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Information-theoretic principles for Quantum Theory and for Quantum Field theory

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2014 Sydney Meeting on Quantum Foundations

April 3rd 2014

G. M. D'Ariano and P. Perinotti, arXiv:1306.1934

A. Bibeau-Delisle, A. Bisio, G. M. D'Ariano, P. Perinotti, A. Tosini, arXiv:1310.6760

A. Bisio, G. M. D'Ariano, A. Tosini, arXiv:1212.2839

Historical background

- The experience in Quantum Information has led us to look at Quantum Theory (QT) under a completely new angle
- QT is a *theory of information*
- Address now the *Mechanics* side of the *Quantum*
- *Mechanics* via *Quantum Field Theory* (QFT)
- QFT as countably many quantum systems in interaction

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification*

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Book from CUP (by the end of 2014)

Principles for Quantum Theory

The *informational* framework

Logic ⊂ Probability ⊂ OPT

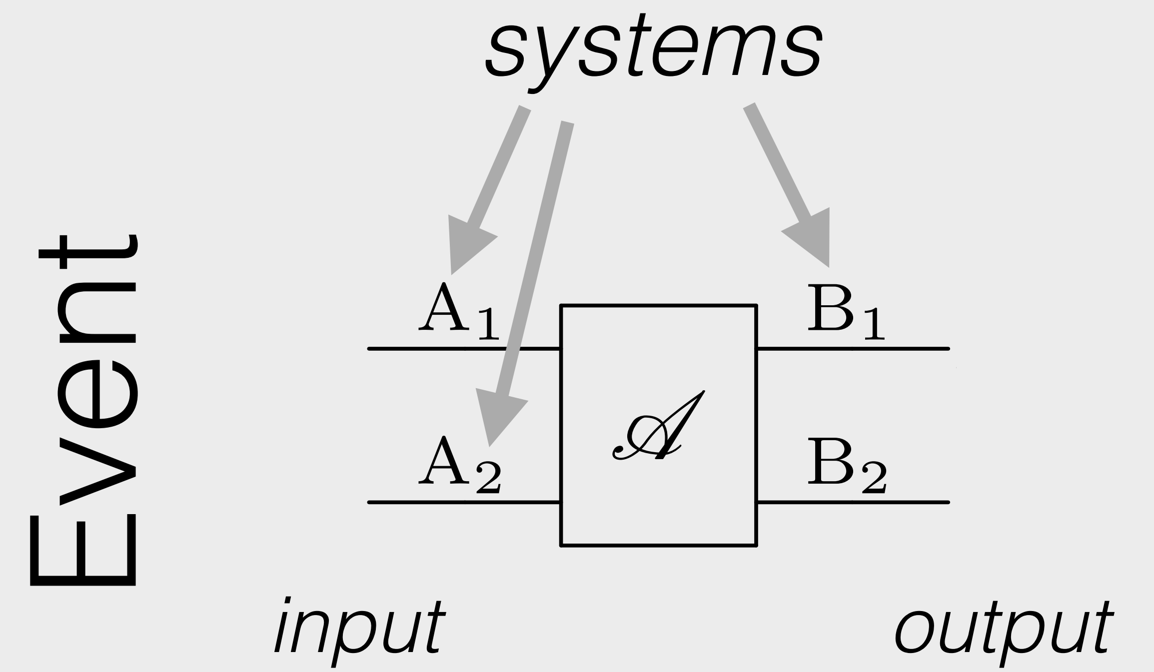
joint probabilities + connectivity

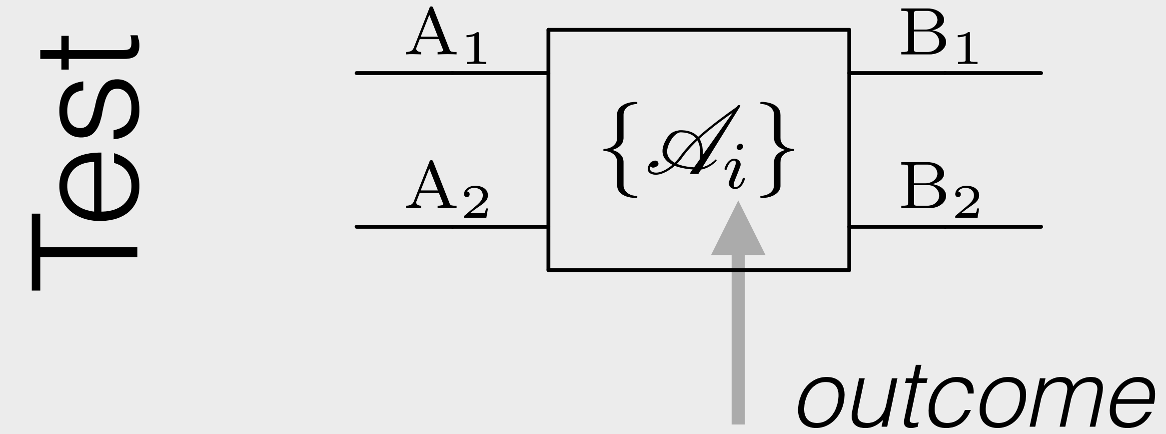
$$p(i, j, k, ...|\text{circuit})$$

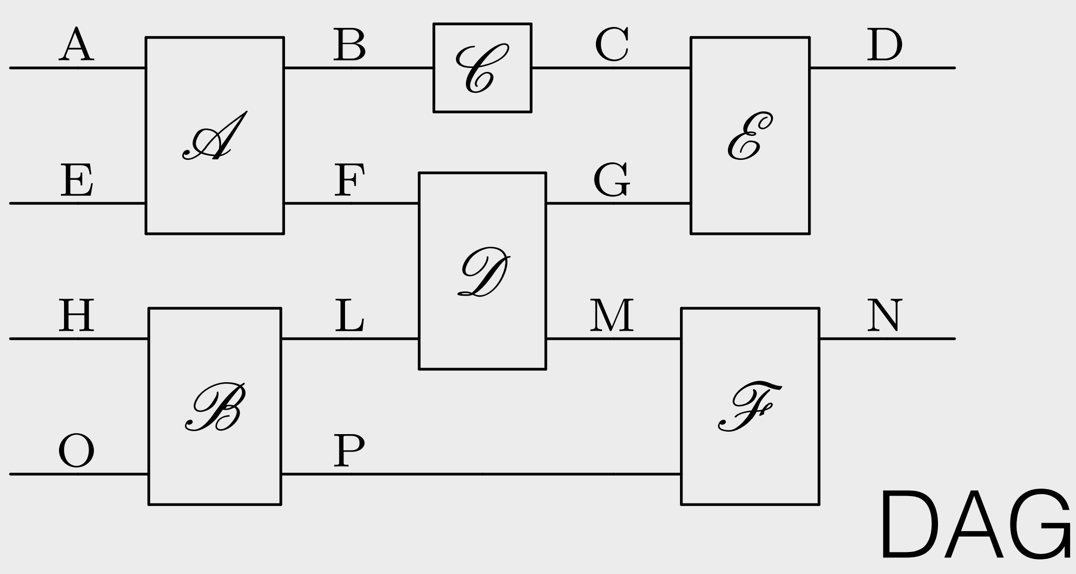
Marginal probability

$$\sum_{i,k,...} p(i, j, k, ...|\text{circuit}) =$$

$$p(j|\text{circuit})$$







Principles for Quantum Theory

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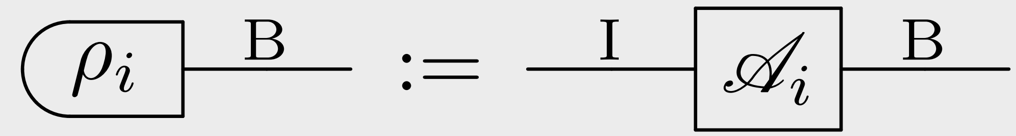
Logic ⊂ Probability ⊂ OPT

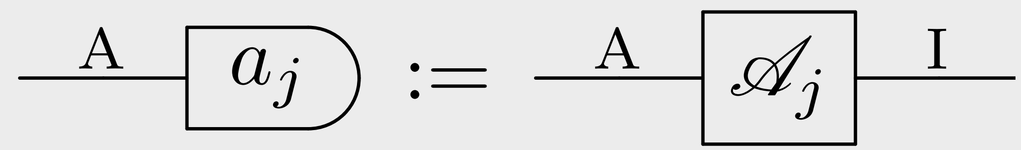
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Maximal set of
<u>NOT</u> independent systems
= *"leaf"*

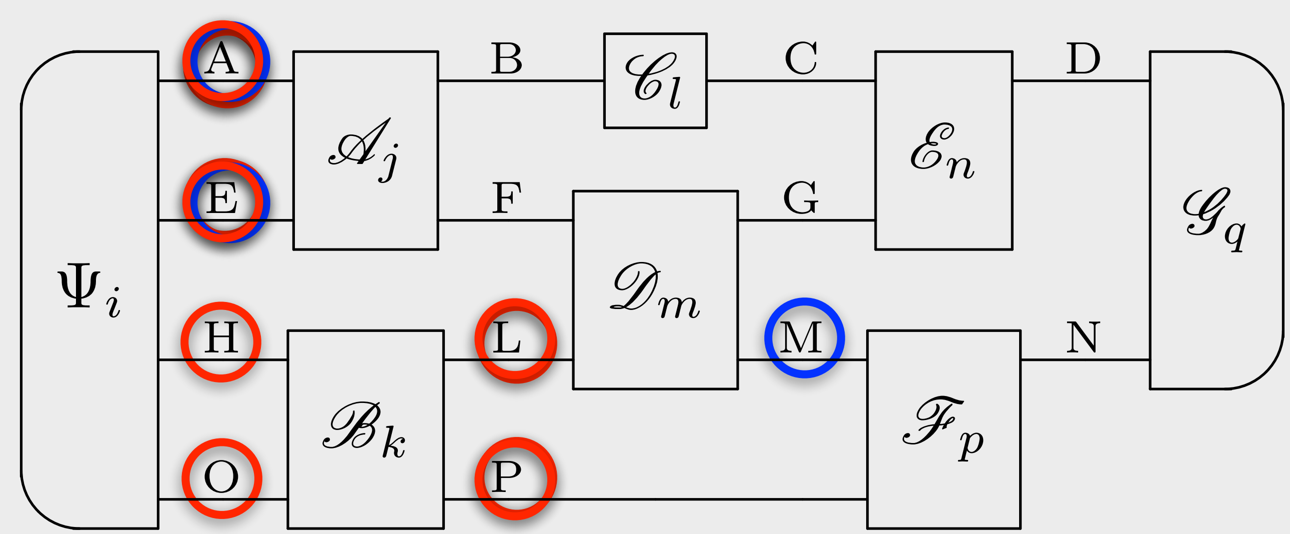


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Principles for Quantum Theory

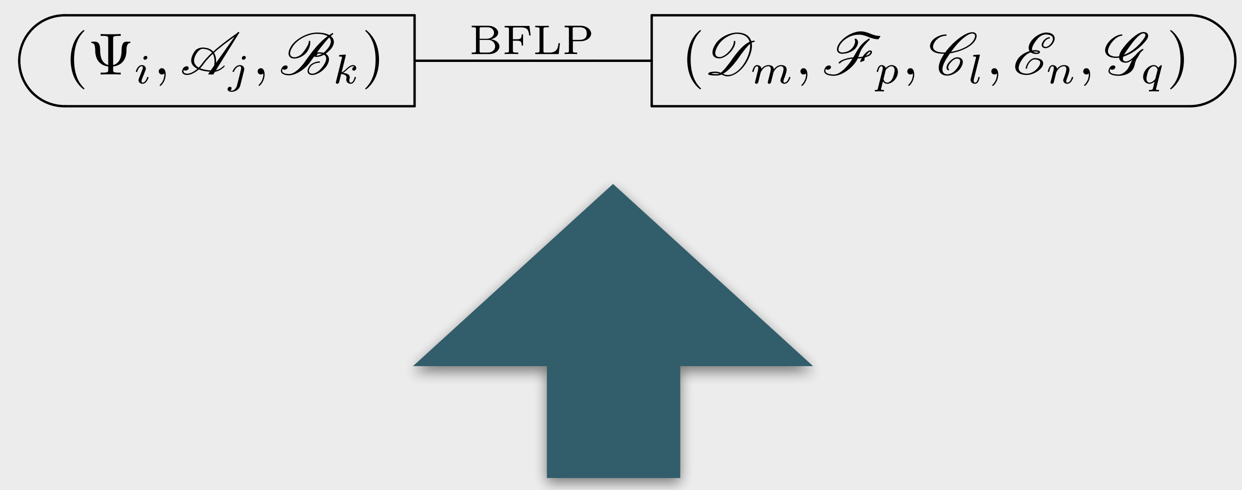
The *informational* framework

Logic ⊂ Probability ⊂ OPT

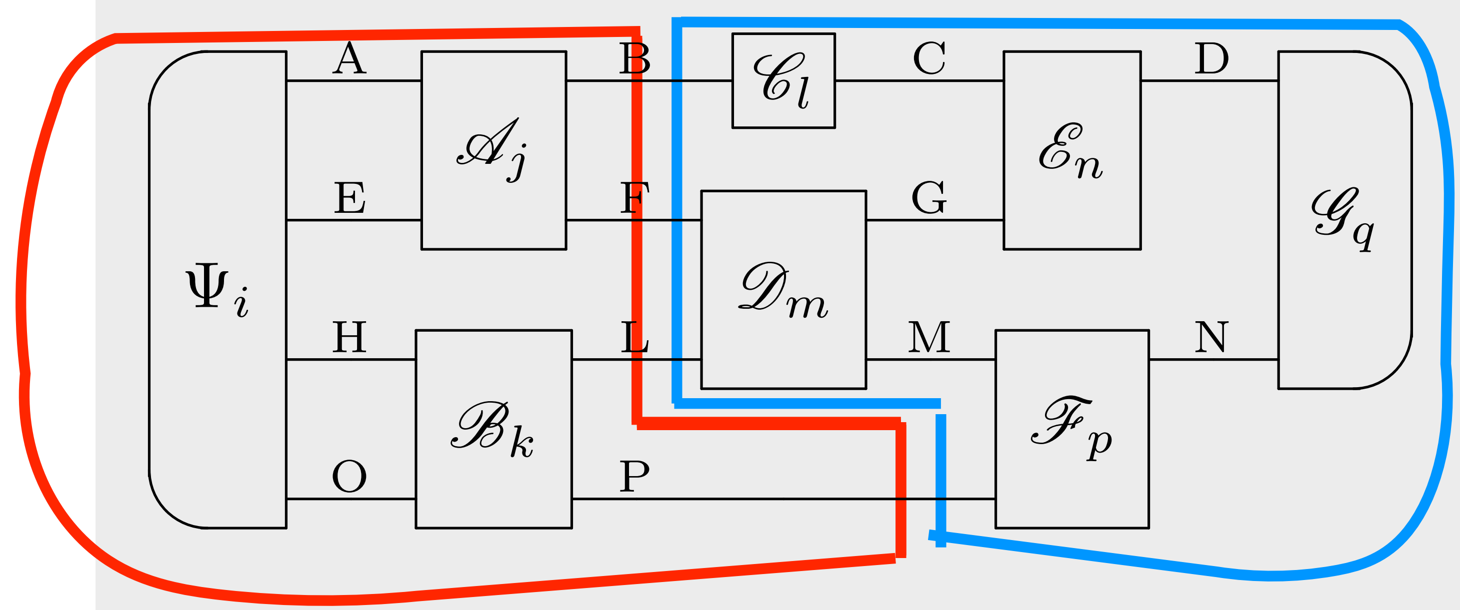
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Maximal set of
<u>NOT</u> independent systems
= "leaf"



$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Principles for Quantum Theory

The *informational* framework

Logic ⊂ Probability ⊂ OPT

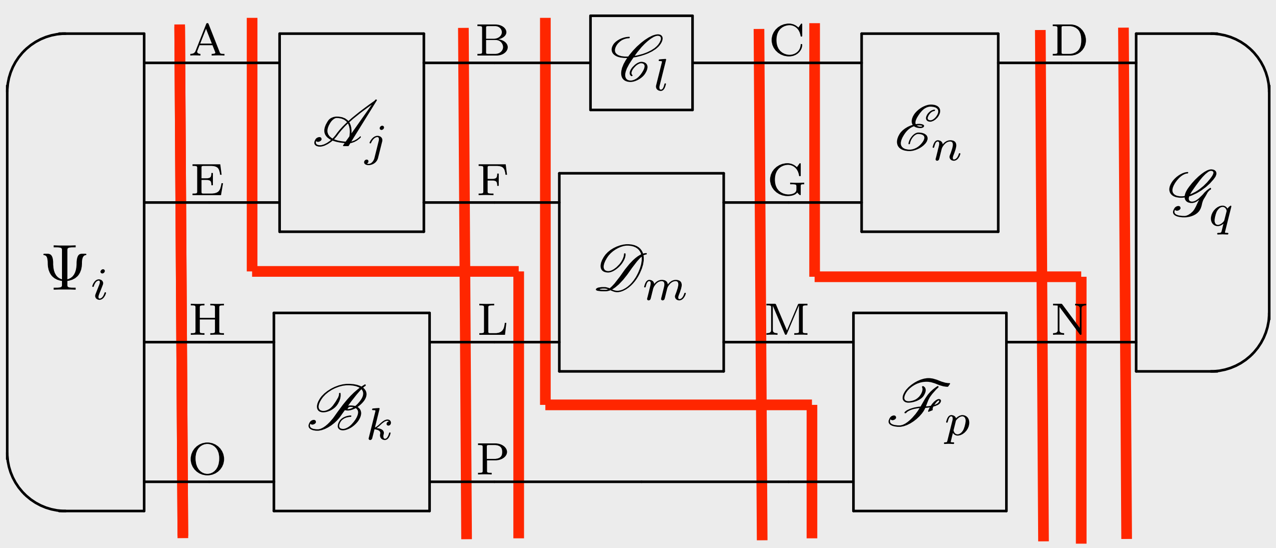
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Maximal set of independent systems = *"leaf"*

Foliation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



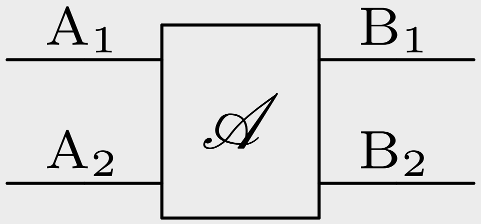
Principles for Quantum Theory

The *informational* framework

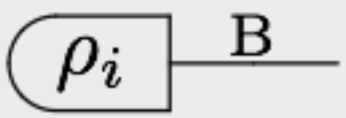
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

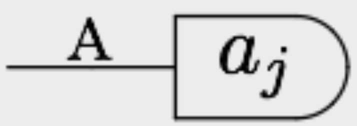
Probabilistic equivalence classes



transformation

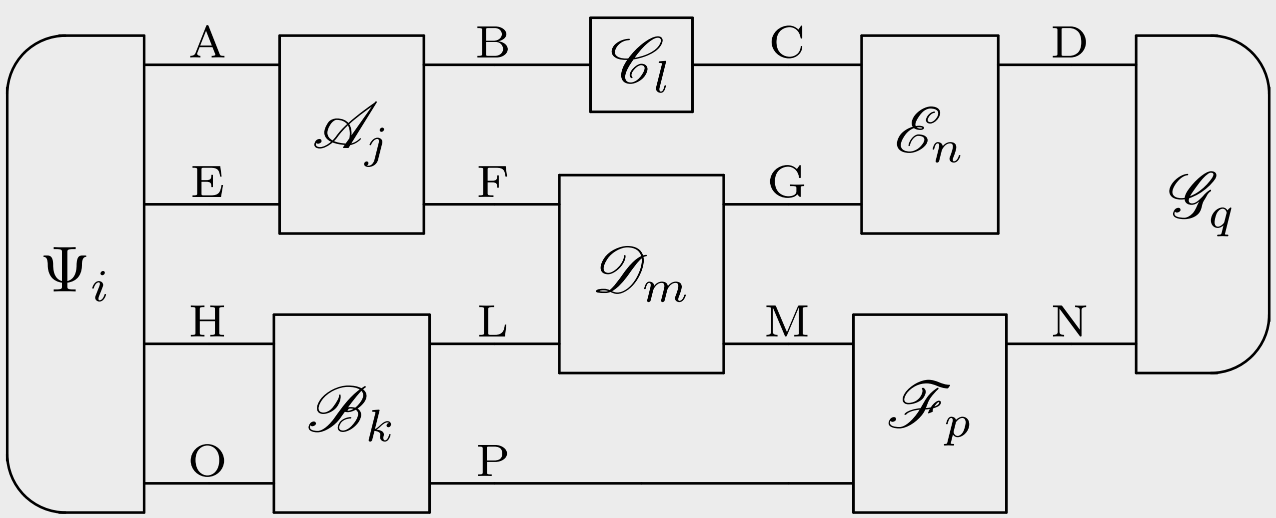


state



effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Principles for Quantum Theory

P1. Causality → convexity

P2. Local discriminability

P3. Purification

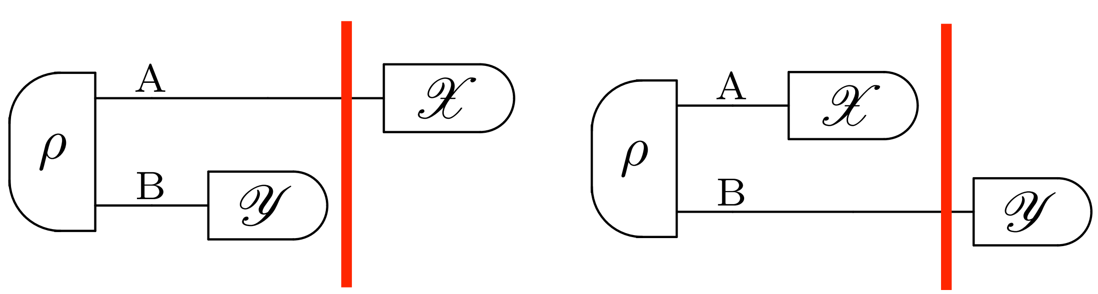
P4. Atomicity of composition

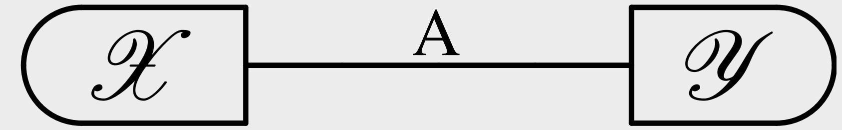
P5. Perfect distinguishability

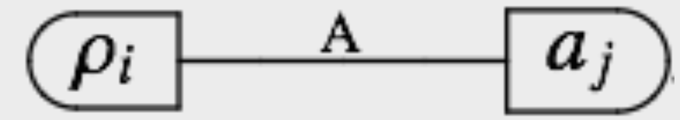
P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

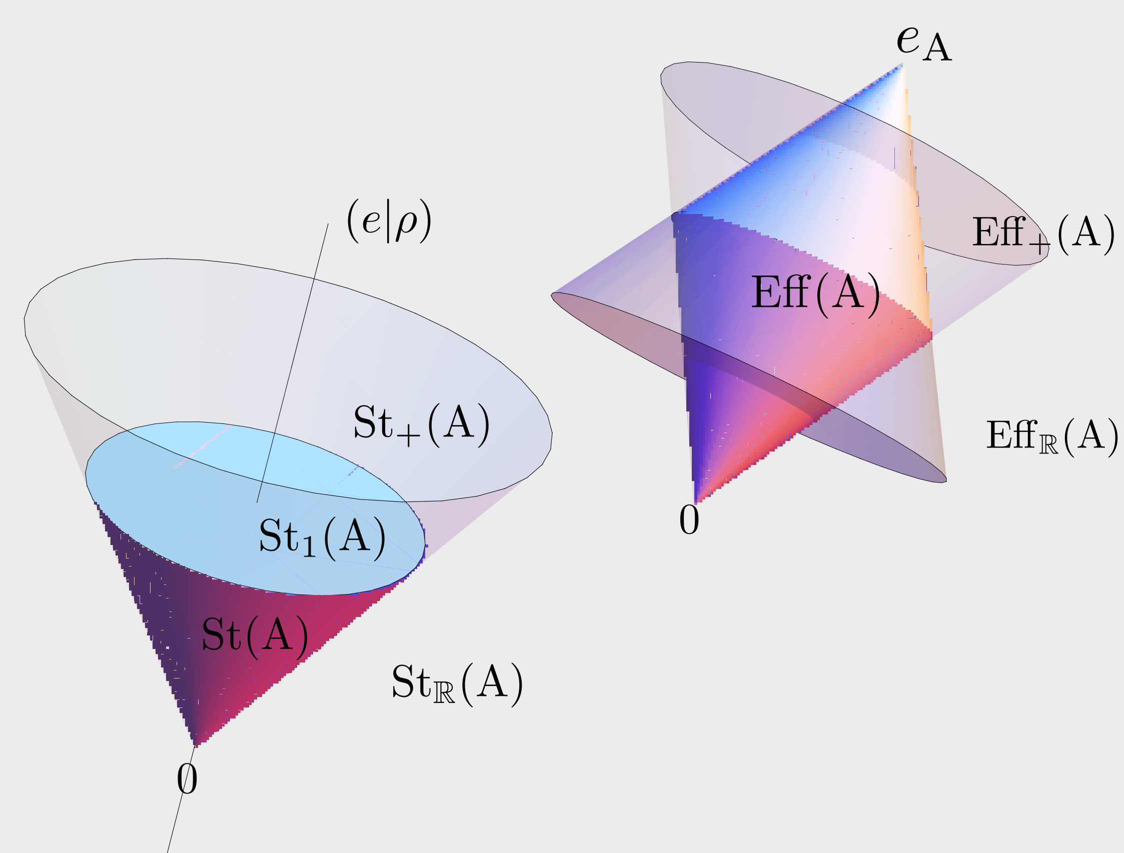
no signaling without interaction



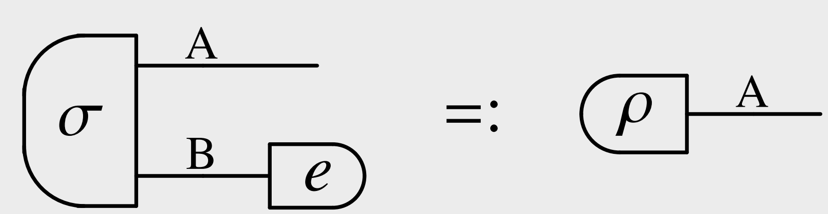


$p(i, j|\mathscr{X}, \mathscr{Y}) :=$ 

$$p(i|\mathscr{X}, \mathscr{Y}) = p(i|\mathscr{X}, \mathscr{Y}') = p(i|\mathscr{X})$$



marginal state



Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
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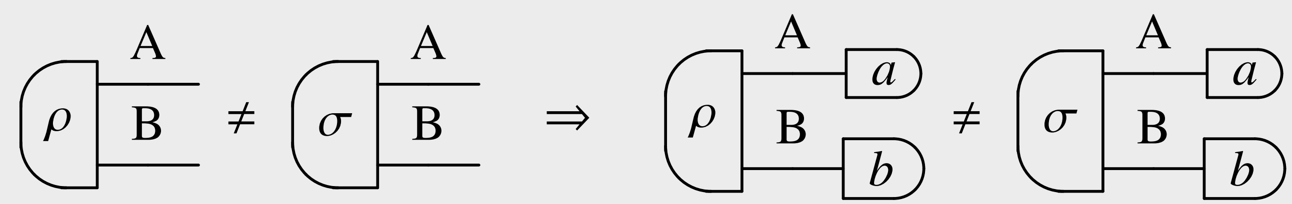
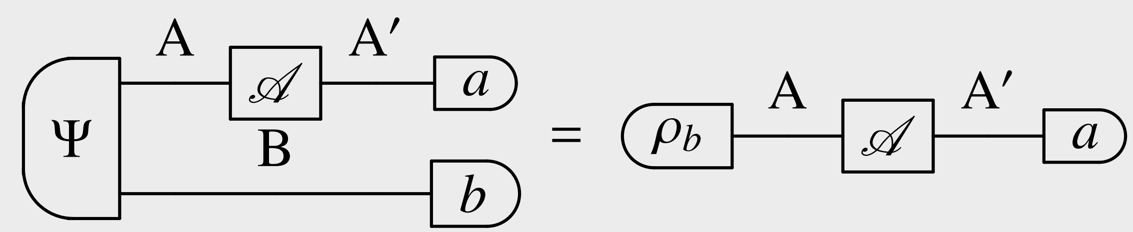
P6. Lossless Compressibility

It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product



Local characterization of transformations



Principles for Quantum Theory

P1. Causality
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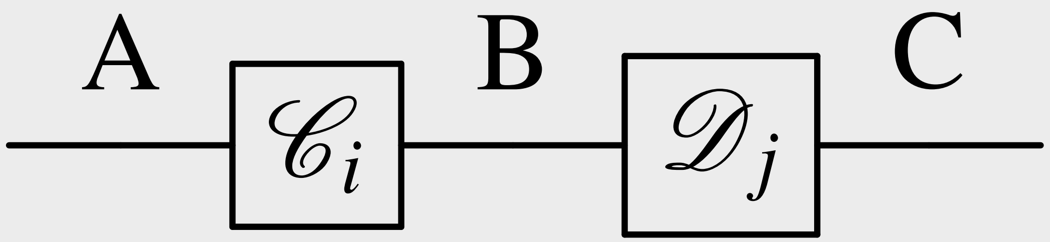
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis



Principles for Quantum Theory

P1. Causality
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P3. Purification
P4. Atomicity of composition
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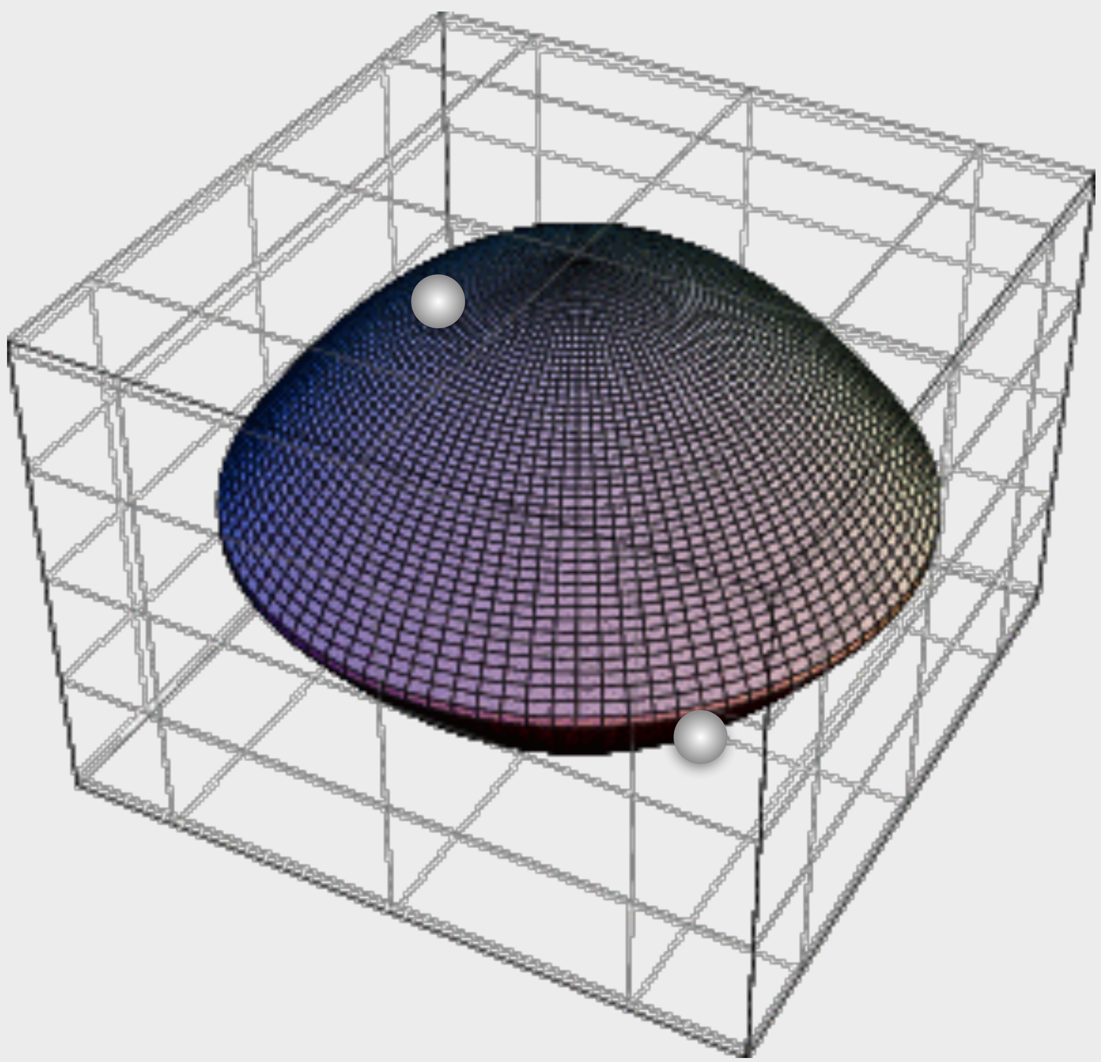
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory



Principles for Quantum Theory

P1. Causality
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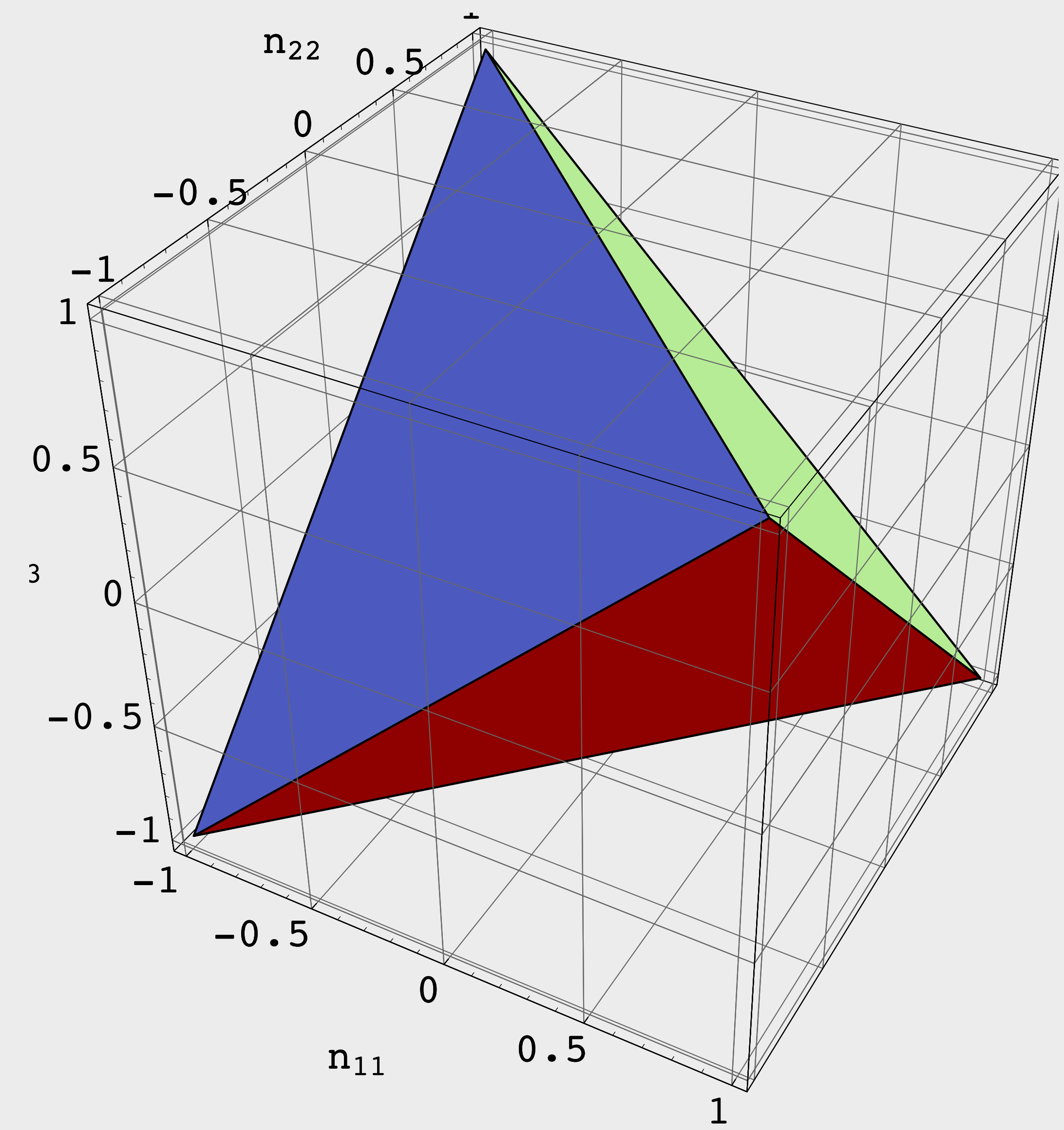
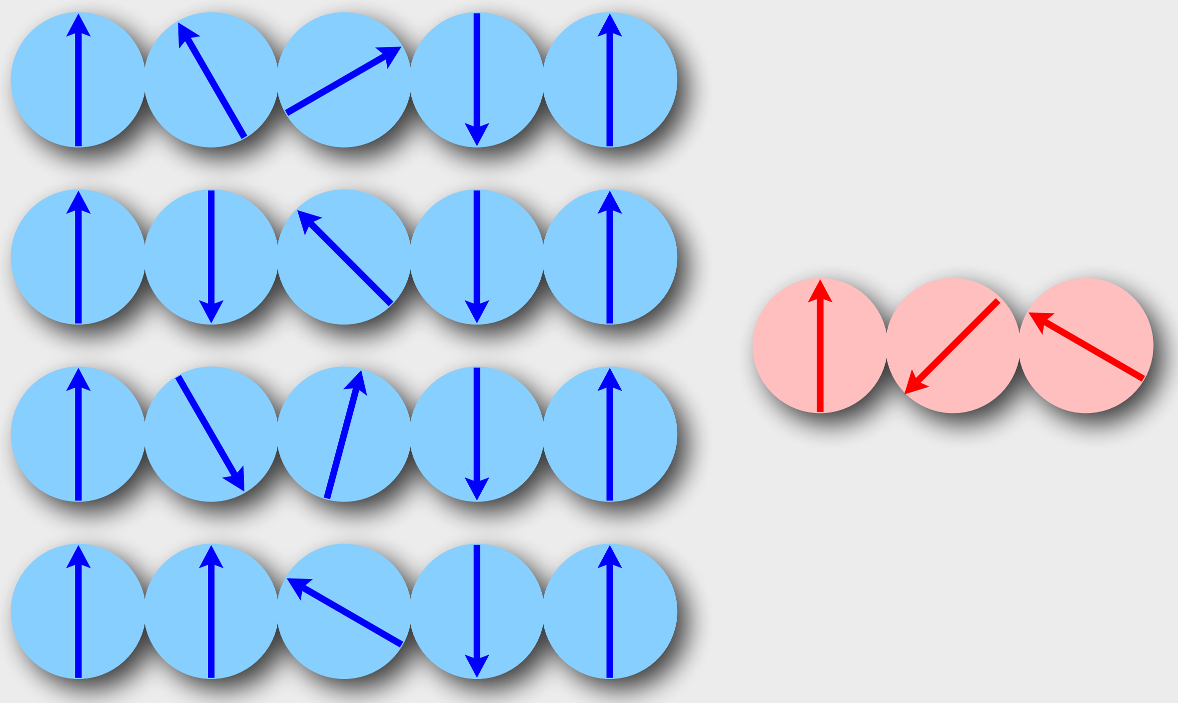
For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system





Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

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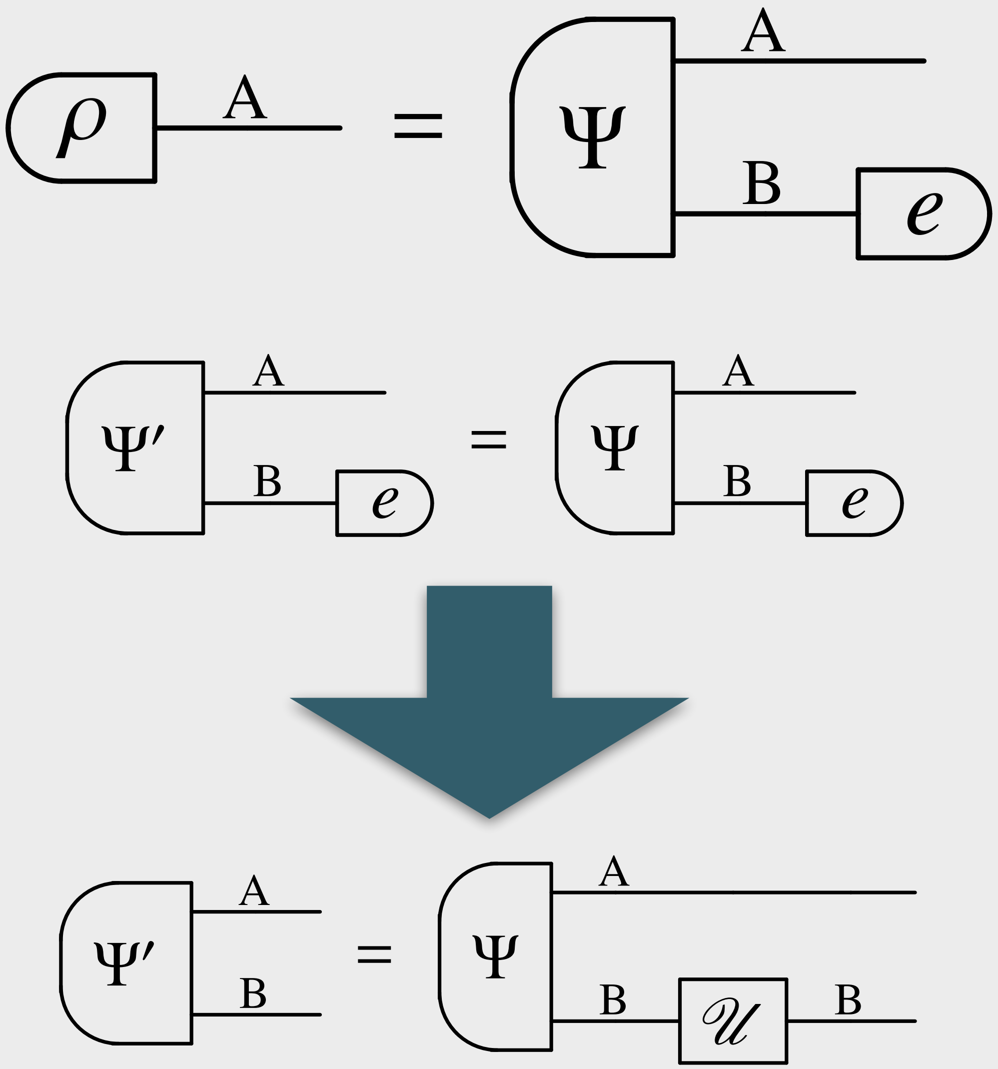
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. $\boxed{\text{Purification}}$

P4. Atomicity of composition

P5. Perfect distinguishability

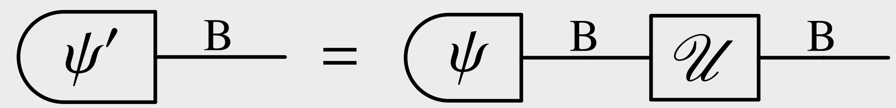
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

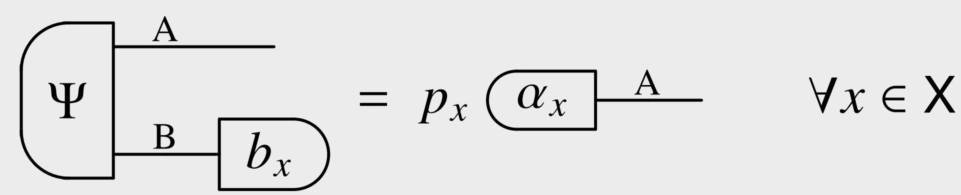
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

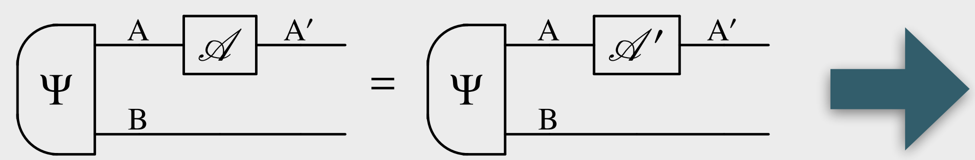
2. *Every two normalized pure states of the same system are connected by a reversible transformation*



3. ***Steering:*** Let Ψ purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



4. ***Process tomography (faithful state):***



$$\mathscr{A}\rho = \mathscr{A}'\rho \quad \forall \rho$$

5. ***No information without disturbance***

Principles for Quantum Theory

P1. Causality

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P3. Purification

P4. Atomicity of composition

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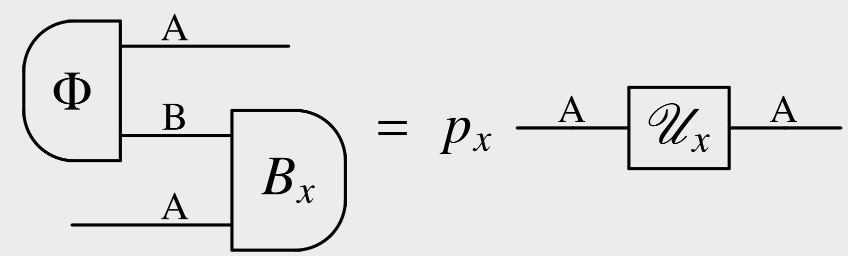
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

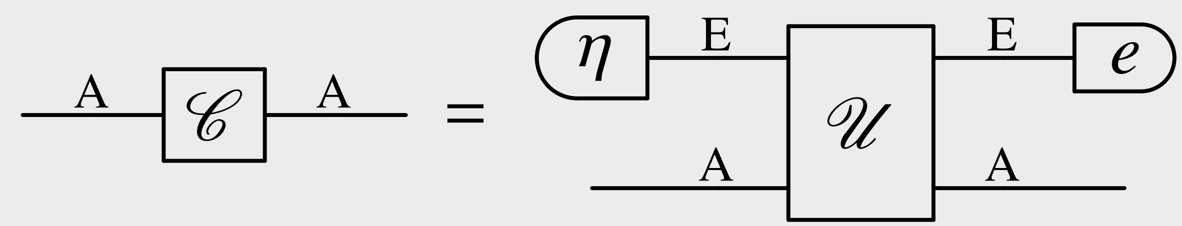
Consequences

6. ***Teleportation***

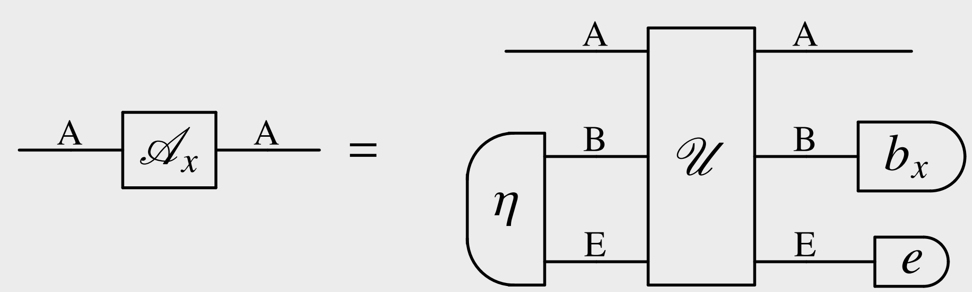


$\forall x \in \mathrm{X}$

7. ***Reversible dilation of "channels"***



8. ***Reversible dilation of "instruments"***



$\forall x \in \mathrm{X}$

9. ***State-transformation cone isomorphism***

10. ***Rev. transform. for a system make a group***

Moving to the *Mechanics*

- The Weyl, Dirac, and Maxwell equations are derived from information-theoretic principles only, <u>without assuming SR</u>

- QCA theory to be regarded as a theory unifying scales from Planck to Fermi (no continuum limit!)

- QFT is recovered in the *relativistic limit* ($k \ll 1$)
- In the *ultra-relativistic limit* (Planck scale) Lorentz covariance is an approximate symmetry, and one has the *Doubly Special Relativity* of Amelino-Camelia/Smolin/Magueijo

Additional principles

- linearity
- unitarity
- locality
- homogeneity
- isotropy
- minimal-dimension and transitivity

Quantum Cellular Automaton (QCA)

Minimal algorithmic complexity of the information processing

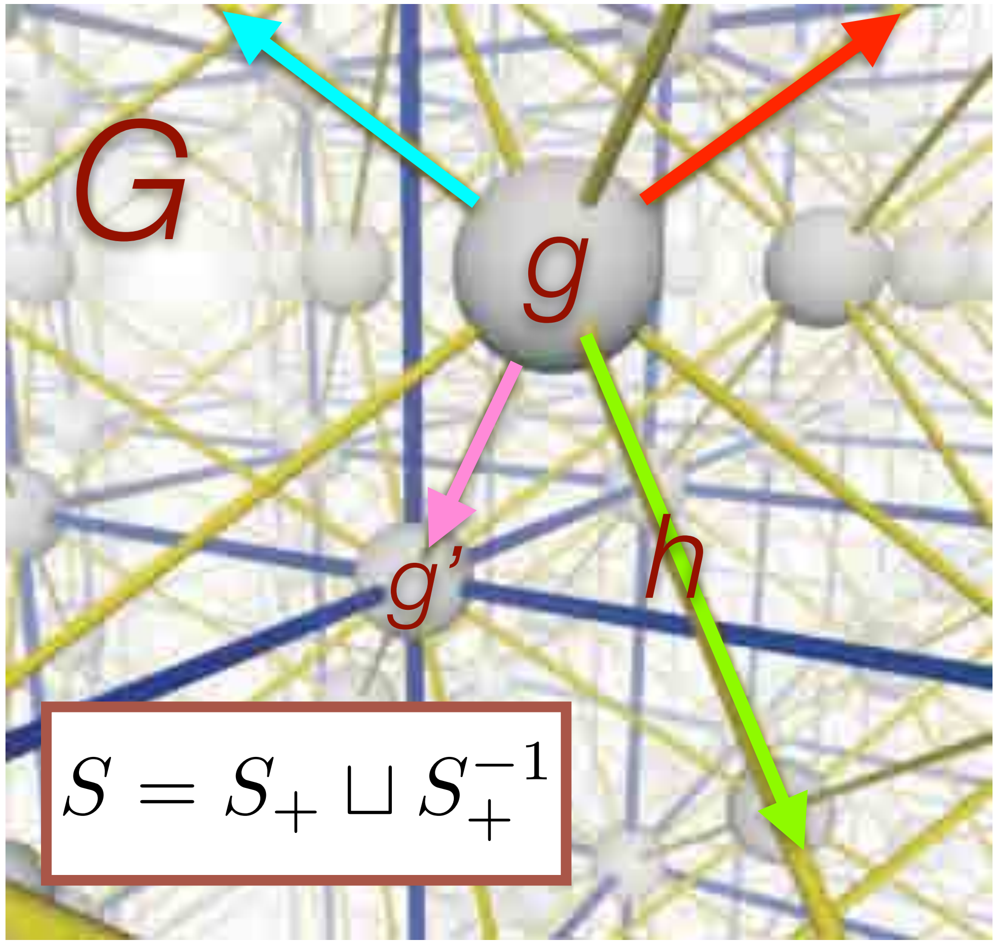
GOOD FEATURES

1. **no SR assumed**: emergence of relativistic quantum field and space-time
2. **quantum *ab-initio***
3. no divergencies and all the problems from the continuum
4. no “violations” of causality
5. computable
6. dynamics stable (dispersive Schrödinger equation for narrow-band states valid at all scales)
7. solves the problem of localization in QFT
8. natural scenario for the *holographic principle*

Quantum Cellular Automaton

Reduce QFT to just interactions among identical quantum systems in a denumerable (infinite) set G (no background and no SR assumed)

- System g is $\psi(g)$, ψ s-dimensional *field operator*, $g \in G$
- ***Minimal-dimension:*** $s>1$ ($s=1$ trivial evolution)
- ***linearity:*** Interactions described by transition matrices $A_{gg'} \in M_s(\mathbb{C})$ between systems $g \in G$:
  single evolution step $\psi(g) \to \psi(g) = \sum_{g' \in S_g} A_{gg'} \psi(g')$
  $S_g \subseteq G$ *set* of systems interacting with g
- ***locality:*** $|S_g| < \infty$, $\forall g$
- ***isotropy:*** we require that $A_{gg'} \neq 0$ *iff* $A_{g'g} \neq 0$ (isotropy to be defined later)
- ***homogeneity and transitivity:***
  - $\{A_{gg'}\}_{g' \in S_g}$ independent of g,
  - G group, $S_g = \{hg\} =: S$, $G = \langle h_1, h_2, \ldots, h_N, | r_1, r_2, \ldots, r_M \rangle$



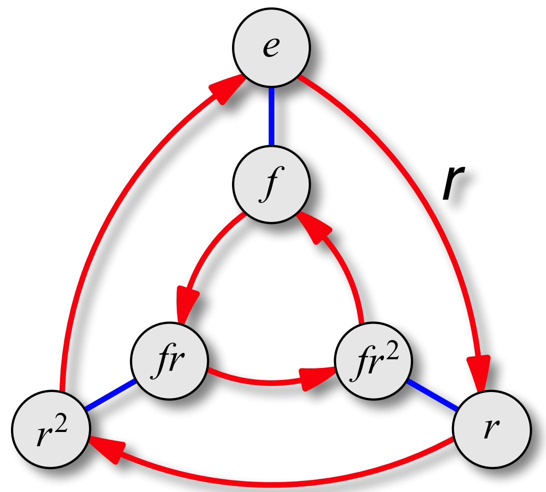
- Finitely generated group G: locality and universality of the physical law
- The physical law has finite quantum algorithmic complexity

Quantum Cellular Automaton

The QCA is:

- a Cayley graph $K(G,S_+)$ of an (infinite) group G with finite generating set

$$S = S_+ \sqcup S_+^{-1}$$



- to each node $g \in G$ it corresponds an evaluation of a quantum field $\psi(g) \in C^s$
- to each generator $h \in S$ of G it corresponds an interaction matrix $A_h \in M_s(C)$

$$\boxed{A = \sum_{h \in S_+ \sqcup S_+^{-1}} T_h \otimes A_h} \Rightarrow \boxed{\text{unitary}}$$

T_h unitary repr. of G on $l^2(G)$

$$\boxed{\psi(x)\psi^\dagger(y) + e^{i\theta(x,y)}\psi^\dagger(y)\psi(x) = \delta_{xy}}$$

linear evolution

$$U^\dagger \psi(x) U = A\psi(x)$$

$$U^\dagger \{\psi(x_1) \otimes \psi(x_2) \otimes \ldots \otimes \psi(x_N)\}_{sym} U =$$

$$A^{\otimes N} \{\psi(x_1) \otimes \psi(x_2) \otimes \ldots \otimes \psi(x_N)\}_{sym}$$

- Vacuum state $|\Omega\rangle$

$$\psi_s(k)|\Omega\rangle = 0$$

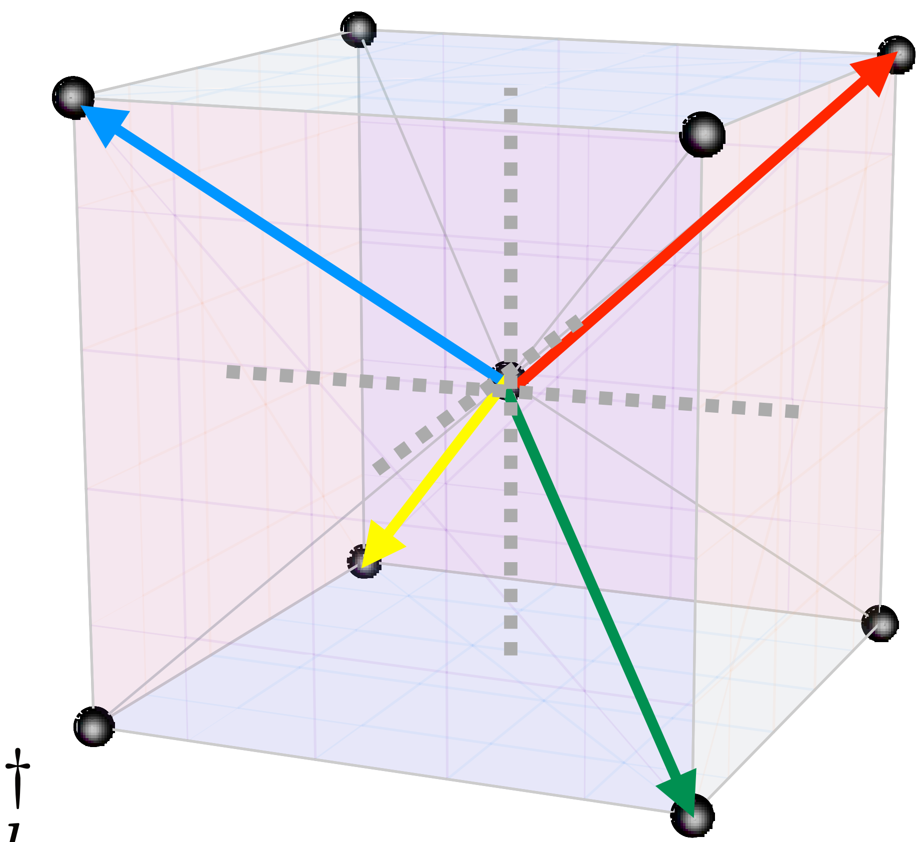
$$U|\Omega\rangle = |\Omega\rangle$$

- N-particle states

$$\psi_{s_1}^\dagger(k_1)\psi_{s_2}^\dagger(k_2)\ldots\psi_{s_N}^\dagger(k_N)|\Omega\rangle$$

Field QCA: isotropy

- There exists a group L of permutations of S_+, <u>transitive</u> over S_+ that leaves $K(G,S_+)$ invariant
- a nontrivial unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:



$$A = \sum_{h \in S} T_h \otimes A_h = \sum_{h \in S} T_{lh} \otimes L_l A_h L_l^\dagger$$

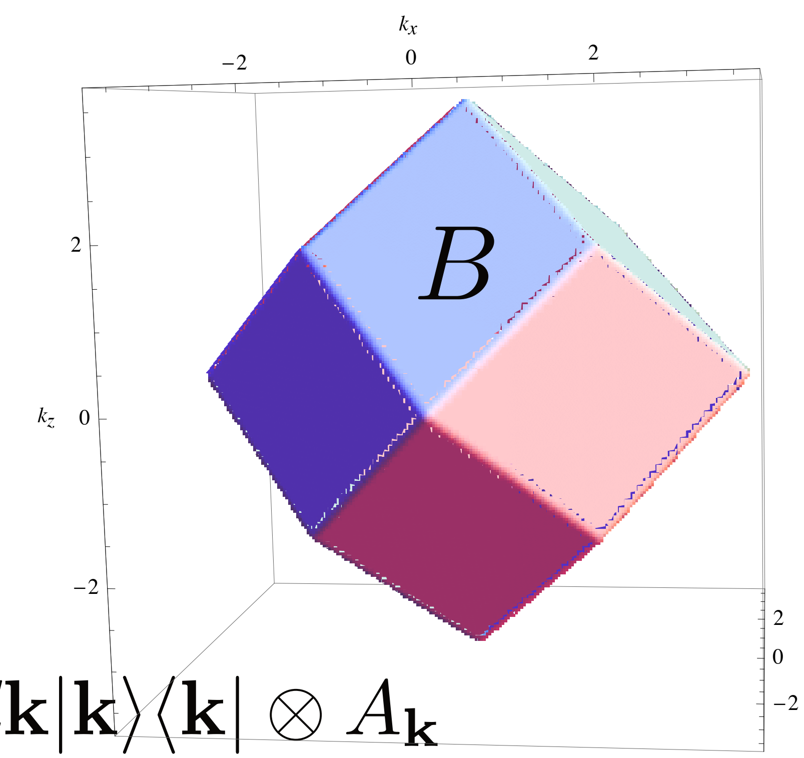
$$S = S_+ \sqcup S_+^{-1}$$

- Conjecture: unitarity + isotropy $\Rightarrow G$ Abelian

D'Ariano, Perinotti,
arXiv:1306.1934

The Weyl QCA

☞ Minimal dimension for nontrivial unitary *A:* s=2



- Unitarity ⇒ the only possible *G* is the BCC!!
- ⇒ A_h are proportional to rank-one projectors

$$A = \int_B d\mathbf{k} |\mathbf{k}\rangle\langle\mathbf{k}| \otimes A_{\mathbf{k}}$$

Two QCAs connected by CPT

$$\boxed{\begin{aligned} A_{\mathbf{k}}^{\pm} = & -i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ & -i(\pm\sigma_y)(c_x s_y c_z \mp s_x c_y s_z) \\ & -i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ & +I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

The Weyl QCA

$$i\partial_t \psi(t) \simeq \tfrac{i}{2}[\psi(t+1) - \psi(t-1)] = \tfrac{i}{2}(A - A^\dagger)\psi(t)$$

$$\begin{aligned}\frac{i}{2}(A^{\pm}_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) =& \sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\pm \sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &+ \sigma_z(c_x c_y s_z \pm s_x s_y c_z)\end{aligned}$$

“Hamiltonian”

$$k \ll 1$$

$$i\partial_t \psi = \frac{1}{\sqrt{3}}\boldsymbol{\sigma} \cdot \mathbf{k}\psi$$ ☜ Weyl equation!

D'Ariano, Perinotti, arXiv:1306.1934

Dirac emerging from the QCA

fidelity with Dirac evolution for a narrowband packet in the relativistic limit $k \simeq m \ll 1$

$$F = |\langle \exp[-iN\Delta(\mathbf{k})]\rangle| \qquad \omega^E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$$

$$\Delta(\mathbf{k}) := (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k})$$

$$= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \Rightarrow t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \Rightarrow N \simeq k^{-2} = 10^{16} \Rightarrow 5 * 10^{-28}$ s

Getting other automata from Weyl

- Direct-sum coupling → Dirac automata
- Tensor with adjoint → Maxwell automaton

D'Ariano, Perinotti, arXiv:1306.1934

The Dirac QCA

The only way of coupling two Weyl automata locally is to couple $A_{\boldsymbol{k}}$ with its inverse with an identity matrix as follows:

$$E_{\mathbf{k}}^{\pm} = \begin{pmatrix} nA_{\mathbf{k}}^{\pm} & imI \\ imI & nA_{\mathbf{k}}^{\pm\dagger} \end{pmatrix}$$

$$\boxed{n^2 + m^2 = 1}$$

$$\omega_{\pm}^{E}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \pm s_x s_y s_z)]$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}} \qquad c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

- m≤1: bound for mass
- n^{-1}:vacuum refraction index

$E_{\mathbf{k}}^{\pm}$ CPT-connected!

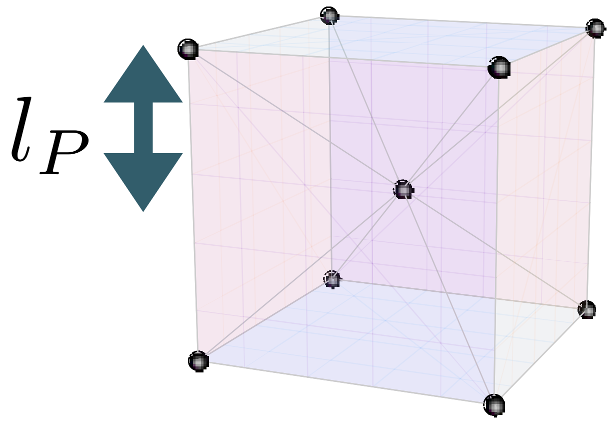
Universal constants of QCA theory

Conversion to dimensional units

l_P	t_P	m_P	*fundamental system* (Wilczek)
[L]	[T]	[M]	

t_P: automaton time-step

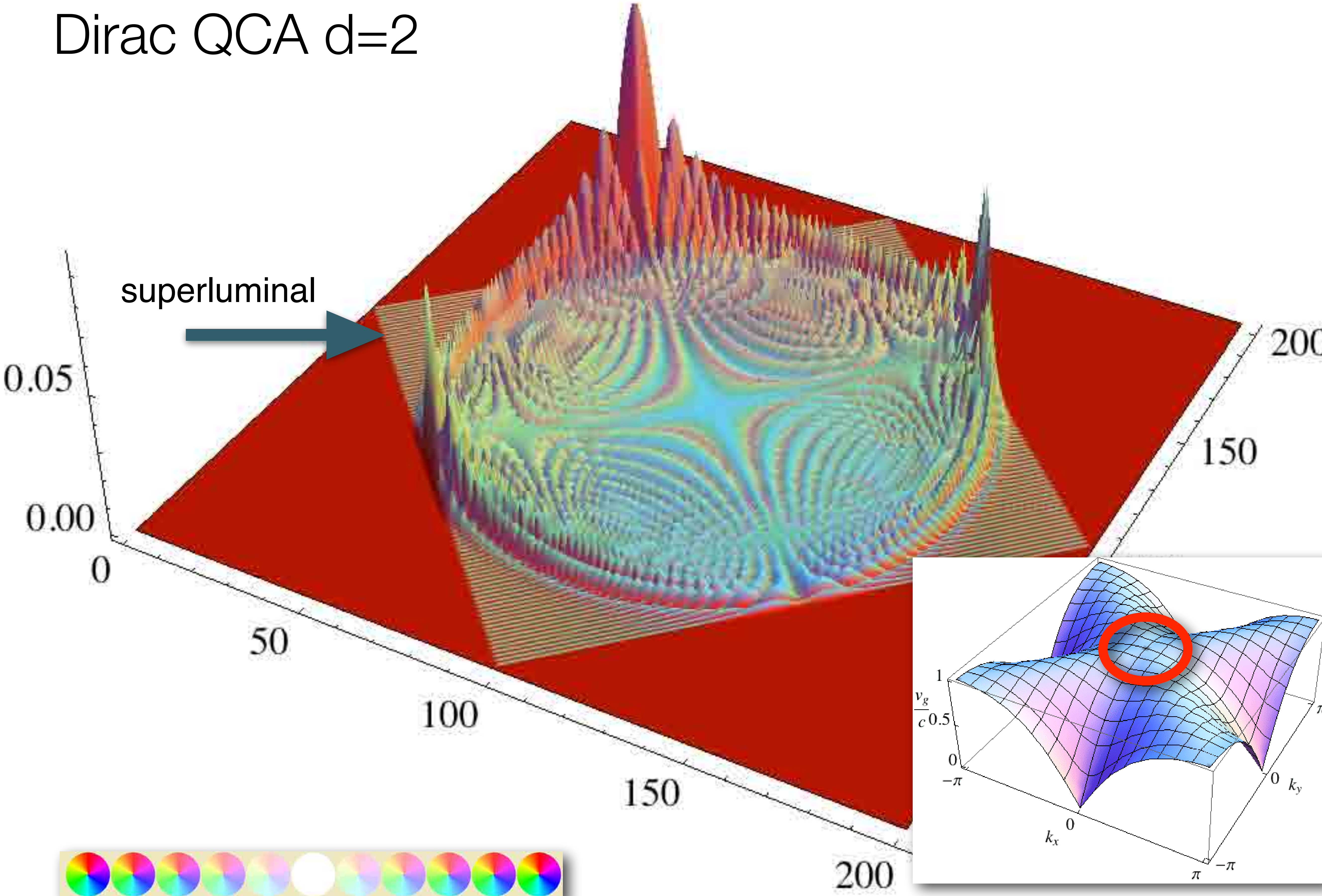
m_P: bound for particle mass



$$m_g = m\, m_P \qquad p = \frac{\hbar k}{\sqrt{3} l_P}$$

$$c := \frac{l_P}{t_P} \qquad \hbar = m_P l_P c \qquad G = \frac{l_P c^2}{m_P}$$

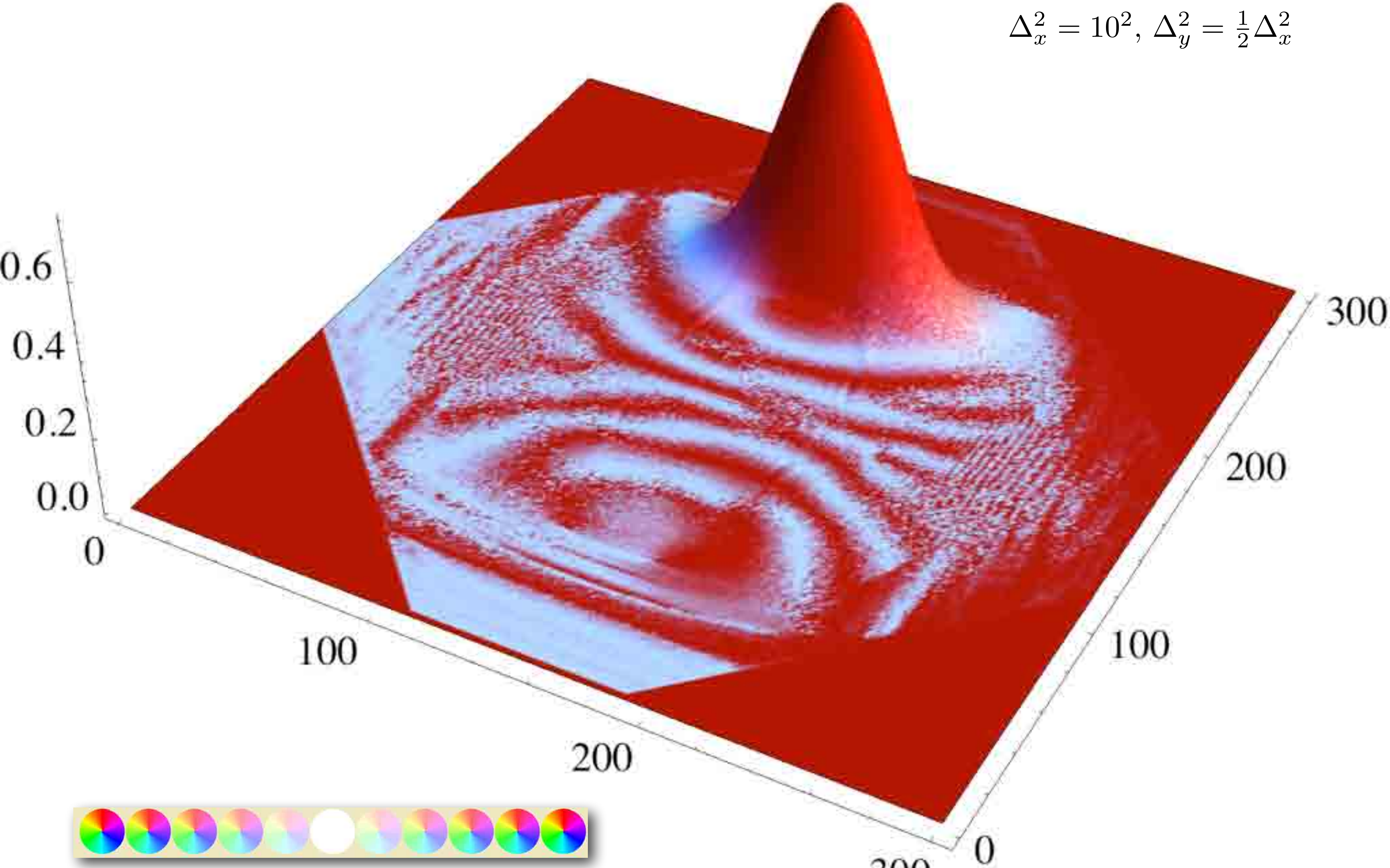
Dirac QCA d=2


superluminal
0.05
0.00
0
50
100
150
200
150
200
1
$\frac{v_g}{c}$ 0.5
0
$-\pi$
0
π
k_x
$-\pi$
0
π
k_y

Dirac QCA d=2

$\mathbf{k} = (\frac{\pi}{10}, 0),\ m = .1, N = 120$

$\Delta_x^2 = 10^2,\ \Delta_y^2 = \frac{1}{2}\Delta_x^2$



Bisio, D'Ariano, Perinotti (unpub)

Maxwell automaton

$$\vec{F}_{\mathbf{k}} = \vec{E}_{\mathbf{k}} + i\vec{B}_{\mathbf{k}} = \sum_{ij} \vec{\sigma}_{ij} (\psi_i \varphi_j^\dagger)_{\mathbf{k}}$$

"neutrino theory of photon"

$$\vec{F}_{\mathbf{k}}(t+1) = A_{\mathbf{k}} \vec{F}_{\mathbf{k}}(t) A_{\mathbf{k}}^\dagger$$

Maxwell equations in relativistic limit $k \ll 1$

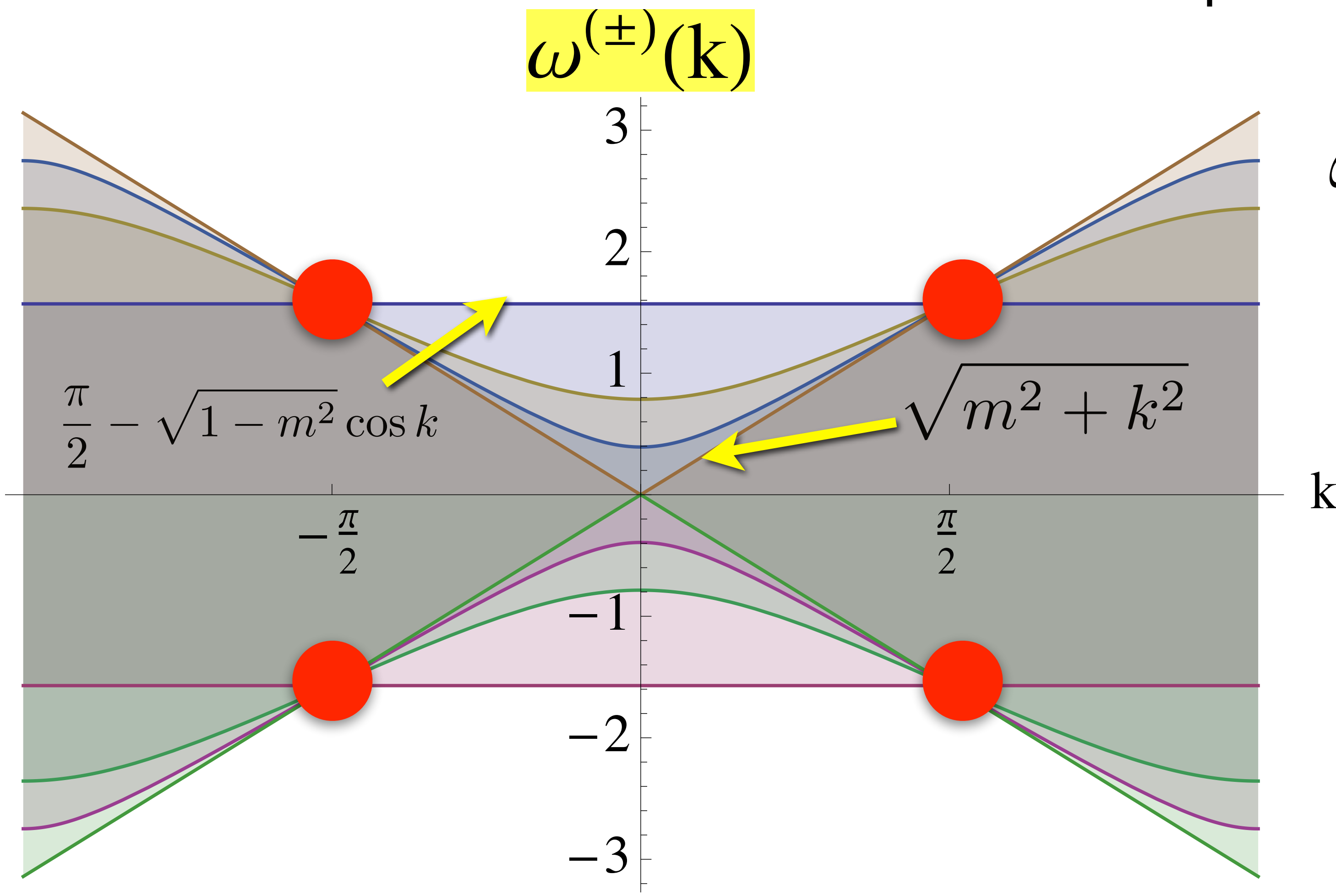
Boson: emergent from convolution of fermions

Tradeoff between violation of Maxwell dynamics and good bosons

Bibeau-Delisle, Bisio, D'Ariano,
Perinotti, Tosini, arXiv:1310.6760

Planck-scale effects: Lorentz covariance distortion

Transformations that leave the dispersion relation invariant



$$\omega_E(k) := \pm\cos^{-1}(\sqrt{1-m^2}\cos k)$$

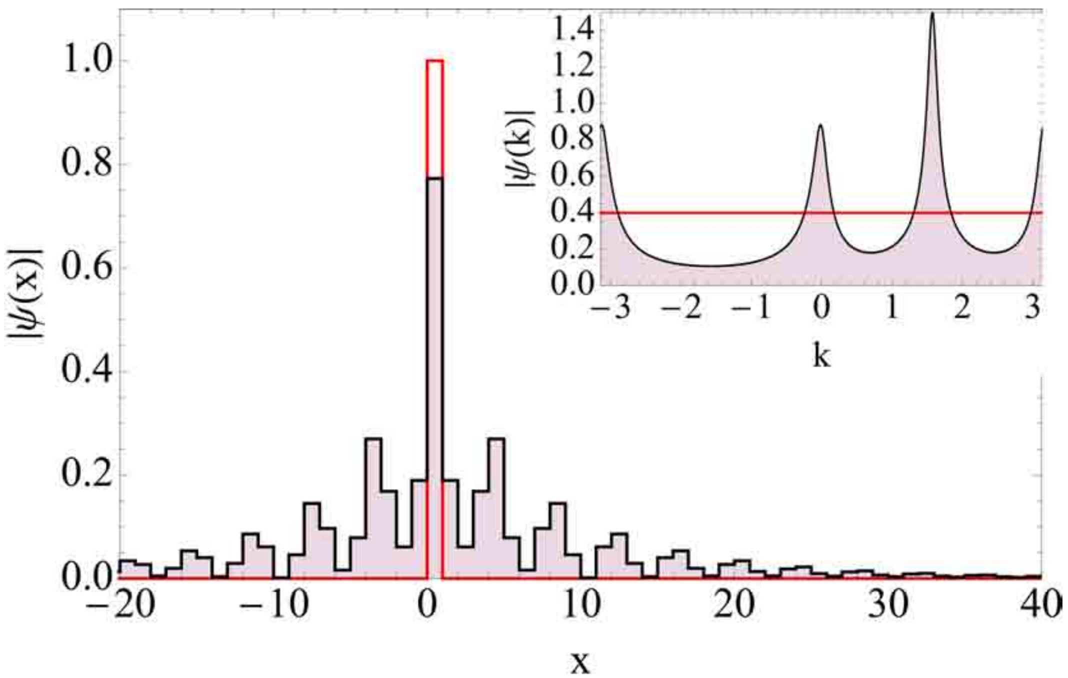
$$\omega' = \arcsin\left[\gamma\left(\sin\omega/\cos k - \beta\tan k\right)\cos k'\right]$$

$$k' = \arctan\left[\gamma\left(\tan k - \beta\sin\omega/\cos k\right)\right]$$

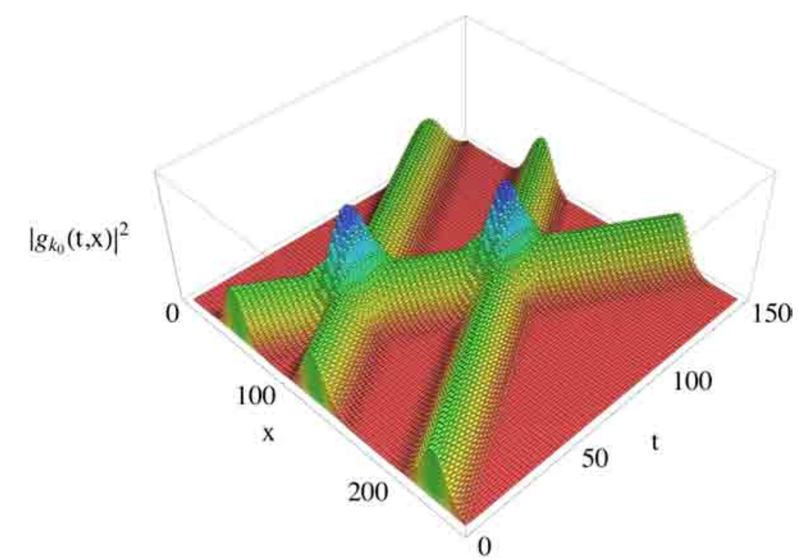
$$\gamma := (1-\beta^2)^{-1/2}$$

Bibeau-Delisle, Bisio, D'Ariano,
Perinotti, Tosini, arXiv:1310.6760

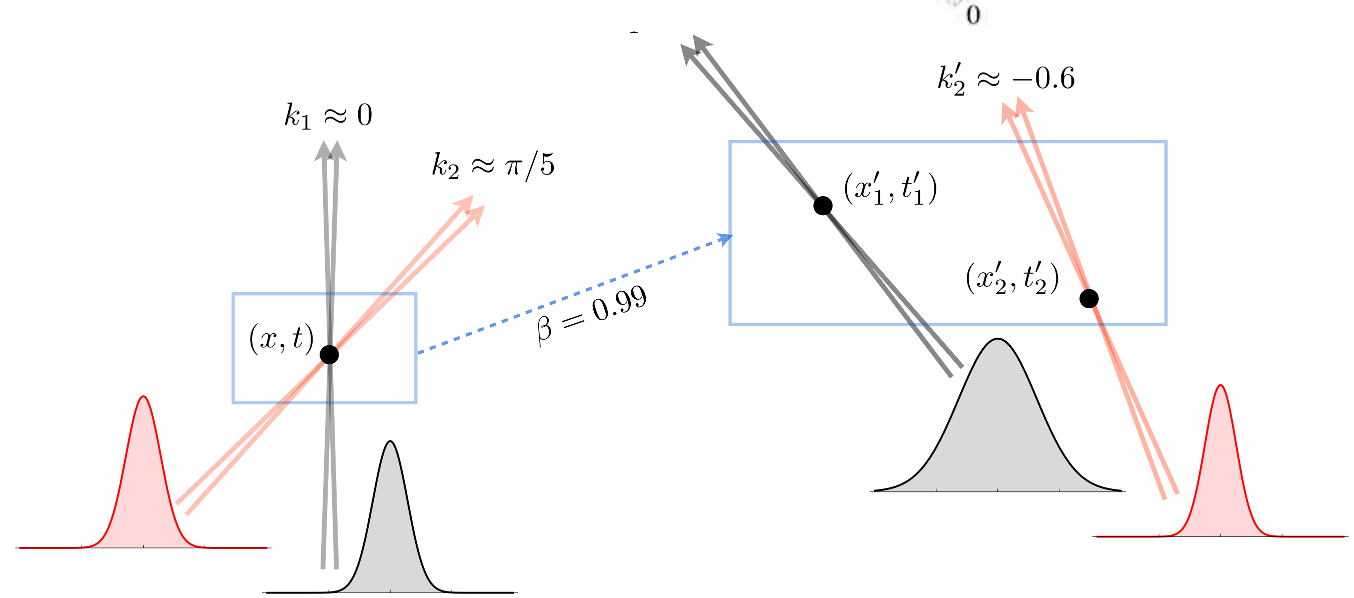
Planck-scale effects: Lorentz covariance distortion



For narrow-band states we can linearize Lorentz transformations around $k=k_0$ and we get k-dependent Lorentz transformations



Delocalization under boost



$$|\psi\rangle = \int \mathrm{d}k\mu(k)\hat{g}(k)|k\rangle \xrightarrow{L_\beta^D} \int \mathrm{d}k\mu(k)\,\hat{g}(k)|k'\rangle =$$

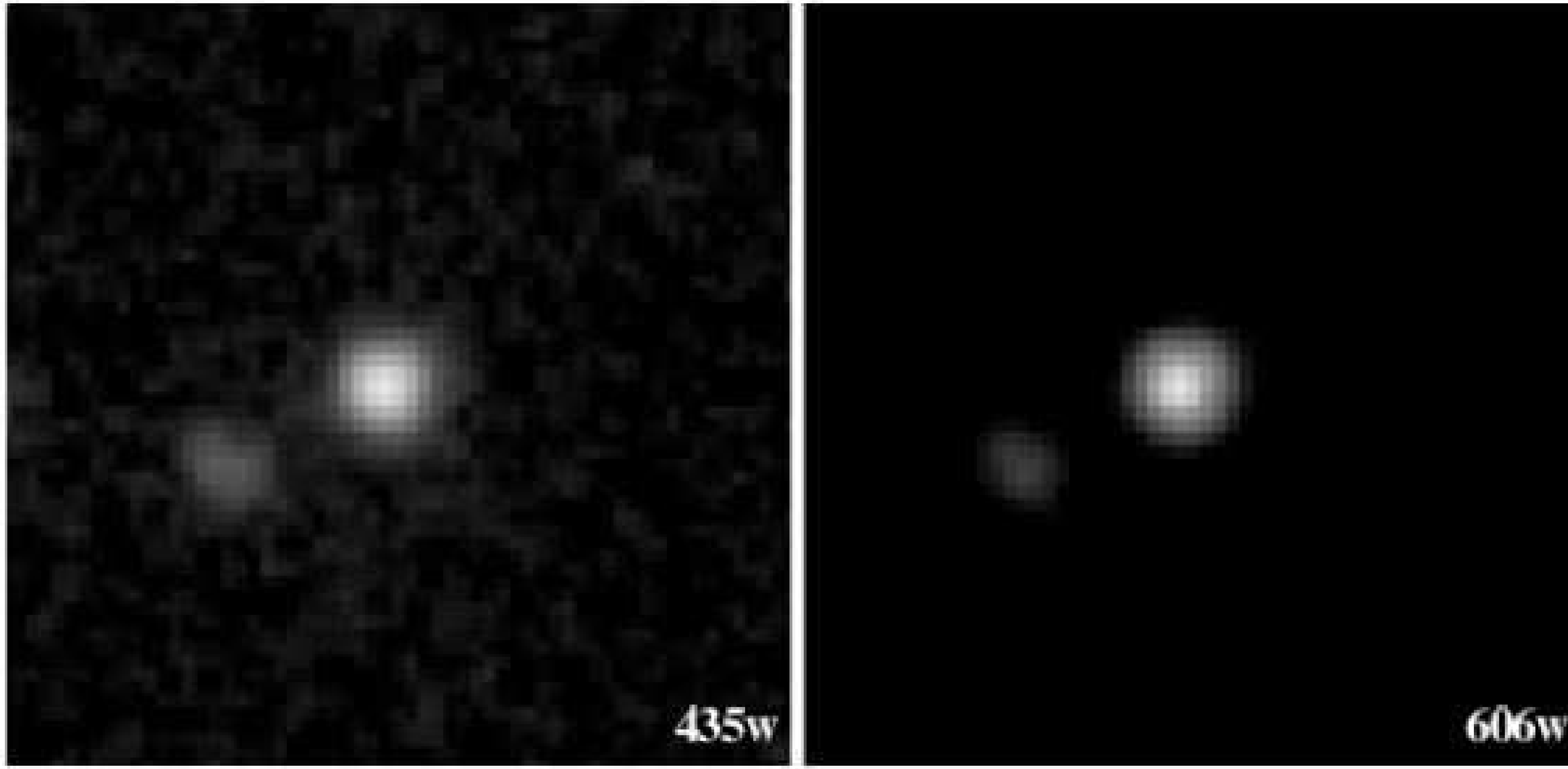
$$= \int \mathrm{d}k\mu(k')\,\hat{g}(k(k'))|k'\rangle$$

Relative locality

R. Schützhold and W. G. Unruh, J. Exp. Theor. Phys. Lett. **78** 431 (2003)
G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, and L. Smolin, arXiv:1106.0313 (2011)

Astrophysical tests?

- Blurring of the image of very far quasars



Caption: An aspect of the analysis that motivated PhysicalReviewLetters96,051301 (paper also mentioned in parts of this project proposal). Here shown are copies of Hubble telescope Ultra-Deep-Field images for quasar 6732 (redshift 3.2), with left panel showing a B-filter ('blue') image and right panel showing a V-filter ('visible') image. It is noticeable from the original images that this quasar appears somewhat blurred in the shorter-wavelength B filter. This illustrates qualitatively the effect expected in some quantum-spacetime models: propagation of photons in the fuzzy spacetime should produce blurring of images, with more blurring found for larger distances (greater 'accumulation' of tiny Planck-length effects) and for shorter wavelengths (more sensitive to the fundamental short-distance structure of spacetime). The available database of quasar images does show some preliminary evidence in favour of these qualitative features, but we are presently unable to exclude that the blurring be due entirely to conventonal-physics mechanisms. We can nonetheless use these data to place limits on spacetime fuzziness: if any blurring is caused by spacetime fuzziness it must not be more than what shown by our quasar images.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Alexandre Bibeau

arXiv
1212.2839
1306.1934
1310.6760

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Thank you!