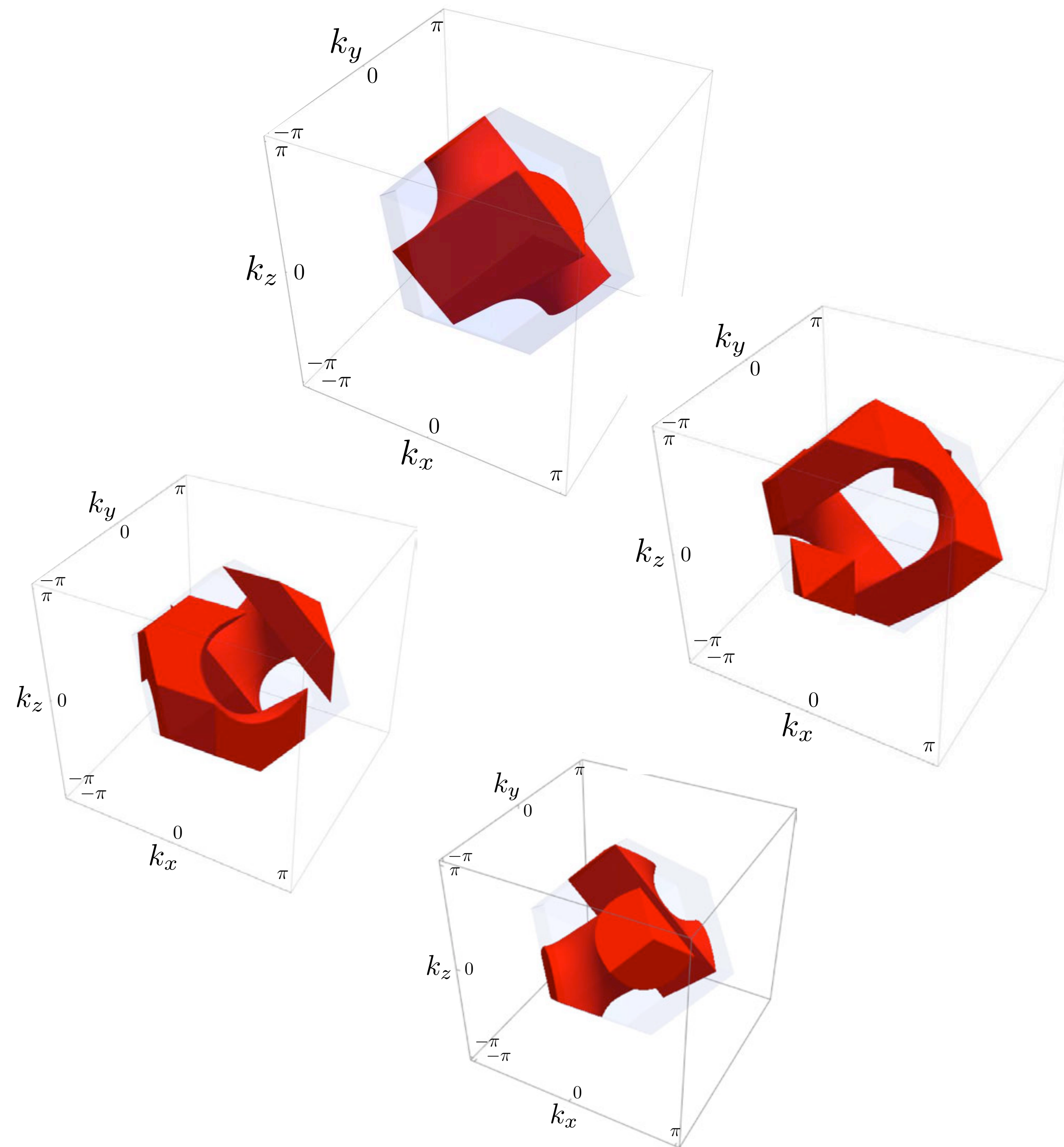


repeated zone

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Lorentz covariance: Weyl automaton

- Therefore:
    - upon defining an inertial frame as a decomposition into irreps
    - requiring the change of frame leaves the dynamics invariant
- we found four Lorentz-invariant *k*-domains
- translations in space and time leave the dynamics invariant:
    - Lorentz-invariance ➞ Poincaré invariance
- We have four different particles.



Bisio, D'Ariano, Perinotti, arXiv:1503.01017

De Sitter covariance: Dirac automaton

Covariance for Dirac QCA

$$[p_\mu(\omega, \mathbf{k}, m)\gamma^\mu - mI]\psi(\omega, \mathbf{k}, m) = 0$$

covariance cannot leave m invariant

invariance of de Sitter norm:

$$\sin^2\omega - (1 - m^2)|\mathbf{n}(\mathbf{k})|^2 - m^2 = 0$$

➡ $SO(1,4)$ invariance

One has $SO(1,4) \longrightarrow SO(1,3)$ for $m \to 0$ $\mathcal{O}(m^2)$

Conclusions

- Free QFT derived from principles (denumerable interacting quantum systems)
    - without assuming Special Relativity
    - quantum ab-initio (mechanics emergent)
- Discrete QCA theory to be regarded as a theory unifying scales from Planck to Fermi
- Fundamental notions surviving at all scales:
    - Nonlinear Lorentz group
    - Notion of particle as Poincaré invariant

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D'Ariano and Perinotti, *Derivation of the Dirac Equation from Principles of Information processing*, Phys. Rev. A **90** 062106 (2014)

Bisio, D'Ariano,Tosini, *Quantum Field as a Quantum Cellular Automaton: the Dirac free evolution in 1d*, Annals of Physics **354** 244 (2015)

D'Ariano, Mosco, Perinotti, Tosini, *Path-integral solution of the one-dimensional Dirac quantum cellular automaton,* PLA **378** 3165 (2014)

D'Ariano, Mosco, Perinotti, Tosini, *Discrete Feynman propagator for the Weyl quantum walk in 2 + 1 dimensions,* EPL **109** 40012 (2015)

D'Ariano, Manessi, Perinotti, Tosini, *The Feynman problem and Fermionic entanglement …,* Int. J. Mod. Phys. **A17** 1430025 (2014)

Bibeau-Delisle, Bisio, D'Ariano, Perinotti, Tosini, *Doubly-Special Relativity from Quantum Cellular Automata,* EPL (in press)

Bisio, D'Ariano, Perinotti, *Quantum Cellular Automaton Theory of Light,* arXiv:1407.6928

Bisio, D'Ariano, Perinotti, *Lorentz symmetry for 3d Quantum Cellular Automata,* arXiv:1503.01017

D'Ariano, *A Quantum Digital Universe,* Il Nuovo Saggiatore **28** 13 (2012)

D'Ariano, *The Quantum Field as a Quantum Computer,* Phys. Lett. A **376** 697 (2012)

D'Ariano, *Physics as Information Processing*, AIP CP1327 7 (2011)

D'Ariano, *On the "principle of the quantumness", the quantumness of Relativity, and the computational grand-unification*, in AIP CP1232 (2010)