JOHN TEMPLETON FOUNDATION
SUPPORTING SCIENCE - INVESTING IN THE BIG QUESTIONS
Project: *A Quantum-Digital Universe*, Grant ID: 43796

# Informationalism as a route to quantum gravity, and the unbelievable power of the axiomatic approach

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

*Testing Quantum Gravity*
*Cavallerizza Reale, 26-27 May 2016, Torino*

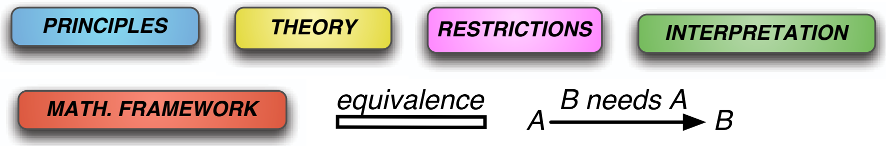
# Program

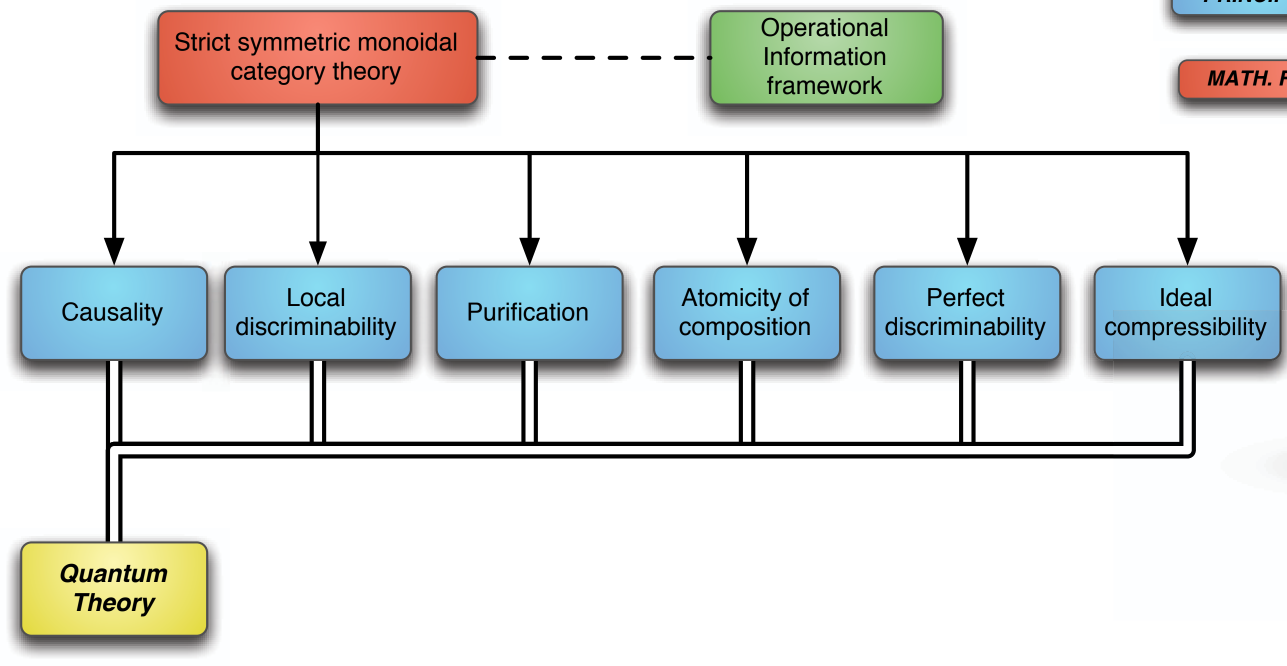
To derive the whole Physics axiomatically

from "principles" stated in form of purely mathematical axioms
without physical primitives,
but having a thorough physical interpretation.

A solution: informationalism

physical primitives: mass, force, rods, clocks,…




# Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

## Informational derivation of quantum theory

Giulio Chiribella*

*Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Ontario, Canada N2L 2Y5*

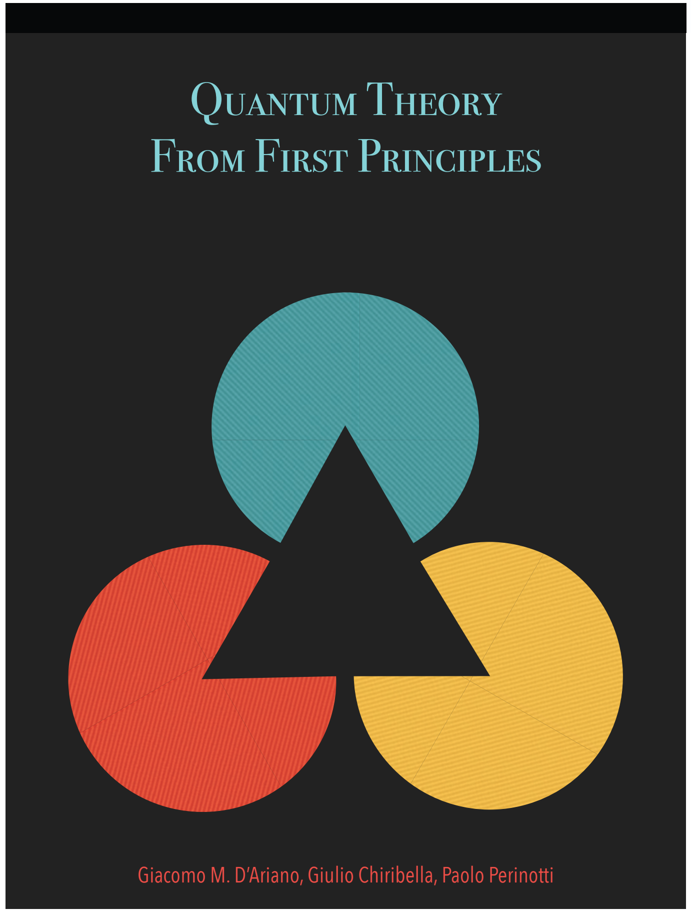
Giacomo Mauro D'Ariano and Paolo Perinotti

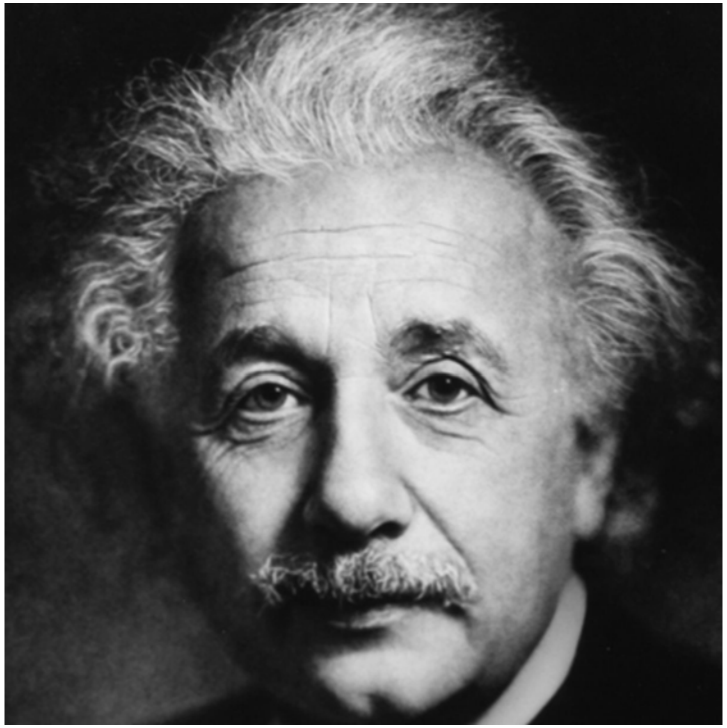
*QUIT Group, Dipartimento di Fisica "A. Volta" and INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy*[‖]

(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03 67 Ac, 03 65 Ta



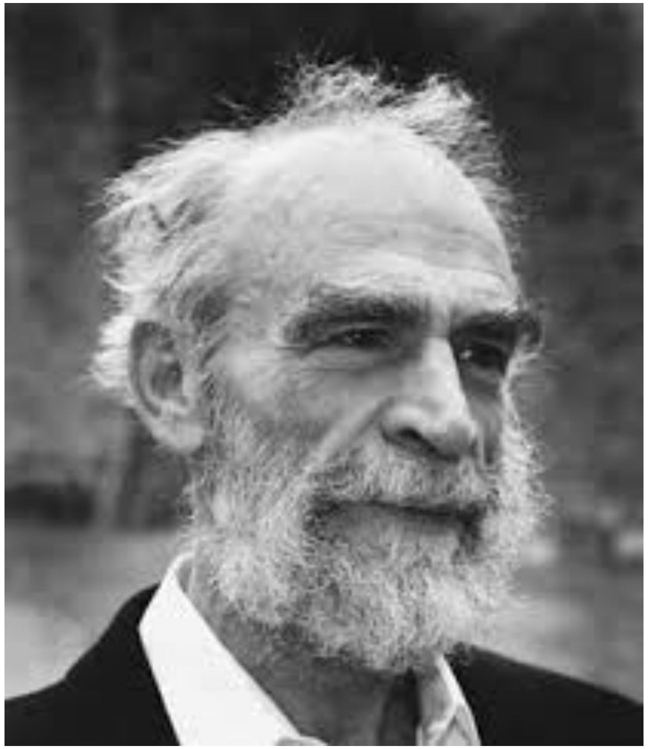


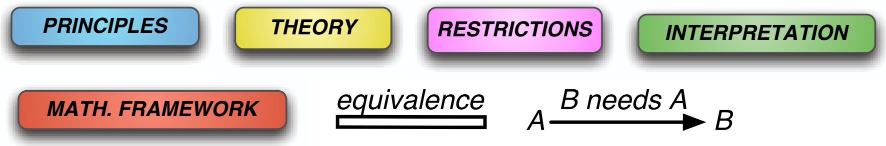
*"But you have correctly grasped <u>the drawback that the continuum brings</u>. If the molecular view of matter is the correct (appropriate) one, i.e., if a part of the universe is to be represented by a finite number of moving points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this too great is responsible for the fact that our present means of description <u>miscarry with the quantum theory</u>. The problem seems to me how one can formulate statements about a discontinuum without calling upon a continuum (space-time) as an aid; the latter should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing "real". <u>But we still lack the mathematical structure unfortunately.</u> How much have I already plagued myself in this way!"*

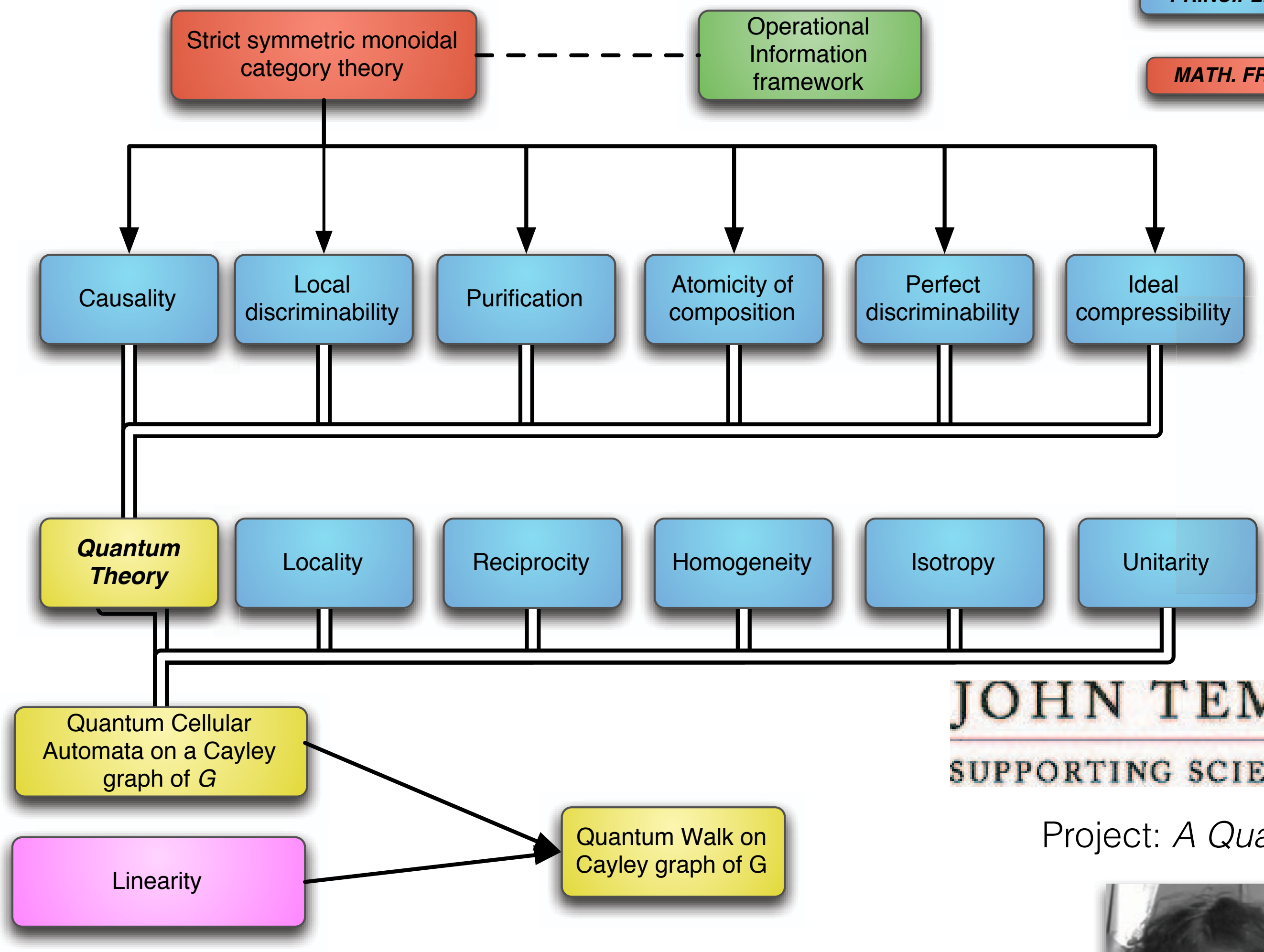
**J. Stachel, in From Quarks to Quasars: Philosophical Problems of Modern Physics**

# A new mathematics for a discrete space-time: *geometric group theory*

Mikhail Gromov






# Principles for Mechanics

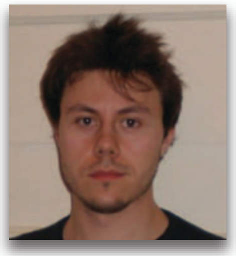
JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

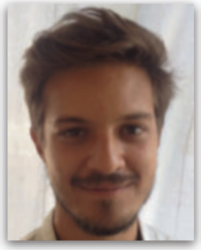
Project: *A Quantum-Digital Universe*, Grant ID: 43796

Paolo Perinotti

Alessandro Bisio



Alessandro Tosini



Marco Erba

Franco Manessi

Nicola Mosco

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

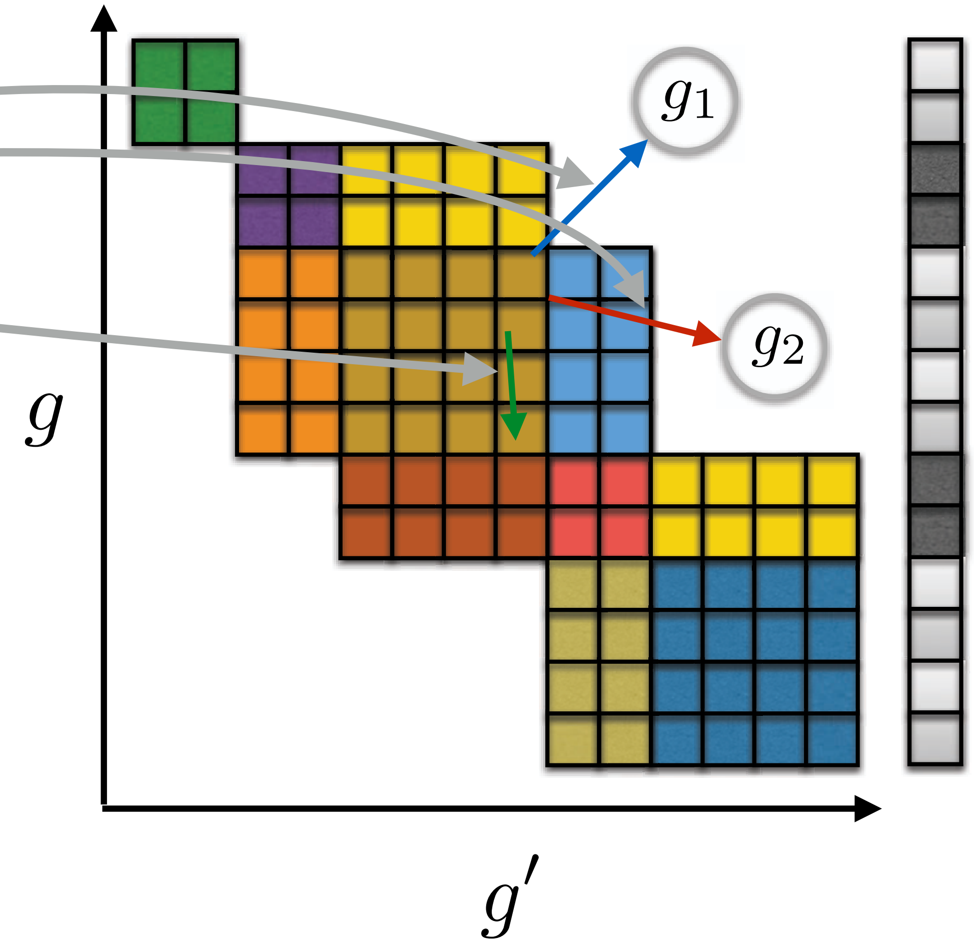
# Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g}$ $G \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$




Build a directed graph with an arrow from $g$ to $g'$ *wherever they* are connected by $A_{gg'} \neq 0$

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

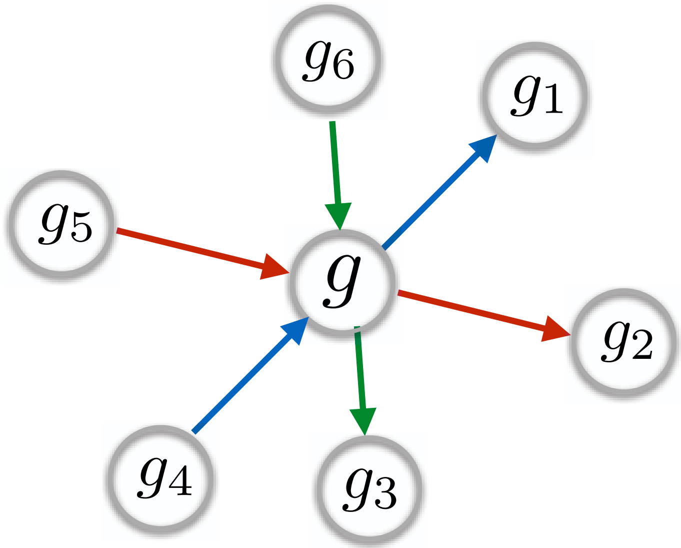
# Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \bigoplus_{g\in G}\mathbb{C}^{s_g}$ $\quad G \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$




1) *Locality*: $S_g$ uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g\in G$ are "equivalent"

$S_g = S,\ s_g = s\ \ldots$ label $A_{gg'} =: A_h,\ h \in: S$

define the "action" on the set of vertices *G*: $gh := g'$ whenever $A_{gg'} = A_h$

skip proof

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

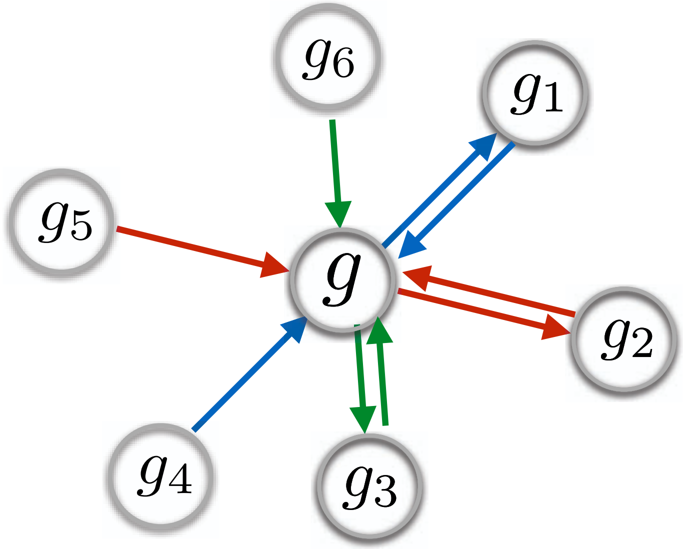
# Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g} \qquad G \leqslant \aleph,\ s_g \in \mathbb{N}$

**Evolution**

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$

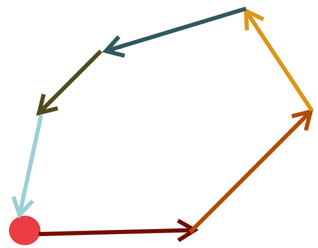


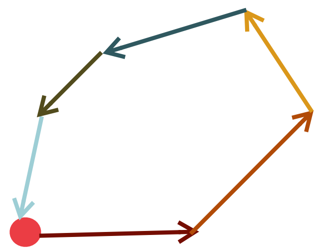

1) *Locality*: $S_g$ uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent

A sequence $A_{h_N}A_{h_{N-1}}\ldots A_{h_1}$ connects $g$ to itself, namely $gh_1h_2\ldots h_N = g$, then it must also connect any other $g'$ to itself, i.e. $g'h_1h_2\ldots h_N = g'$ .





From 2): two-loop $ghh^{-1} = g$ defines uniquely $h^{-1}$ for $h$ and viceversa

$A_{gg'} =: A_h,\ A_{g'g} =: A_{h^{-1}},\ h \in S \equiv S_+ \cup S_-,\ S_- := S_+^{-1}$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

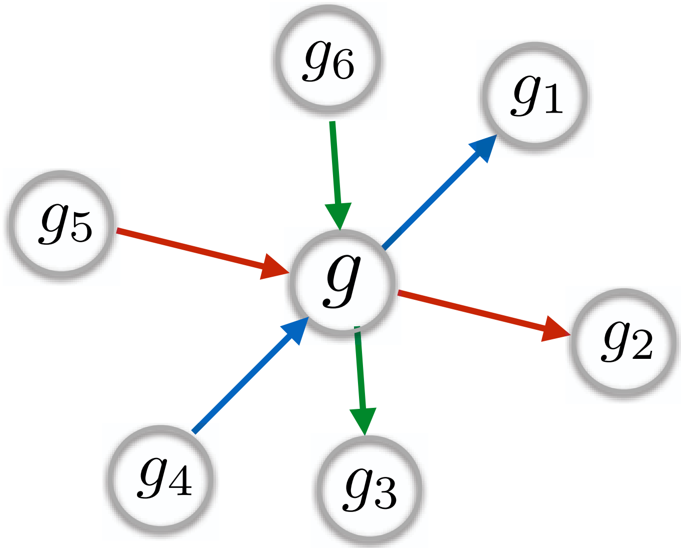
# Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \bigoplus_{g \in G} \mathbb{C}^{s_g}$ $\quad G \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g' \in S_g} A_{gg'} \psi_{g'}(t)$$

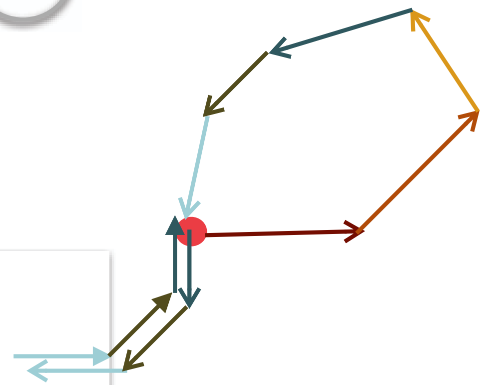
$$\sum_{g'} A_{gg'} A^{\dagger}_{g''g'} = \sum_{g'} A^{\dagger}_{gg'} A_{g''g'} = \delta_{gg''} I_{s_g}$$




1) *Locality*: $S_g$ uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent



Build the free group *F* of words made with letters:

$$h \in S := S_+ \cup S_-$$

with action on vertices in *G*: $gh := g'$ whenever $A_{gg'} = A_h$

Consider the subgroup *H* of closed paths → *H* normal subgroup of *F*

D'Ariano, Perinotti, PRA **90** 062106 (2014)

# Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g}$ $\quad G \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$

1) *Locality*: $S_g$ uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent

$\Gamma(G, S_+)$ *colored directed graph* with vertices $g \in G$ and edges $(g, g')$ with $g' = gh$

Either the graph is connected, or it consists of disconnected copies.
W.l.g. assume it as connected.

Being $H$ normal, one concludes that:

$G = F/H = \langle S\ R\rangle$ is a group with Cayley graph $\Gamma(G, S_+)$ (the identity any element $e \in G$).

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

# Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g}$ $\quad G \leqslant \aleph,\; s_g \in \mathbb{N}$

$$\boxed{\begin{array}{l}\text{Evolution}\\ \psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)\\ \sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}\end{array}}$$

The following operator over the Hilbert space $\ell^2(G)\otimes\mathbb{C}^s$ is unitary

$$A = \sum_{h\in S} T_h \otimes A_h$$

where $T$ is the right regular representation of $G$ on $\ell^2(G)$ acting as

$$T_g\, g'\rangle = g'g^{-1}\rangle$$

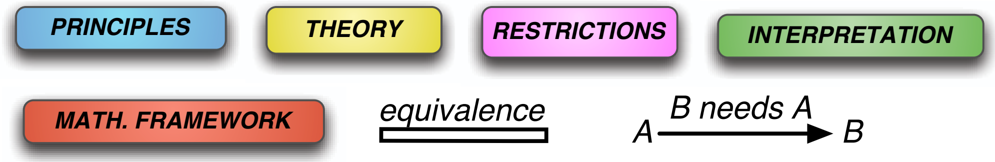
$$\left.\begin{array}{l}\text{1) \textit{Locality}: } S_g \text{ uniformly bounded}\\ \text{2) \textit{Reciprocity}: } A_{gg'} \neq 0 \implies A_{g'g} \neq 0\\ \text{3) \textit{Homogeneity}: all } g \in G \text{ are equivalent}\end{array}\right\} = \text{Quantum Walk on Cayley graph}$$

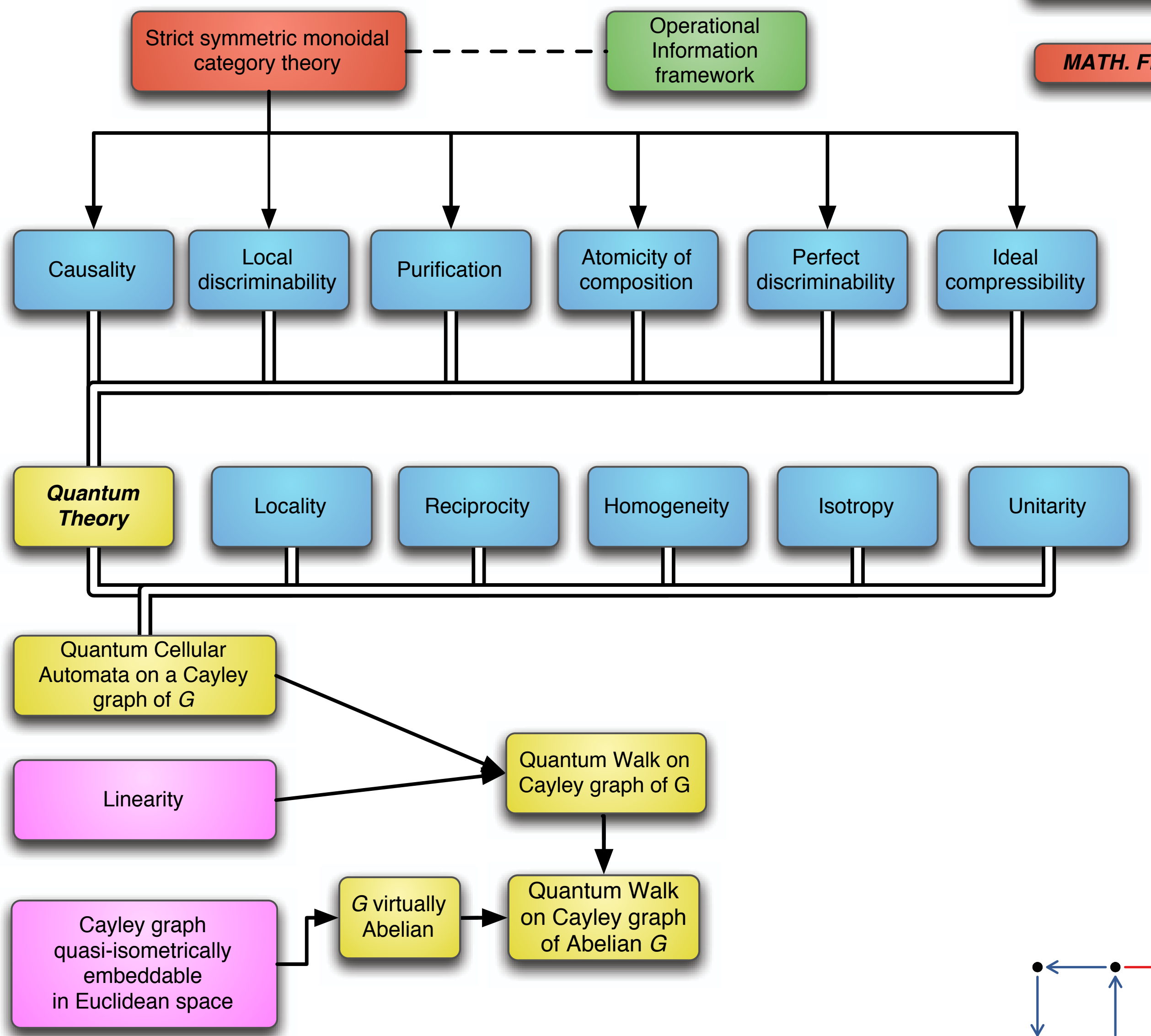
4) *Isotropy:*

There exist:

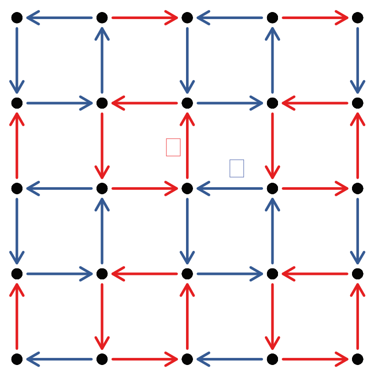
- a group *L* of permutations of *S*₊, transitive over *S*₊ that leaves the Cayley graph invariant
- a unitary *s*-dimensional (projective) representation $\{L_l\}$ of *L* such that:

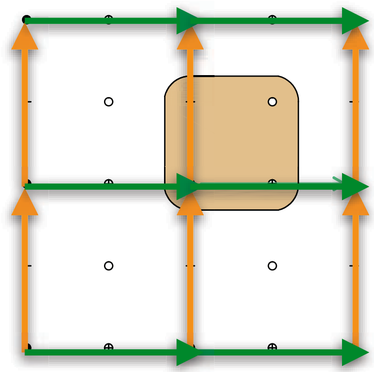
$$\boxed{A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh} \otimes L_l A_h L_l}$$



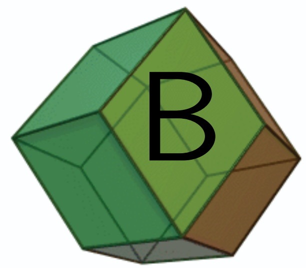

$$\langle\ c, d \mid c^4, d^4, (cd)^2\ \rangle$$



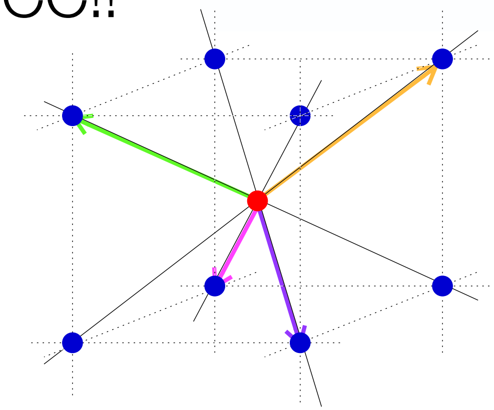


# The Weyl QW

☞ Minimal dimension for nontrivial unitary *A:* s=2




Unitarity + isotropy ⇒ for d=3 the only Cayley is the BCC!!



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QWs connected by P

$$\boxed{\begin{aligned} A^{\pm}_{\mathbf{k}} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ &+ I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

# The Weyl QW

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

$$i\partial_t\psi(t) \simeq \tfrac{i}{2}[\psi(t+1) - \psi(t-1)] = \tfrac{i}{2}(A - A\ \ )\psi(t)$$

$$\begin{aligned}\tfrac{i}{2}(A^{\pm}_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+\sigma_x(s_xc_yc_z \pm c_xs_ys_z) \\ &\pm \sigma_y(c_xs_yc_z \mp s_xc_ys_z) \\ &+\sigma_z(c_xc_ys_z \pm s_xs_yc_z)\end{aligned}$$ *"Hamiltonian"*

$k \ll 1$ $\boxed{i\partial_t\psi = \frac{1}{\sqrt{3}}\boldsymbol{\sigma}^{\pm}\cdot\mathbf{k}\,\psi \quad \text{Weyl equation!}}$ $\boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$

Two QCAs connected by P

$$\boxed{\begin{aligned}A^{\pm}_{\mathbf{k}} = &-i\sigma_x(s_xc_yc_z \pm c_xs_ys_z) \\ &\mp i\sigma_y(c_xs_yc_z \mp s_xc_ys_z) \\ &- i\sigma_z(c_xc_ys_z \pm s_xs_yc_z) \\ &+ I(c_xc_yc_z \mp s_xs_ys_z)\end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

# Dirac QW $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$\boxed{E_{\mathbf{k}} = \begin{pmatrix} nA_{\mathbf{k}} & imI \\ imI & nA_{\mathbf{k}} \end{pmatrix} \qquad n^2 + m^2 = 1 \qquad n, m \in \mathbb{R}}$$

$E_{\mathbf{k}}$ CPT-connected!

$$\omega^E_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll m \ll 1$

m: mass, $m^2 \leq 1$

$n^{-1}$: refraction index

$\pm 1$

$m$

0

!

Bisio, D'Ariano, Perinotti, Ann. Phys. **368** 177 (2016)

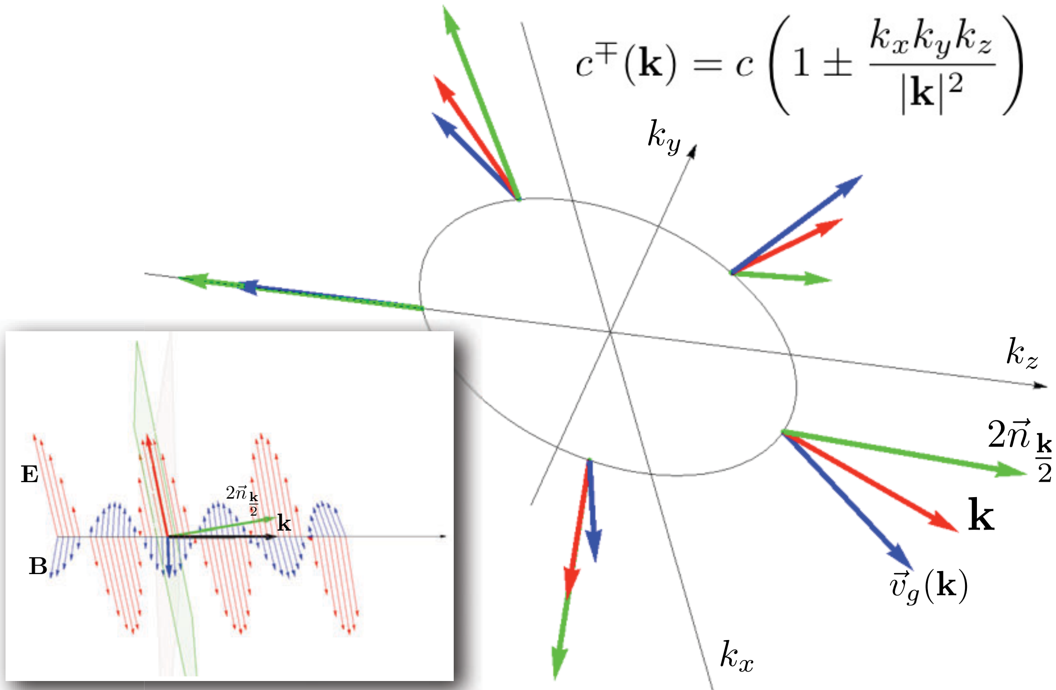
# Maxwell QW $\otimes$

$$\boxed{M^{\pm}_{\mathbf{k}} = A^{\pm}_{\mathbf{k}} \otimes A^{\pm *}_{\mathbf{k}} \qquad F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})}$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions

*(De Broglie neutrino-theory of photon)*



# Dirac emerging from the QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

Fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

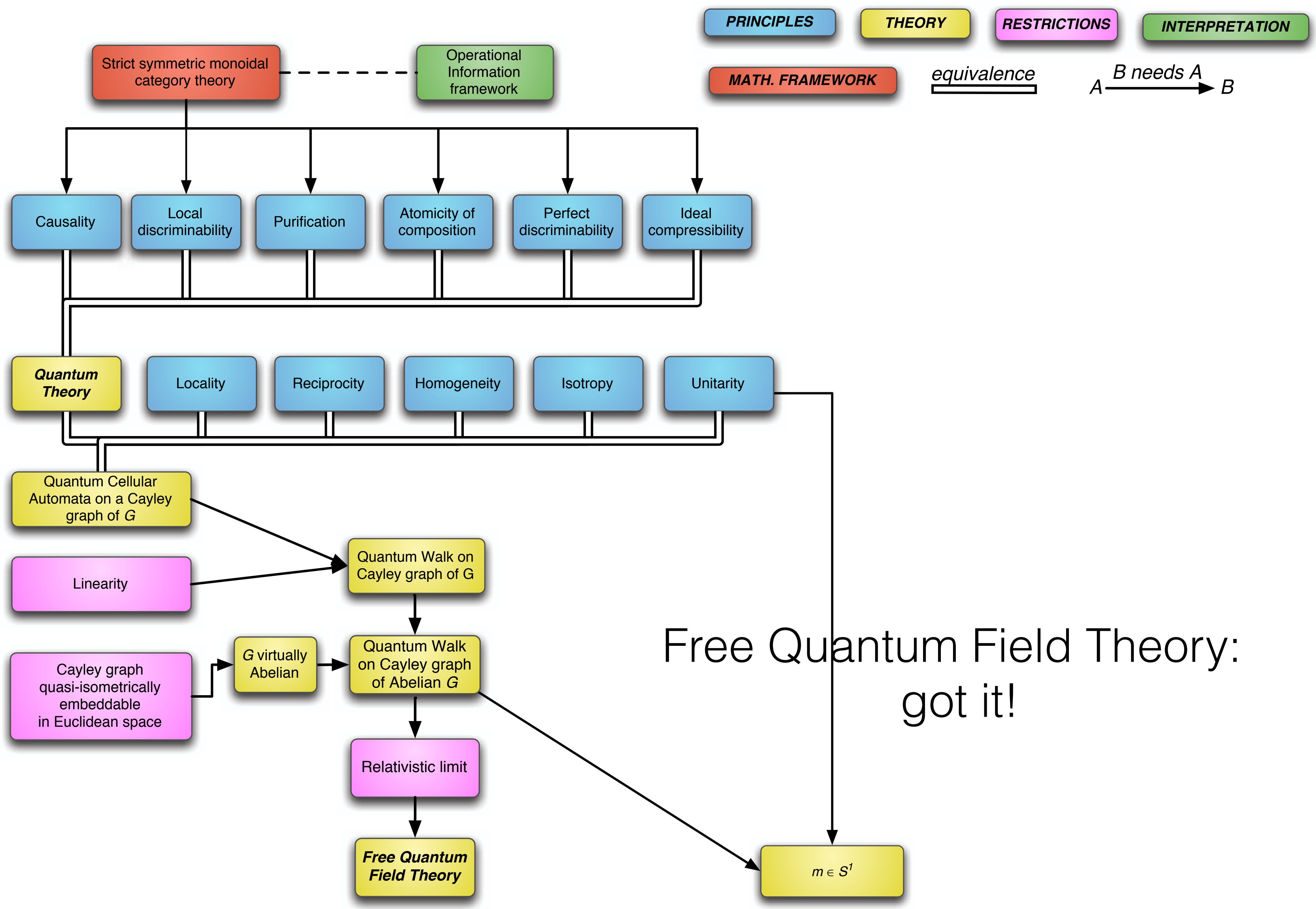
$$F = |\langle \exp\left[-iN\boldsymbol{\Delta}(\mathbf{k})\right]\rangle|$$

$$\boldsymbol{\Delta}(\mathbf{k}) := (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k})$$

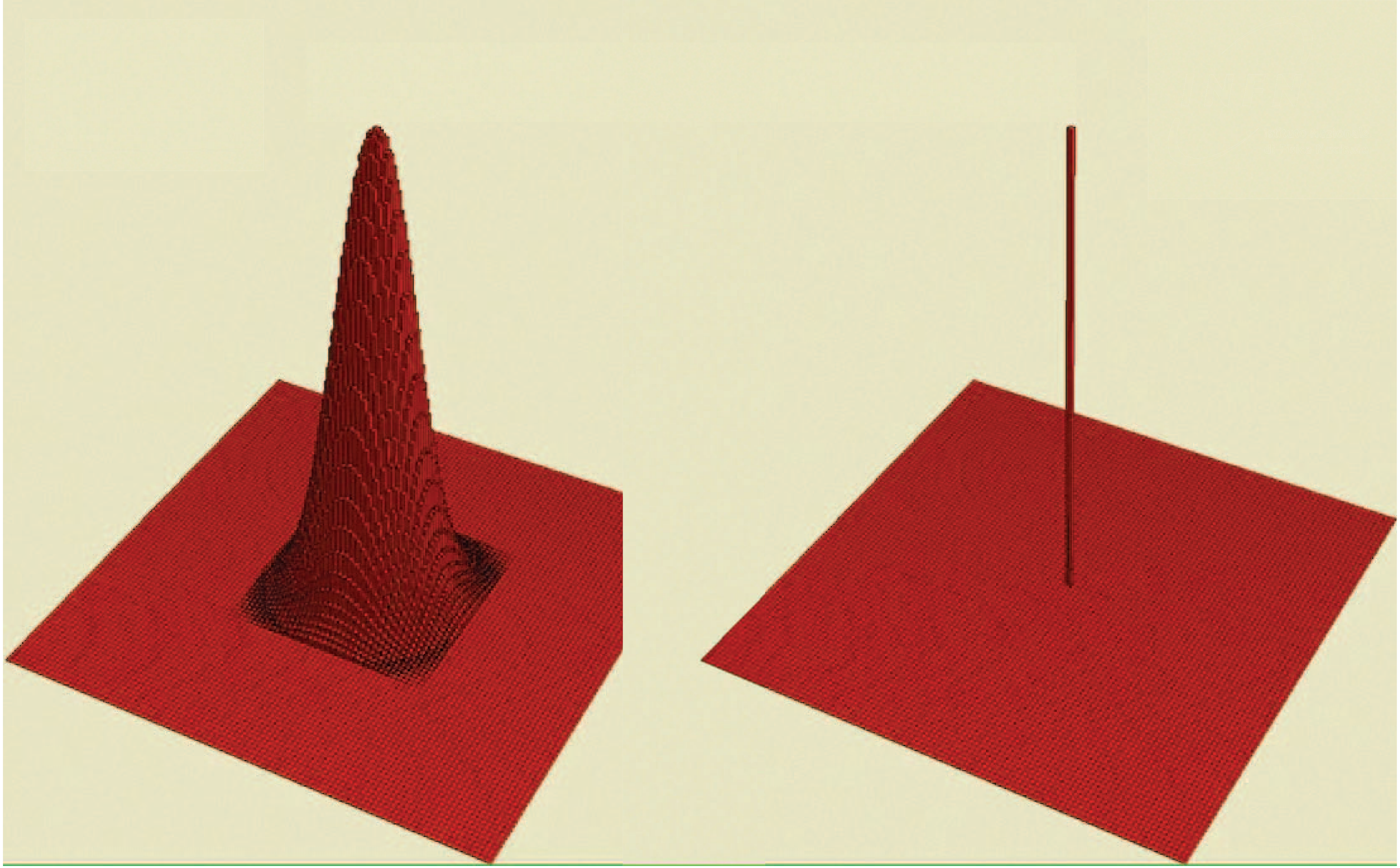
$$= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \frac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \;\Rightarrow\; t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \;\Rightarrow N \simeq k^{-2} = 10^{16} \;\Rightarrow\; 5 * 10^{-28}$ s

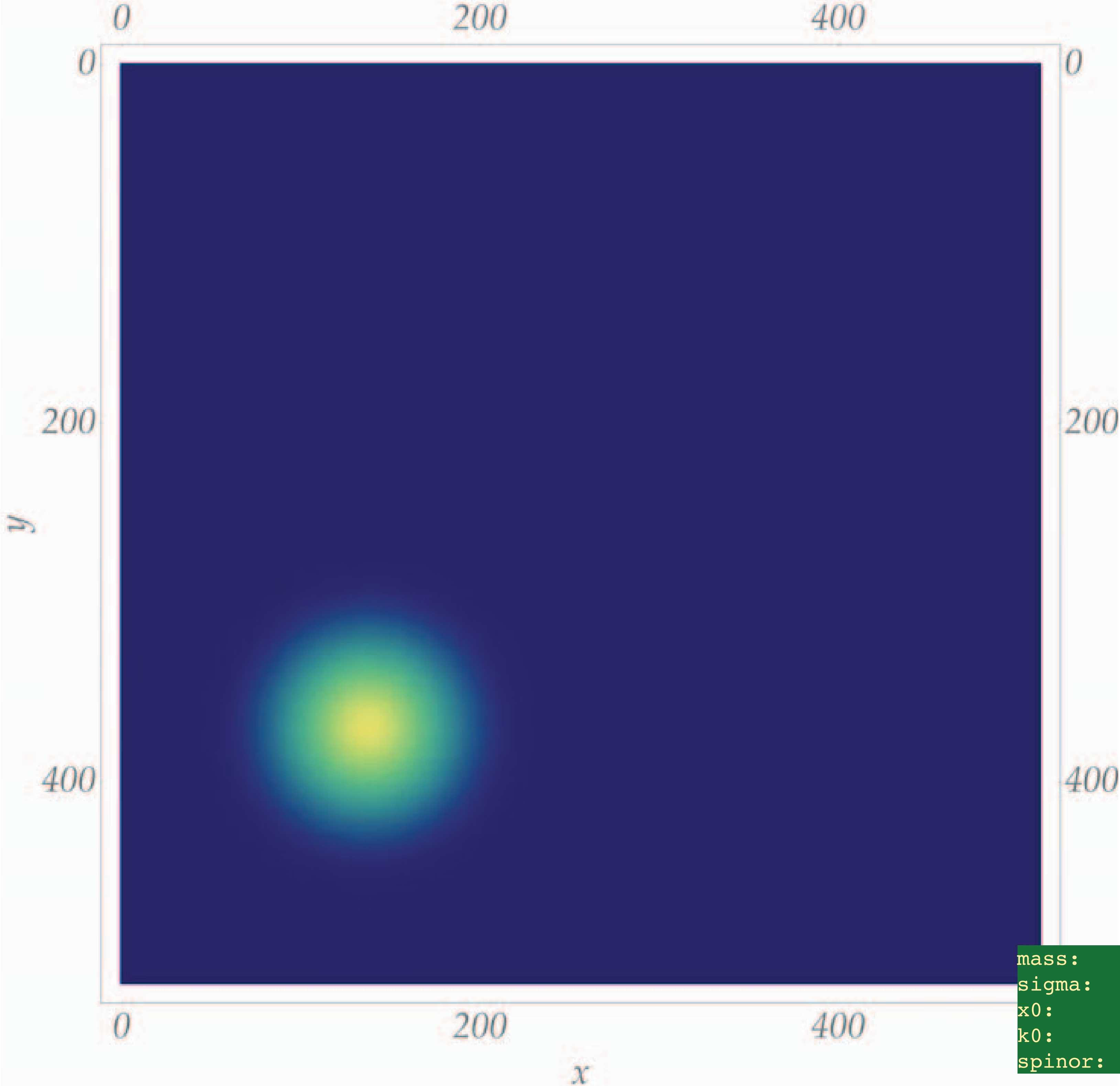



Free Quantum Field Theory: got it!



# 2d Dirac

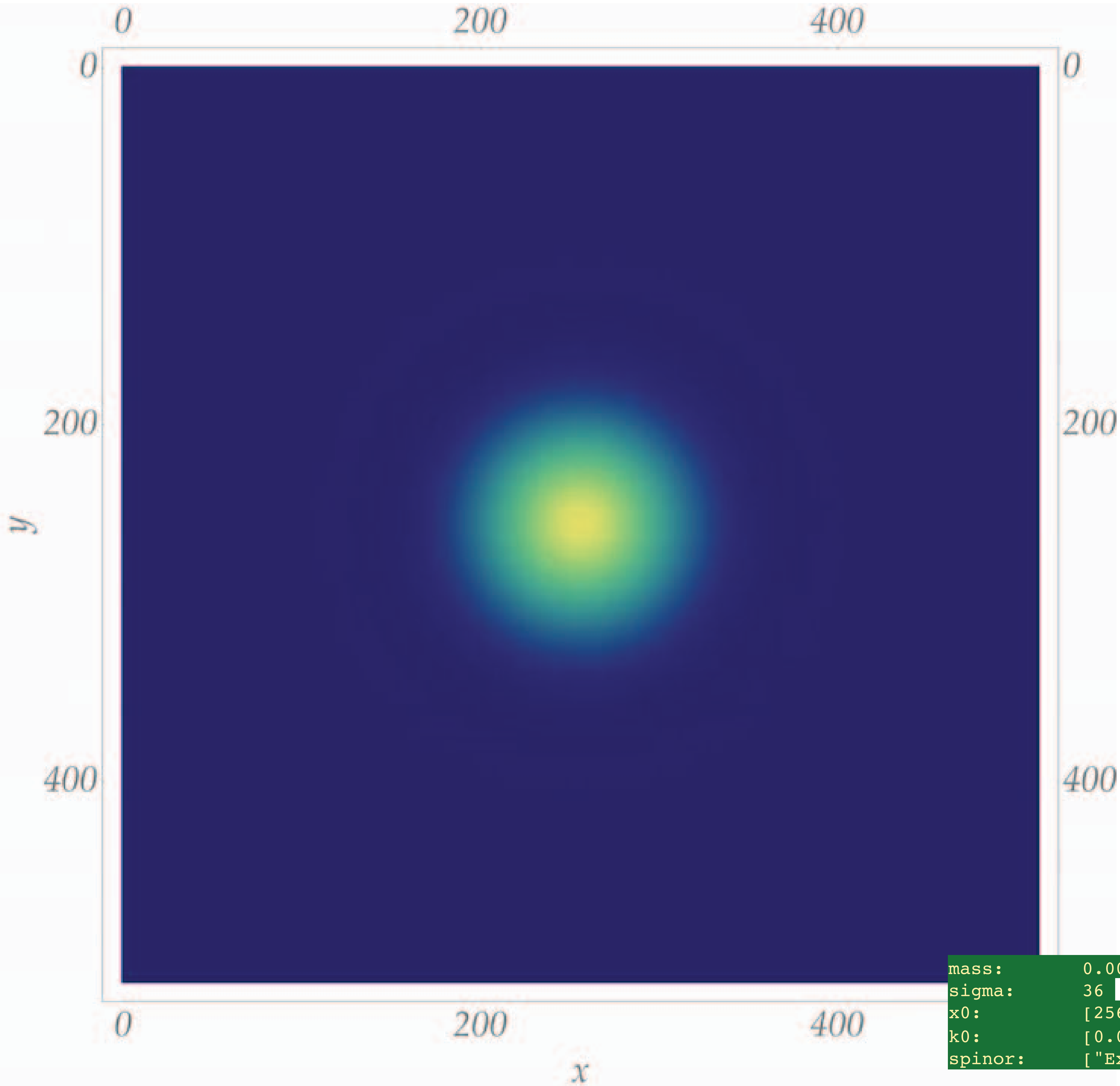
- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*




D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

# Dirac 3d

```
mass:       0.002
sigma:      32
x0:         [140,140,140]
k0:         [0.05,0.05,0.05]
spinor:     ["Exp[I k0.#]",0,0,0]
```




D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

# Dirac 3d

```
mass:           0.008
sigma:          36
x0:             [256,256,256]
k0:             [0.05,0.05,0.05]
spinor:         ["Exp[I k0.#]",0,0,"Exp[I k0.#]"]
```

G. M. D'Ariano, N. Mosco, P. Perinotti, A. Tosini,
PLA **378** 3165 (2014); EPL **109** 40012 (2015)

# Analytical solution of Dirac (d=1) and Weyl (d=1,2,3)

The analytical solution of the Dirac automaton can also be expressed in terms of Jacobi polynomials $P_k^{(\zeta,\rho)}$ performing the sum over $f$ in Eq. (16) which finally gives

$$\psi(x,t) = \sum_y \sum_{a,b\in\{0,1\}} \gamma_{a,b} P_k^{(1,-t)}\left(1 + 2\left(\frac{m}{n}\right)^2\right) A_{ab}\psi(y,0),$$

$$k = \mu_+ - \frac{a\oplus b + 1}{2},$$

$$\gamma_{a,b} = -(\mathrm{i}^{a\oplus b}) n^t \left(\frac{m}{n}\right)^{2+a\oplus b} \frac{k!\left(\mu_{(-)^{ab}} + \frac{\overline{a\oplus b}}{2}\right)}{(2)_k}, \qquad (18)$$

where $\gamma_{00} = \gamma_{11} = 0$ ($\gamma_{10} = \gamma_{01} = 0$) for $t + x - y$ odd (even) and $(x)_k = x(x+1)\cdots(x+k-1)$.

# Dispersive Schrödinger equation

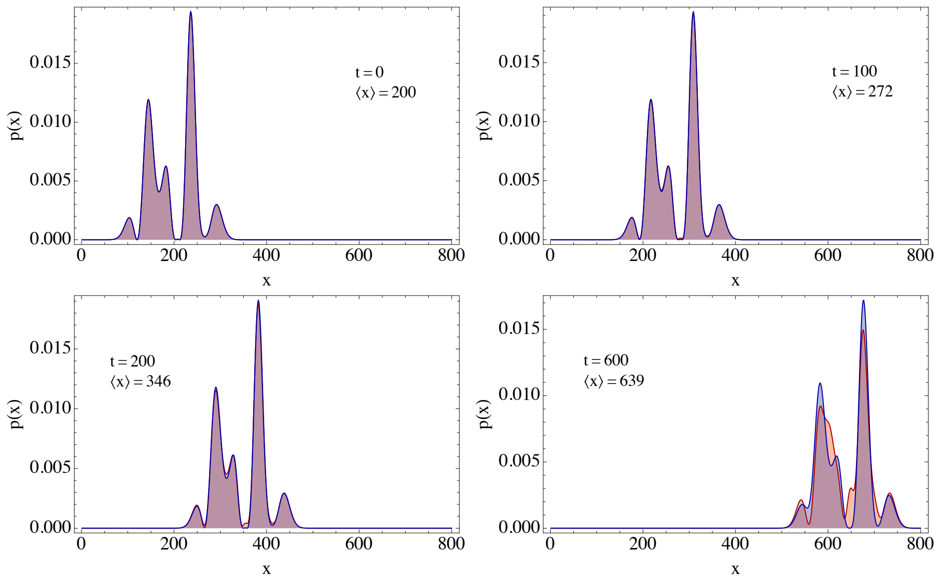
$$i\partial_t e^{-i\mathbf{k}_0\ \mathbf{x}+i\omega_0 t}\psi(\mathbf{k}\ \ t) = s[\omega(\mathbf{k}) - \omega_0]e^{-i\mathbf{k}_0\ \mathbf{x}+i\omega_0 t}\psi(\mathbf{k}\ \ t)$$

$$i\partial_t\psi(\mathbf{k}\ \ t) = s[\omega(\mathbf{k}) - \omega_0]\psi(\mathbf{k}\ \ t)$$

$$i\partial_t\psi(\mathbf{x}\ \ t) = s[\mathbf{v}\ \ \ \boldsymbol{\nabla} + \frac{1}{2}\mathbf{D}\ \ \ \boldsymbol{\nabla}\boldsymbol{\nabla}]\psi(\mathbf{x}\ \ t)$$

$$\mathbf{v} = (\boldsymbol{\nabla}_{\mathbf{k}}\omega)\,(\mathbf{k}_0)$$

$$\mathbf{D} = (\boldsymbol{\nabla}_{\mathbf{k}}\boldsymbol{\nabla}_{\mathbf{k}}\omega)\,(\mathbf{k}_0)$$



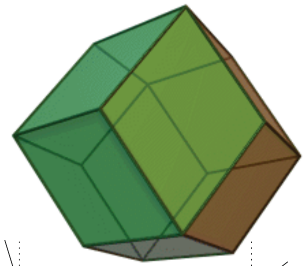

D'Ariano, Perinotti, PRA **90** 062106 (2014)

# The LTM standards of the theory

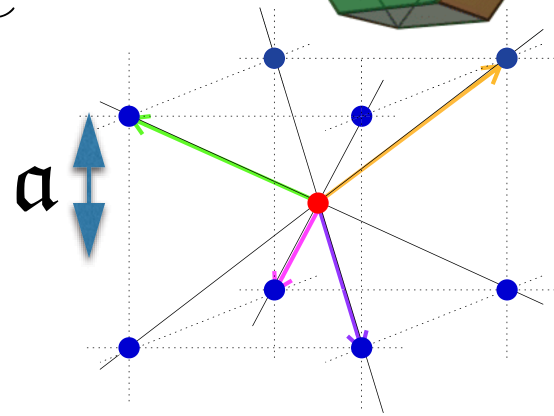
Dimensionless variables

$$x = \frac{x_{[m]}}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_{[sec]}}{\mathfrak{t}} \in \mathbb{Z}, \quad m = \frac{m_{[kg]}}{\mathfrak{m}} \in [0,1]$$

$$k/k_{max} \qquad \omega \quad \omega_{max}$$

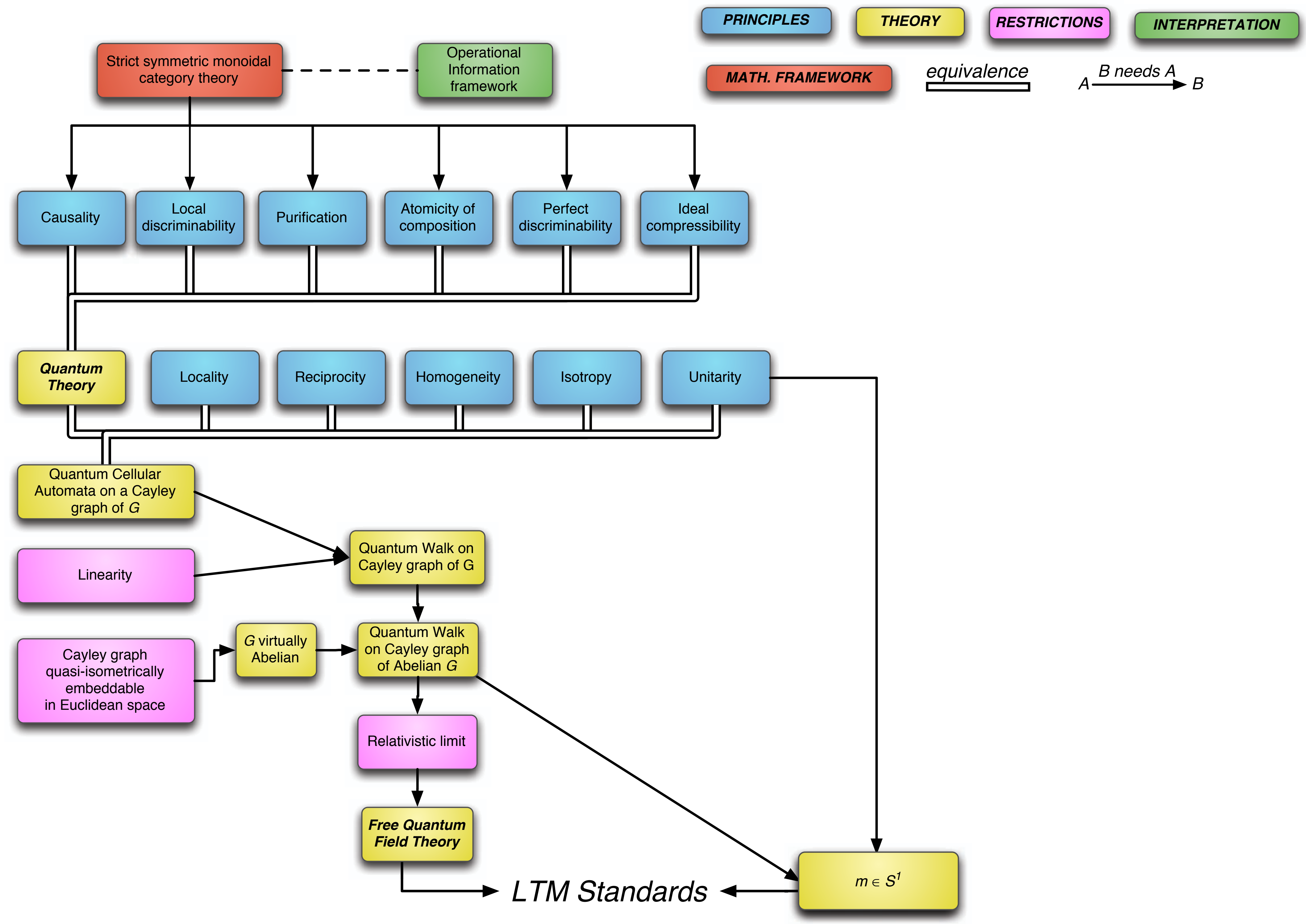


Relativistic limit: $\Rightarrow$ $c = \mathfrak{a}/\mathfrak{t} \qquad \hbar = \mathfrak{m}\mathfrak{a}c$




Measure $\mathfrak{a}$ ($k_{max}$) from light-refraction-index

$$\Rightarrow \quad c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$


PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of $G$
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
$G$ virtually Abelian
Quantum Walk on Cayley graph of Abelian $G$
Relativistic limit
Free Quantum Field Theory
LTM Standards
$m \in S^1$

Bisio, D'Ariano, Perinotti, unpublished

# Special Relativity recovered … and more

Relativity principle: invariance of the physical law under change of inertial reference frame

→ invariance of eigenvalue equation under change of representation.

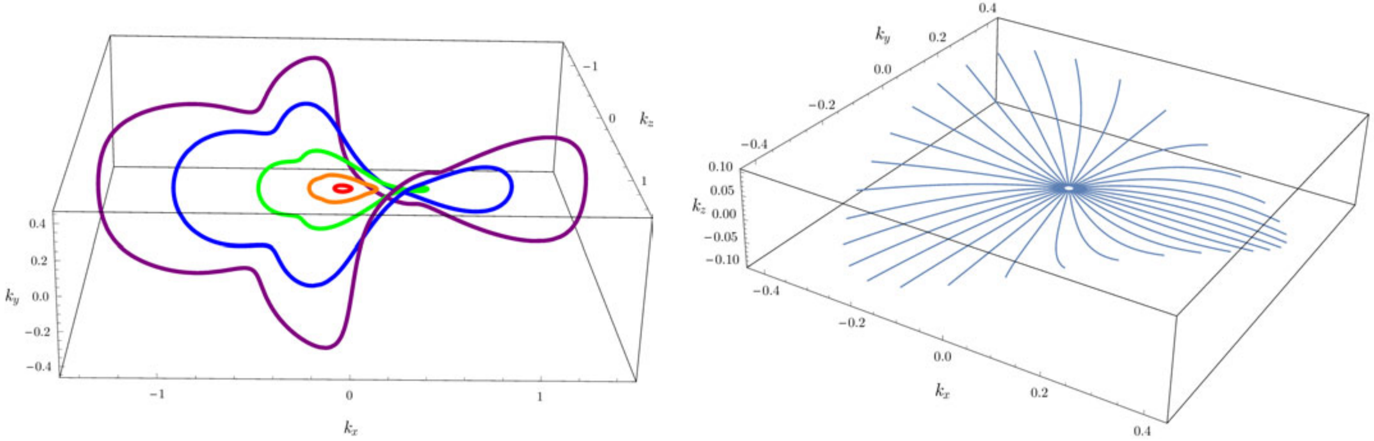



FIG. 2: The distortion effects of the Lorentz group for the discrete Planck-scale theory represented by the quantum walk in Eq. (6). Left figure: the orbit of the wavevectors $\mathbf{k} = (k_x, 0, 0)$, with $k_x \in \{.05, .2, .5, 1, 1.7\}$ under the rotation around the $z$ axis. Right figure: the orbit of wavevectors with $|\mathbf{k}| = 0.01$ for various directions in the $(k_x, k_y)$ plane under the boosts with $\boldsymbol{\beta}$ parallel to $\mathbf{k}$ and $|\boldsymbol{\beta}| \in [0, \tanh 4]$.

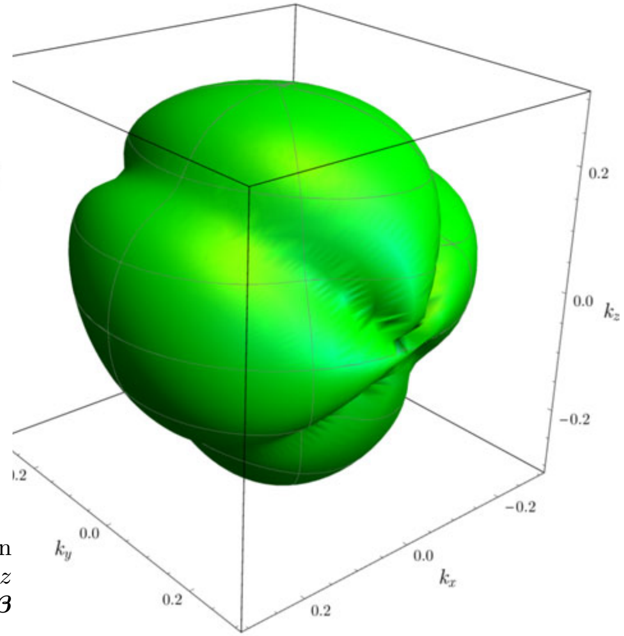



FIG. 3: The green surface represents the orbit of the wavevector $\mathbf{k} = (0.3, 0, 0)$ under the full rotation group $SO(3)$.

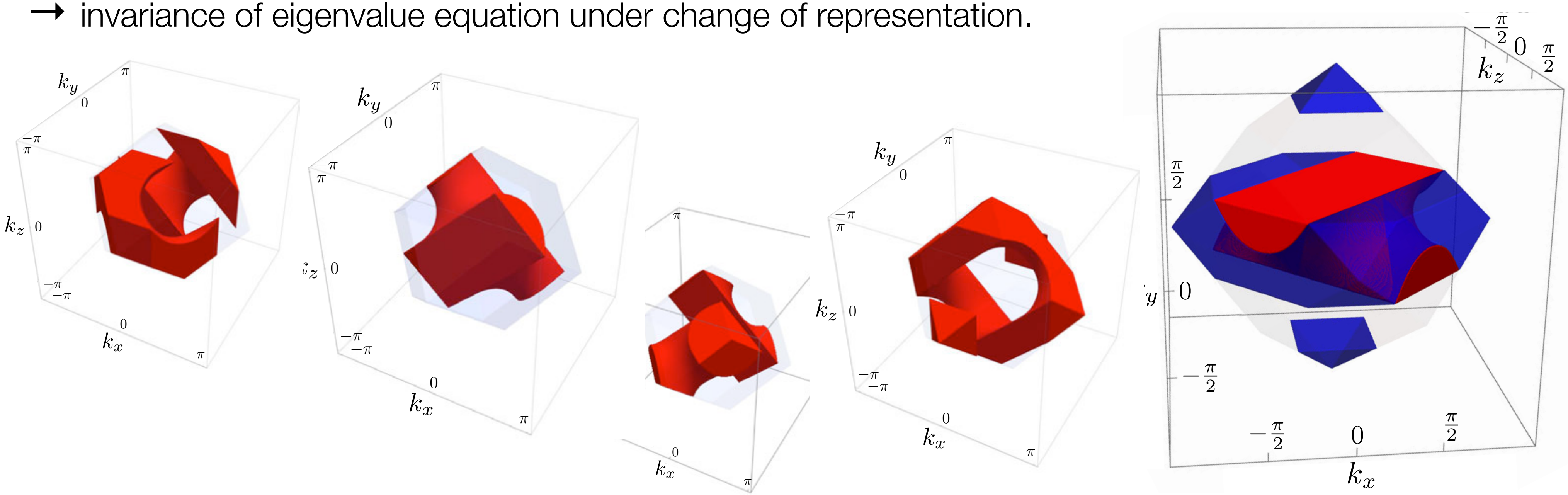
- Deformed Poincaré group
- Lorentz transformations are perfectly recovered for *k≪1.*
- For *k~1*:
  - *Double Special Relativity* (Camelia-Smolin)*.*
  - *Relative locality* (in addition to relativity of simultaneity)

Bisio, D'Ariano, Perinotti, unpublished

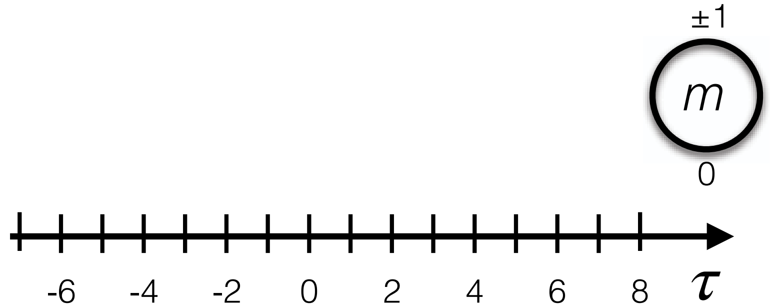
# Special Relativity recovered … and more

Relativity principle: invariance of the physical law under change of inertial reference frame

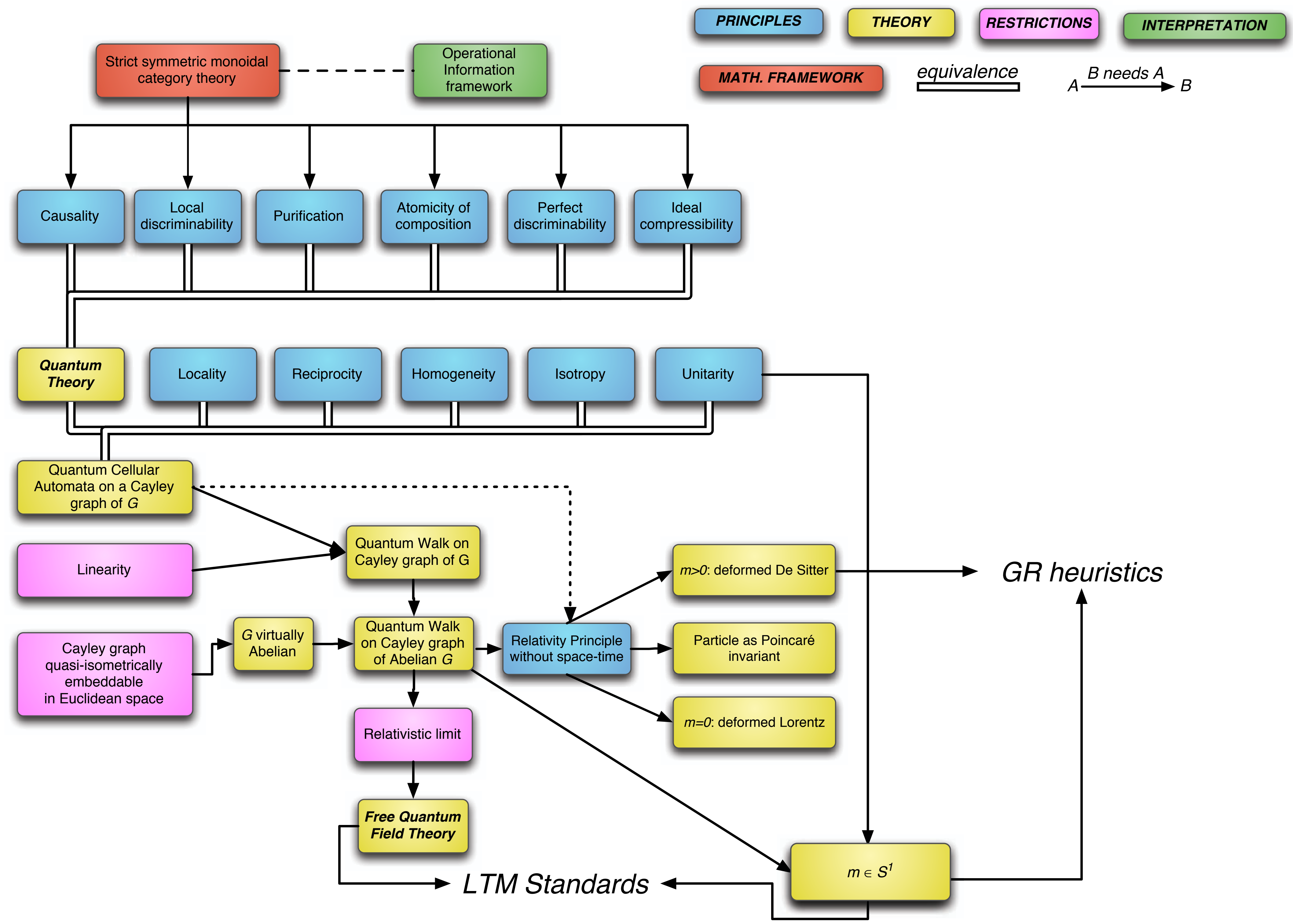
→ invariance of eigenvalue equation under change of representation.




- The Brillouin zone separates into four Poincaré-invariant regions diffeomorphic to balls, corresponding to four different particles.
- *m≠0* De Sitter *SO(1,4)*
- mass *m* and proper-time $\tau$ are conjugated




$$H(q_\alpha \ p_\alpha \ \tau \ m) = \sum_\alpha p_\alpha \dot{q}_\alpha + c^2 m \dot{\tau} - L$$



PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
m ∈ S1

Bisio, D'Ariano, Perinotti, Tosini
unpublished

# Interacting Dirac QCA in 1+1!

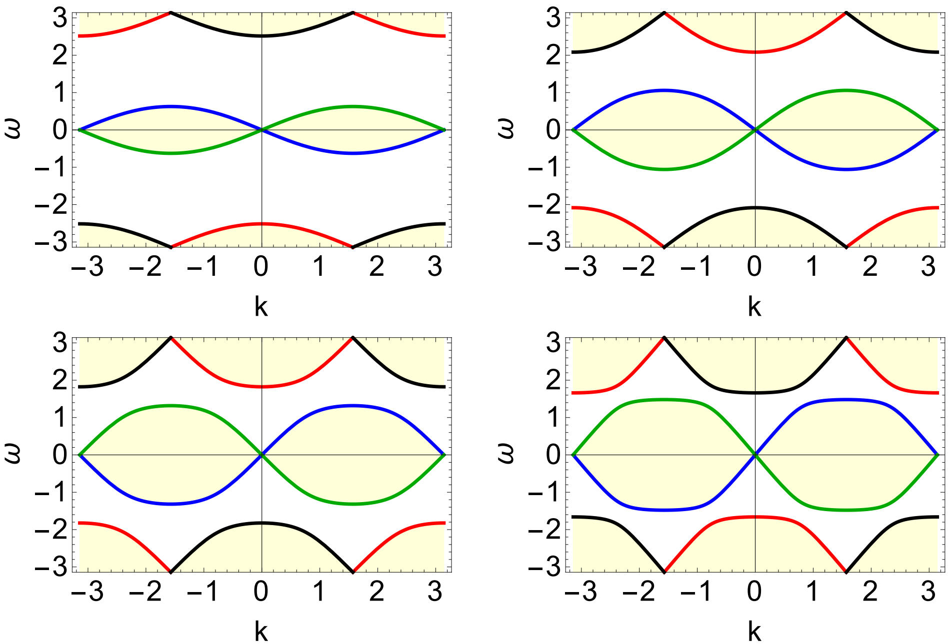



FIG. 1: Dispersion relation of the two particle Dirac Quantum Walk. The eigenvalue of the eigenstates $|++\rangle, |--\rangle, |+-\rangle$ and $|-+\rangle$ are respectively depicted in black, red, blue and green. The eigenvalues are plotted in terms of the relative momentum $k$, while the mass $m$ and the total momentum $p$ are fixes. The mass and total momentum parameters are $m = 0.9, 0.7, 0.5, 0.3$ and $p = -3\pi/4, -\pi/4, \pi/4, 3\pi/4$ from the top left to the bottom right.

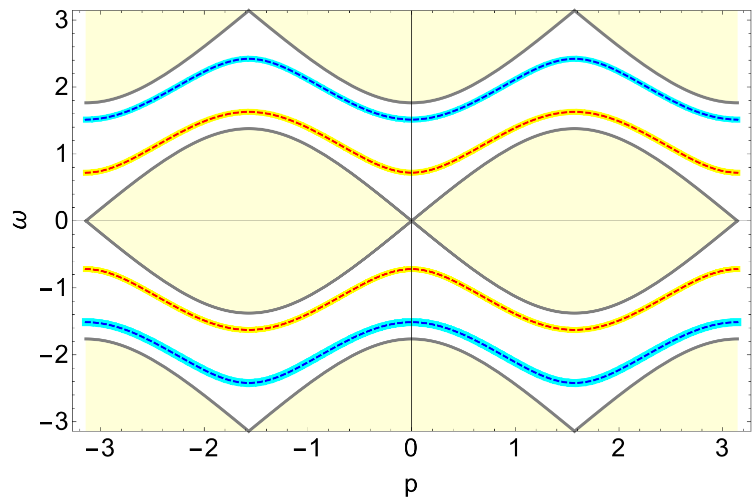
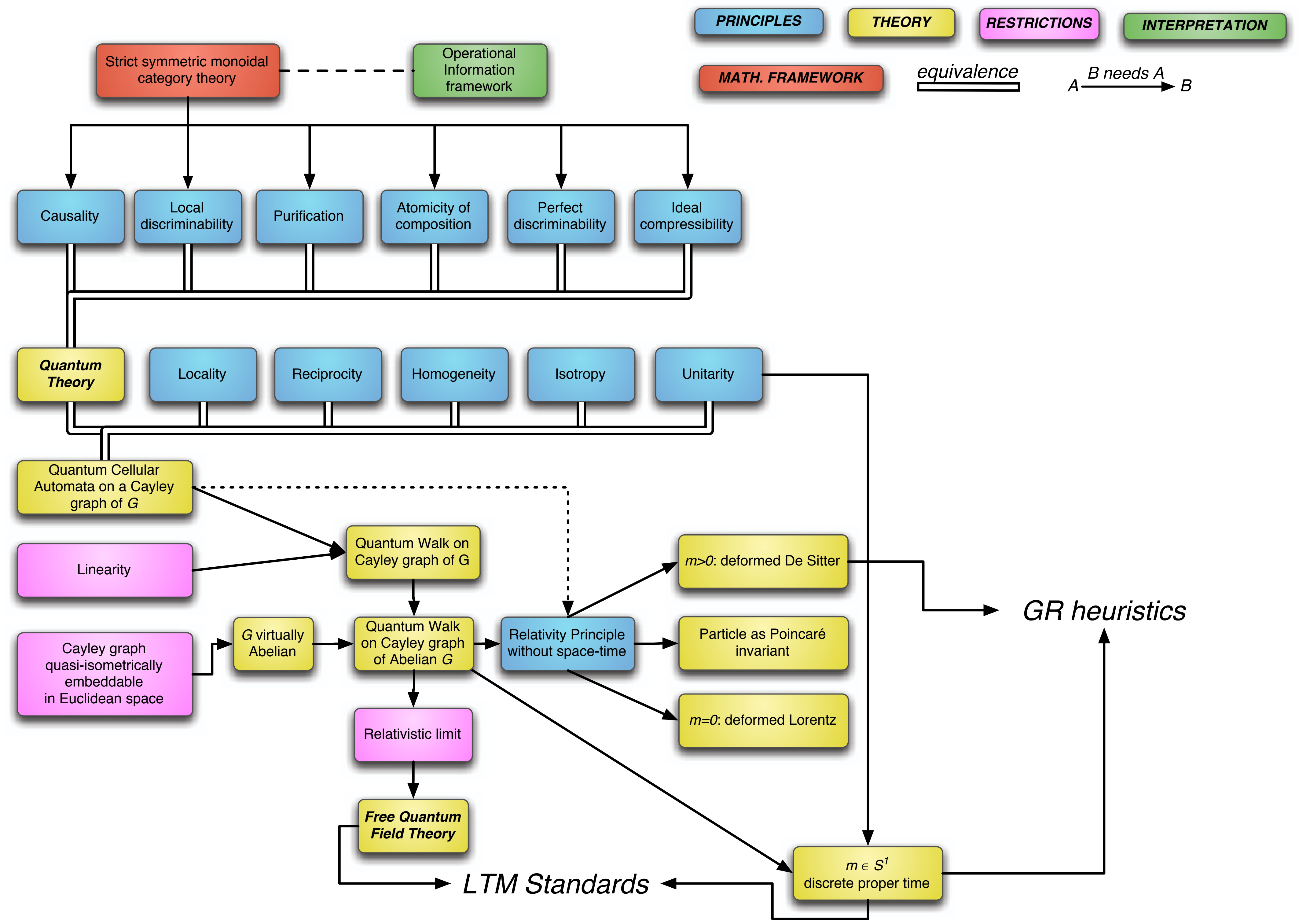



FIG. 2: Discrete spectrum of the Hubbard QCA for $\chi = 1.14$. $\omega_+$ in red and $\omega_-$ in blue.

This is more or less what I wanted to say

Thank you for your attention


PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
A
B needs A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
m ∈ S1 discrete proper time