QUIT quantum information theory group

JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

INFN Istituto Nazionale di Fisica Nucleare

Quantum Theory is an Information Theory

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

48 Symposium on Mathematical Physics
Gorini-Kossakowski-Lindblad-Sudarshan Master Equation - 40 Years After

Toruń, Poland, June 10-12, 2016

Quantum Theory is an Information Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

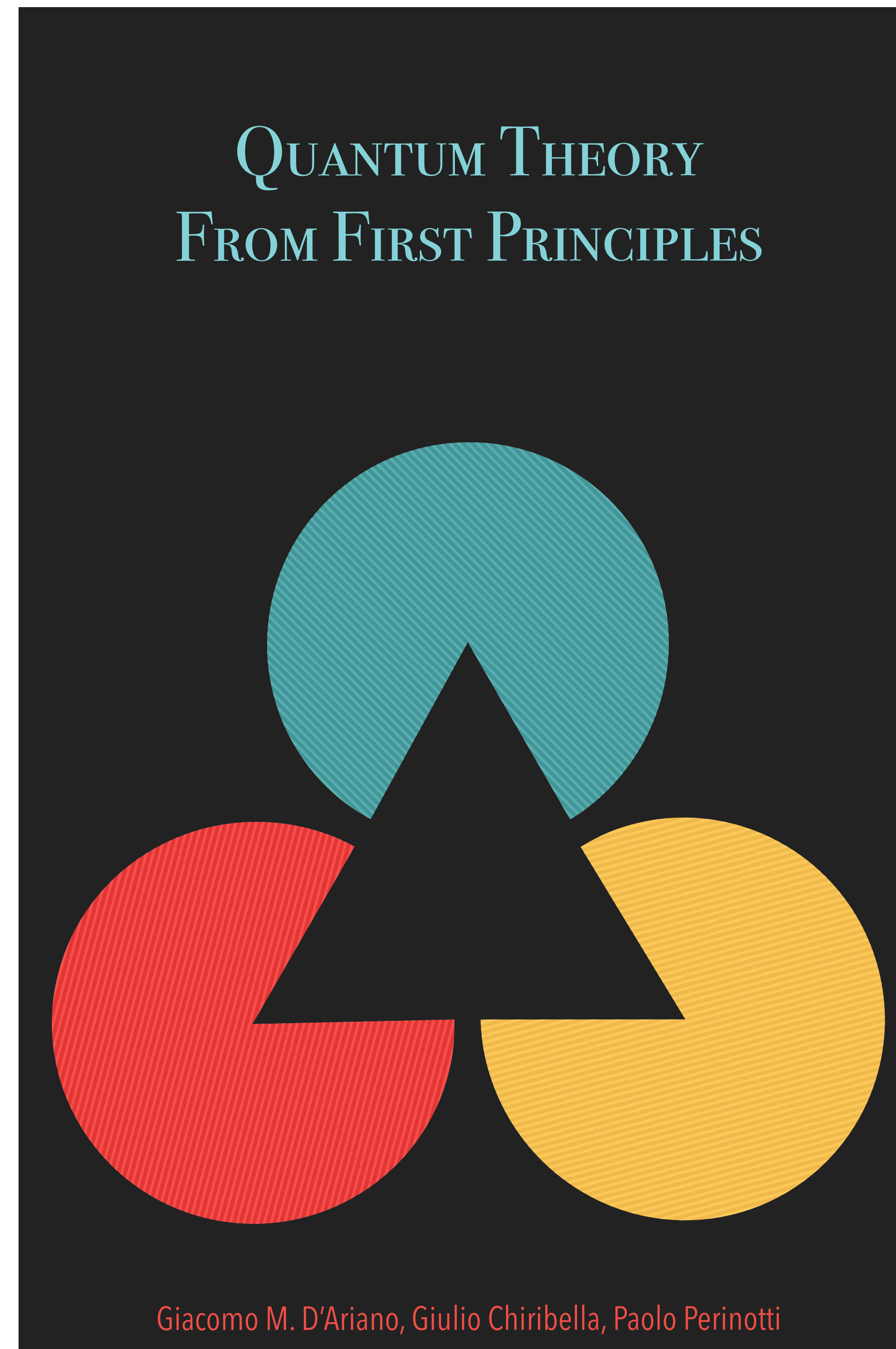
Informational derivation of quantum theory

Giulio Chiribella*
Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Ontario, Canada N2L 2Y5†

Giacomo Mauro D'Ariano‡ and Paolo Perinotti§
QUIT Group, Dipartimento di Fisica "A. Volta" and INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy‖
(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03.67.Ac, 03.65.Ta



Operational Probabilistic Theory

The framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

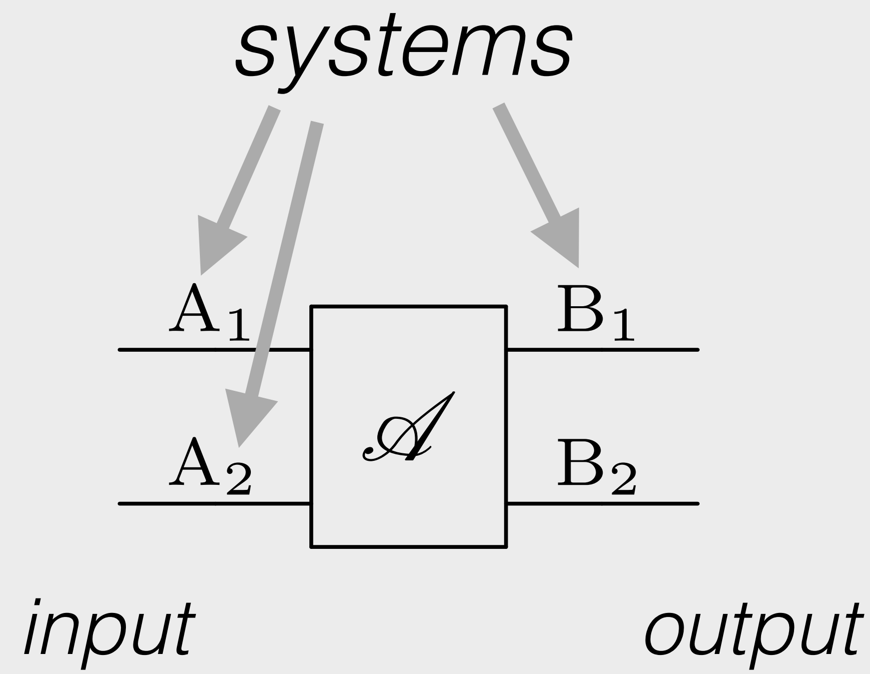
$$p(i, j, k, \ldots | \text{circuit})$$

Marginal probability

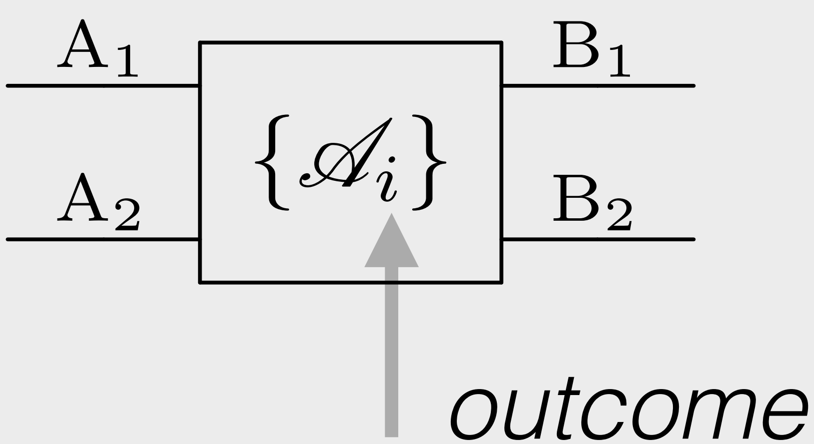
$$\sum_{i,k,\ldots} p(i, j, k, \ldots | \text{circuit}) =$$

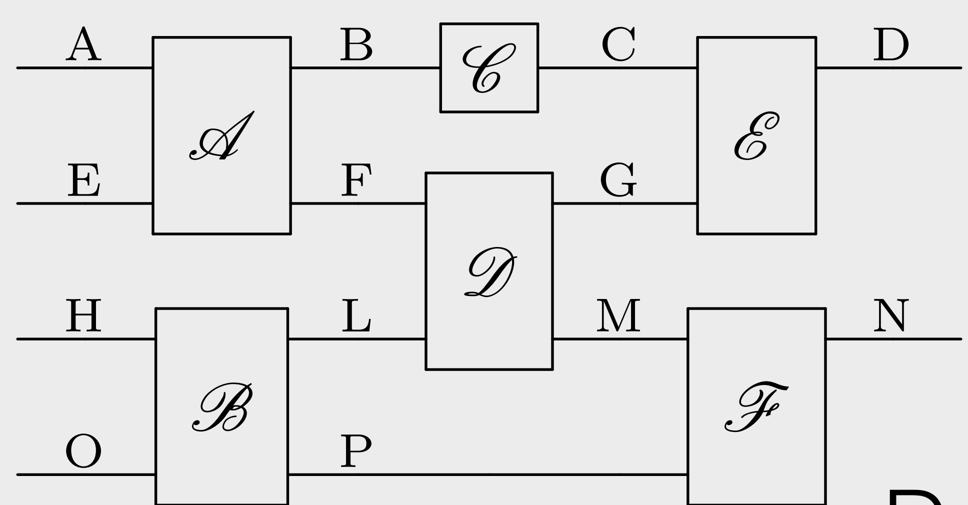
$$p(j | \text{circuit})$$

Event



Test





DAG

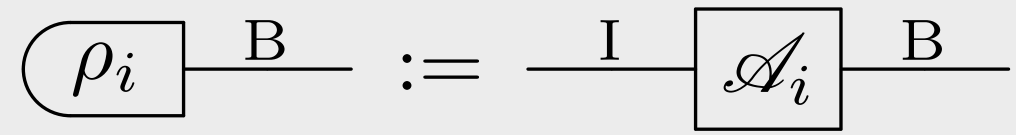
Operational Probabilistic Theory

The framework

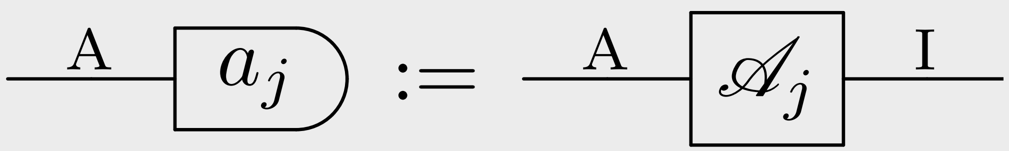
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

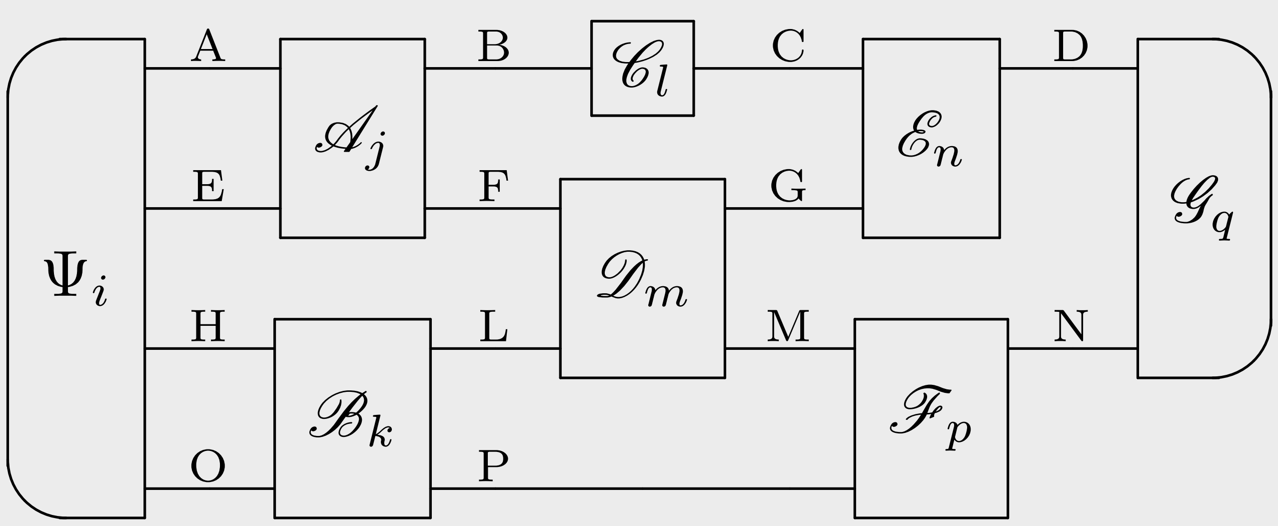


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

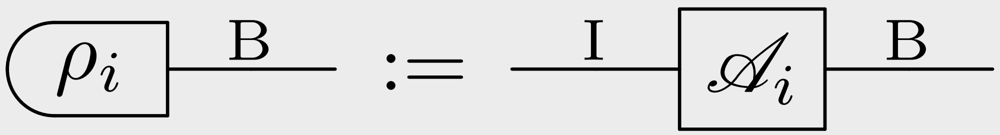
The framework

Logic ⊂ Probability ⊂ OPT

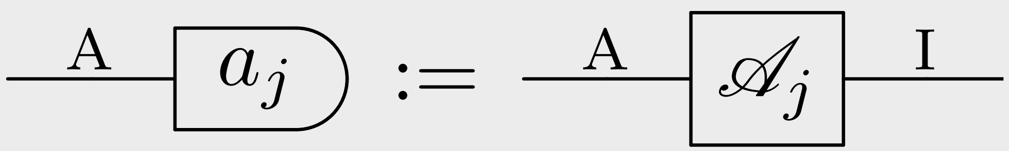
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Notice: the probability of a “preparation” generally depends on the circuit at its output.

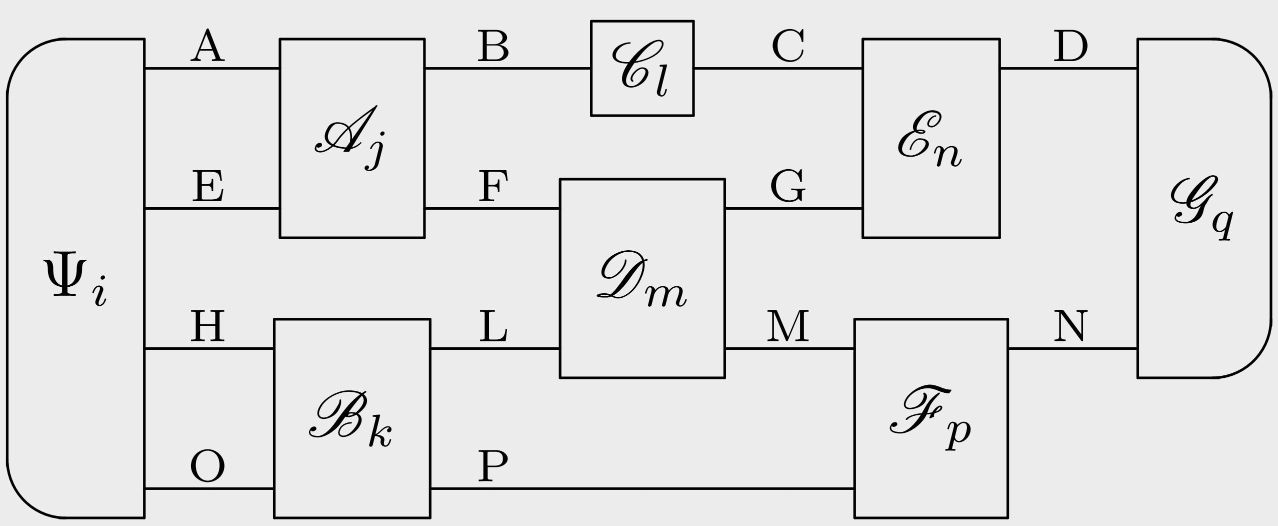


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



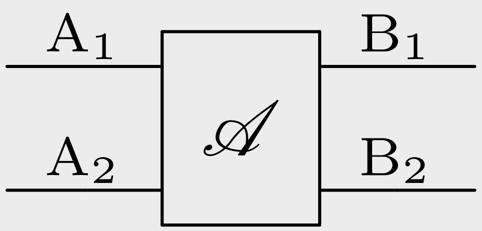
Operational Probabilistic Theory

The framework

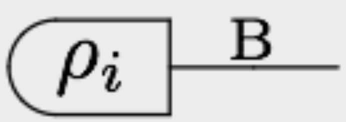
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

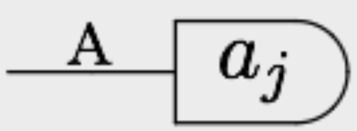
Probabilistic equivalence classes



transformation

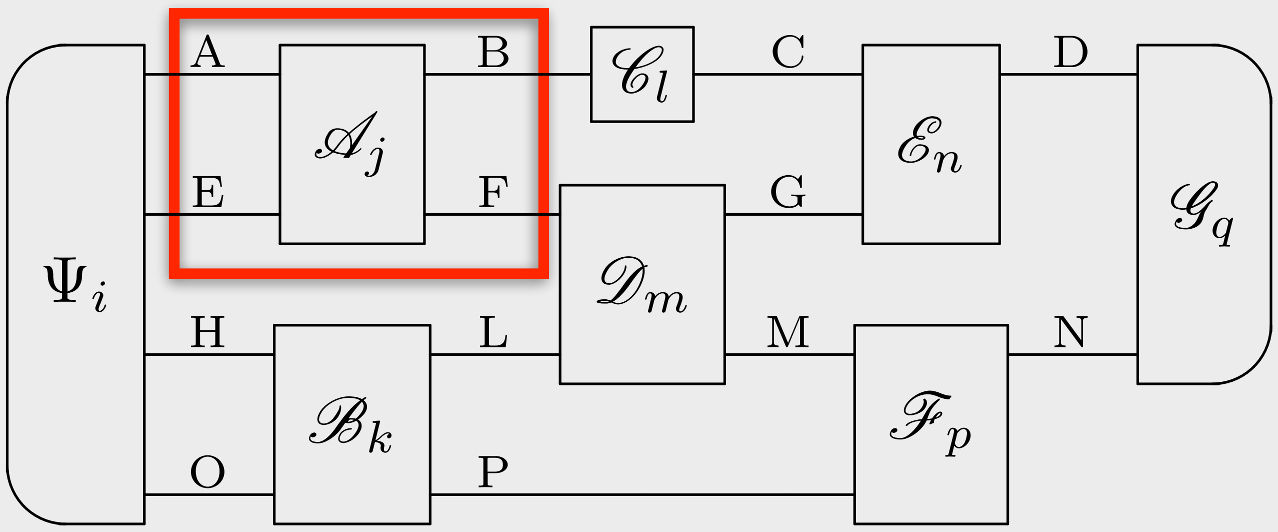


state



effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

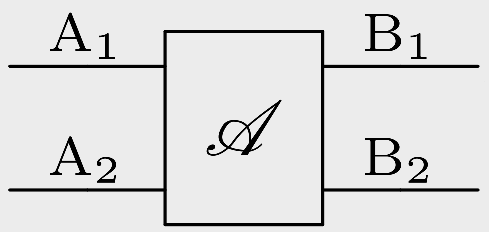
The framework

Logic ⊂ Probability ⊂ OPT

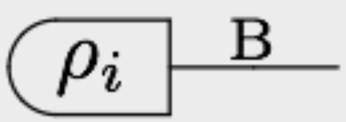
joint probabilities + connectivity

Probabilistic equivalence classes

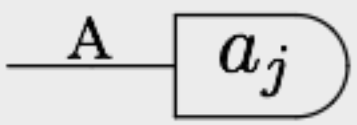
Notice: the probability of a transformation generally depends on the circuit at its output!!



transformation

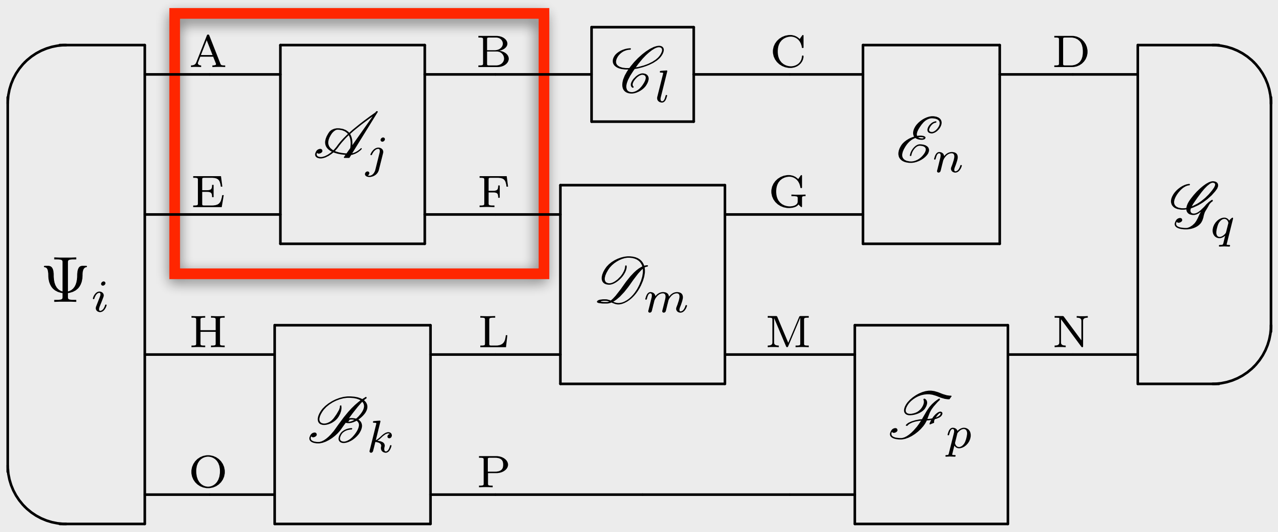


state



effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

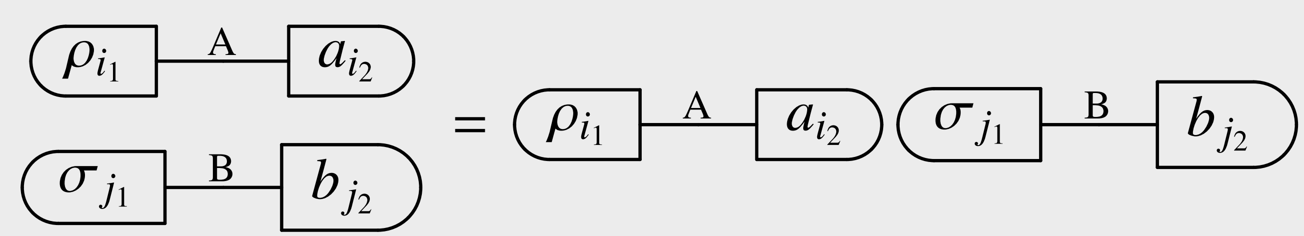
The framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

Probabilistic equivalence classes

Multiplication of closed circuits



$$= p(i_1, i_2)\, q(j_1, j_2)$$

Operational Probabilistic Theory

The framework

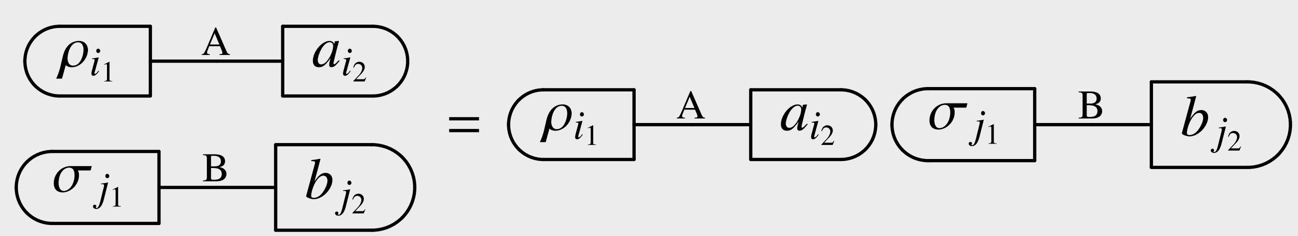
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

Probabilistic equivalence classes

monoidal category theory

Multiplication of closed circuits



$$= p(i_1, i_2)\, q(j_1, j_2)$$

Operational Probabilistic Theory

Sequential composition (associative)

$$\overset{\mathrm{A}}{-\!\!-\!\!-}\boxed{\{\mathscr{A}_x\}_{x\in\mathrm{X}}}\overset{\mathrm{B}}{-\!\!-\!\!-}\boxed{\{\mathscr{B}_y\}_{y\in\mathrm{Y}}}\overset{\mathrm{C}}{-\!\!-\!\!-} \quad =: \quad \overset{\mathrm{A}}{-\!\!-\!\!-}\boxed{\{\mathscr{B}_x \circ \mathscr{A}_y\}_{(x,y)\in\mathrm{X}\times\mathrm{Y}}}\overset{\mathrm{C}}{-\!\!-\!\!-}$$

Operational Probabilistic Theory

Sequential composition (associative)

$$\overset{\mathrm{A}}{\text{---}}\boxed{\{\mathscr{A}_x\}_{x\in \mathrm{X}}}\overset{\mathrm{B}}{\text{---}}\boxed{\{\mathscr{B}_y\}_{y\in \mathrm{Y}}}\overset{\mathrm{C}}{\text{---}} \;=:\; \overset{\mathrm{A}}{\text{---}}\boxed{\{\mathscr{B}_x \circ \mathscr{A}_y\}_{(x,y)\in \mathrm{X}\times\mathrm{Y}}}\overset{\mathrm{C}}{\text{---}}$$

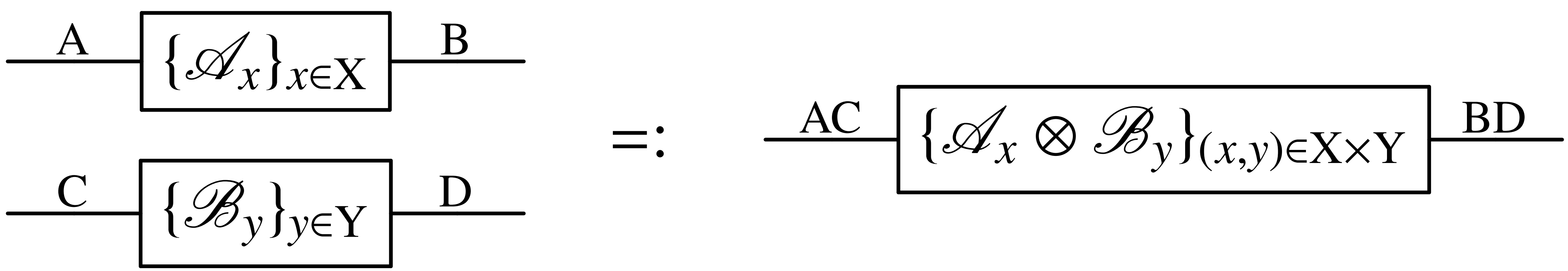
Identity test

$$\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{I}_\mathrm{A}}\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{---}} \;=\; \overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{---}}$$

$$\overset{\mathrm{B}}{\text{---}}\boxed{\mathscr{D}}\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{I}_\mathrm{A}}\overset{\mathrm{A}}{\text{---}} \;=\; \overset{\mathrm{B}}{\text{---}}\boxed{\mathscr{D}}\overset{\mathrm{A}}{\text{---}}$$

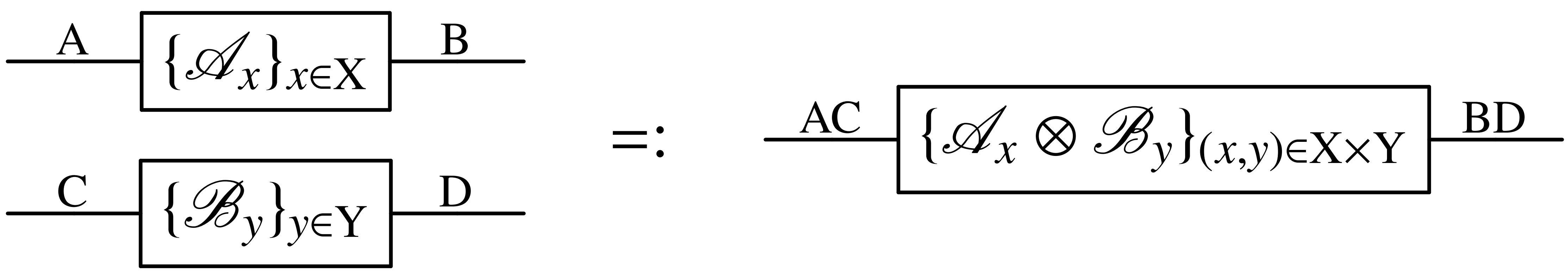
Operational Probabilistic Theory

Parallel composition (associative)



Operational Probabilistic Theory

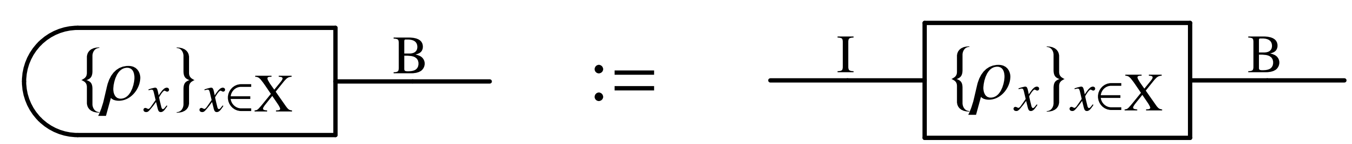
Parallel composition (associative)

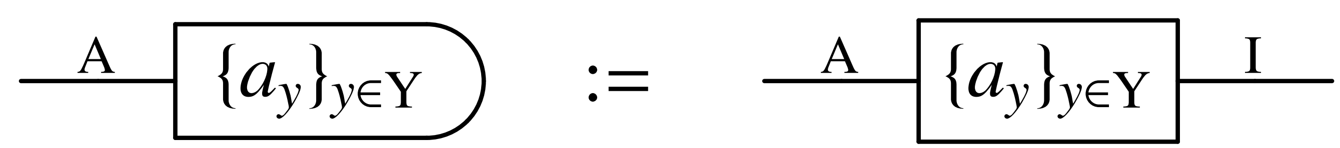


$$\mathrm{AB} = \mathrm{BA}$$

$$\mathrm{AI} = \mathrm{IA} = \mathrm{A}$$

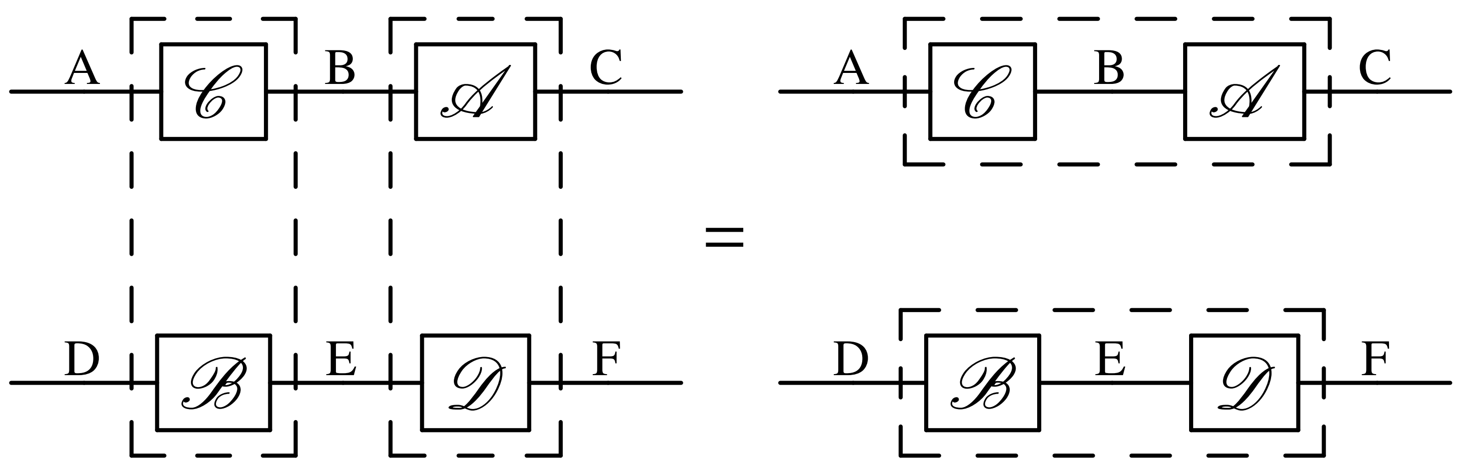
$$\mathrm{A(BC)} = \mathrm{(AB)C}$$





Operational Probabilistic Theory

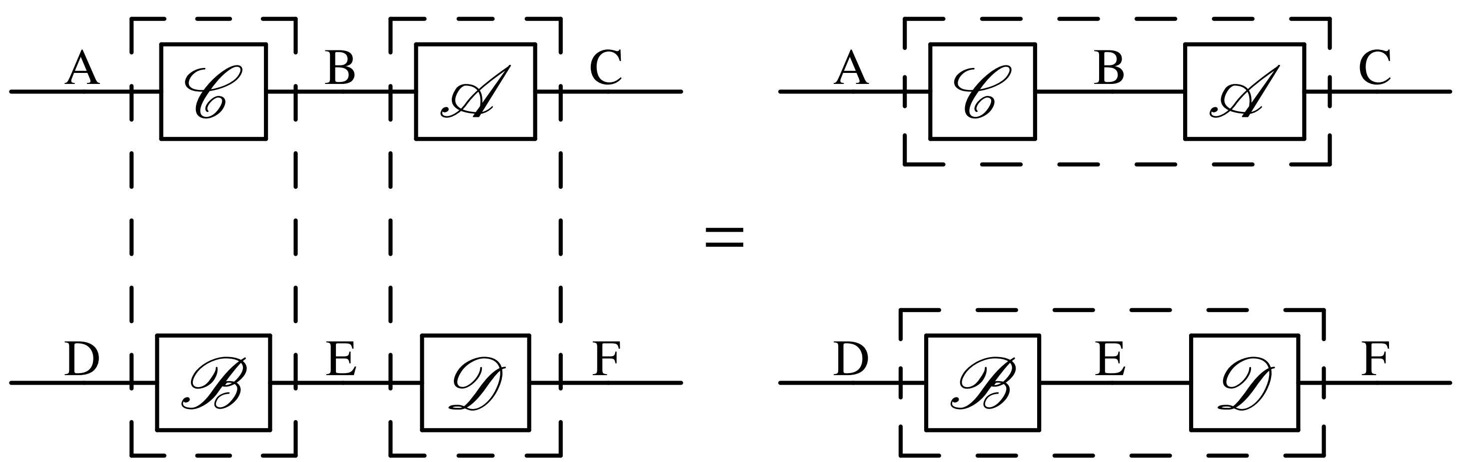
Sequential and parallel compositions commute



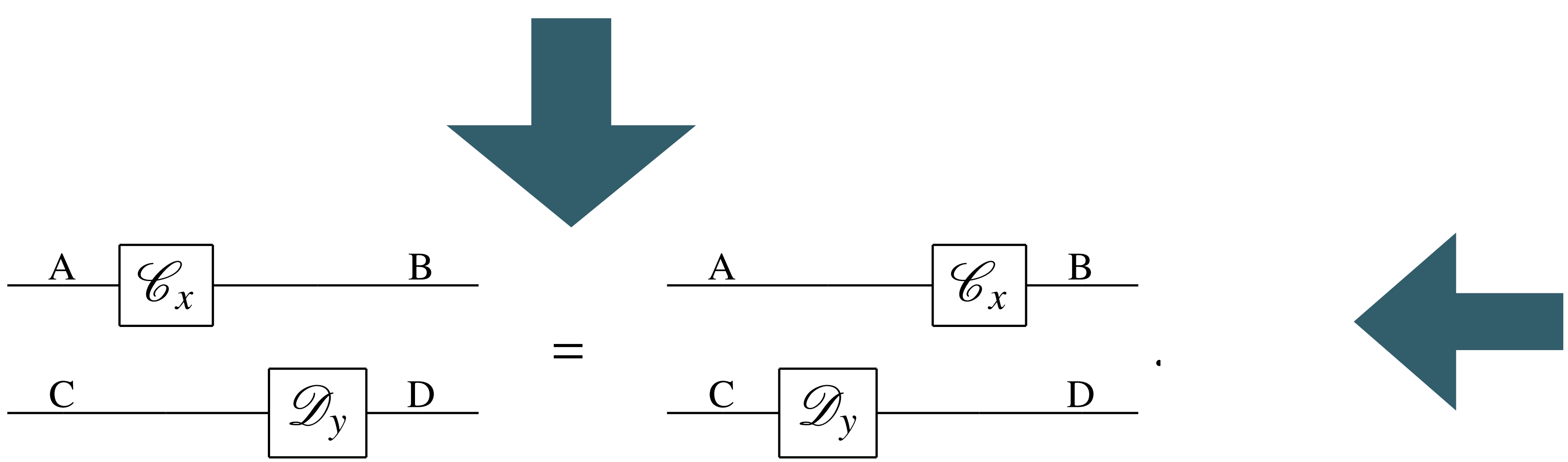
$$(\mathscr{A} \otimes \mathscr{D}) \circ (\mathscr{C} \otimes \mathscr{B}) = (\mathscr{A} \circ \mathscr{C}) \otimes (\mathscr{D} \circ \mathscr{B})$$

Operational Probabilistic Theory

Sequential and parallel compositions commute



$$(\mathscr{A} \otimes \mathscr{D}) \circ (\mathscr{C} \otimes \mathscr{B}) = (\mathscr{A} \circ \mathscr{C}) \otimes (\mathscr{D} \circ \mathscr{B})$$



wire-stretching

(foliations)

Operational Probabilistic Theory

The framework

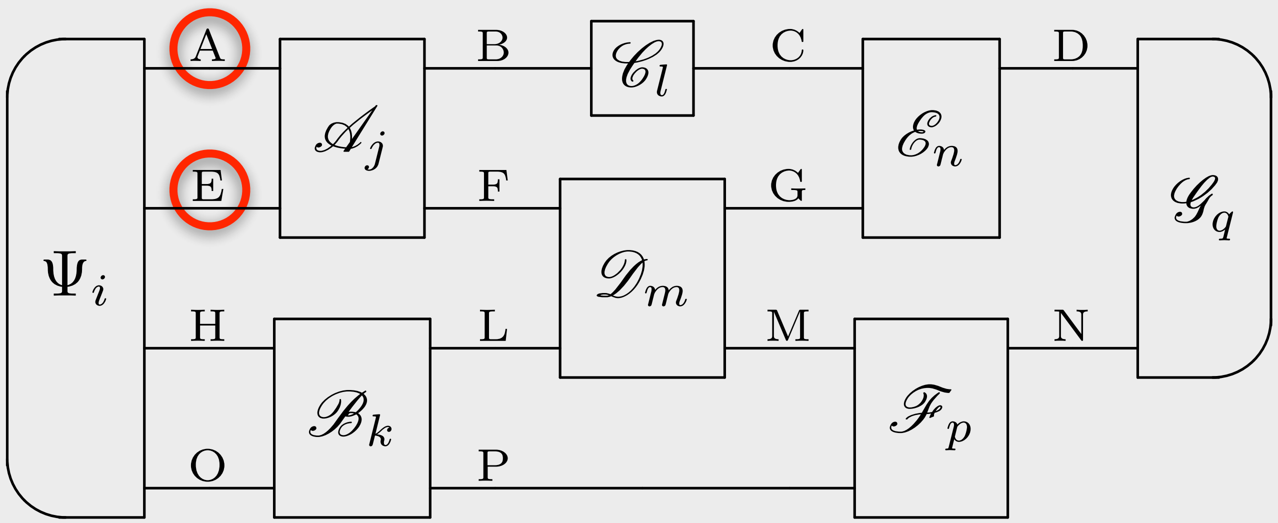
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

independent systems

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

The framework

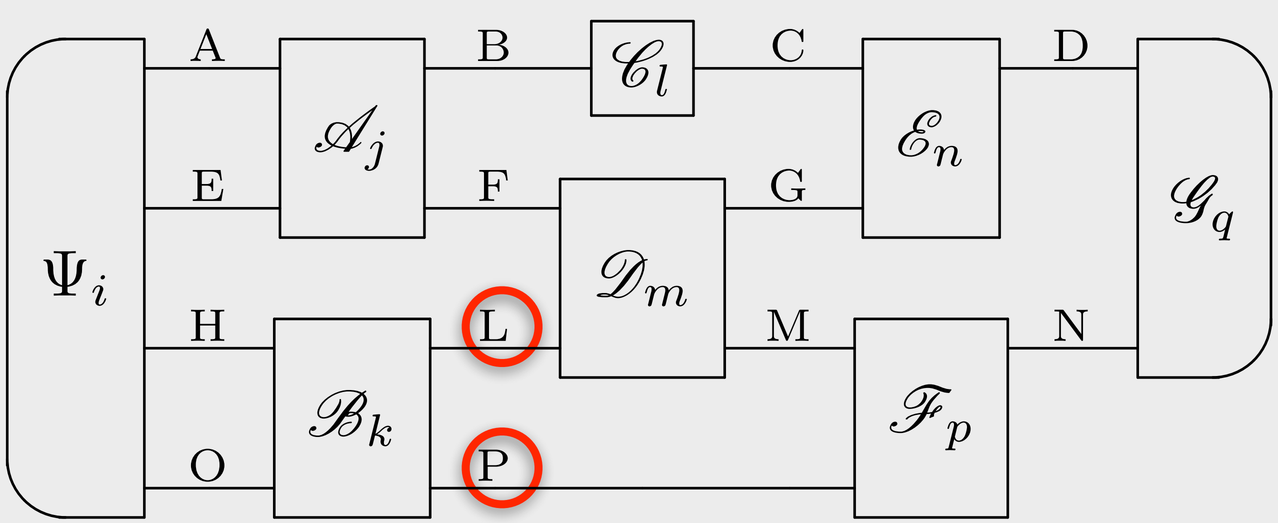
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, ...|\text{circuit})$$

independent systems

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

The framework

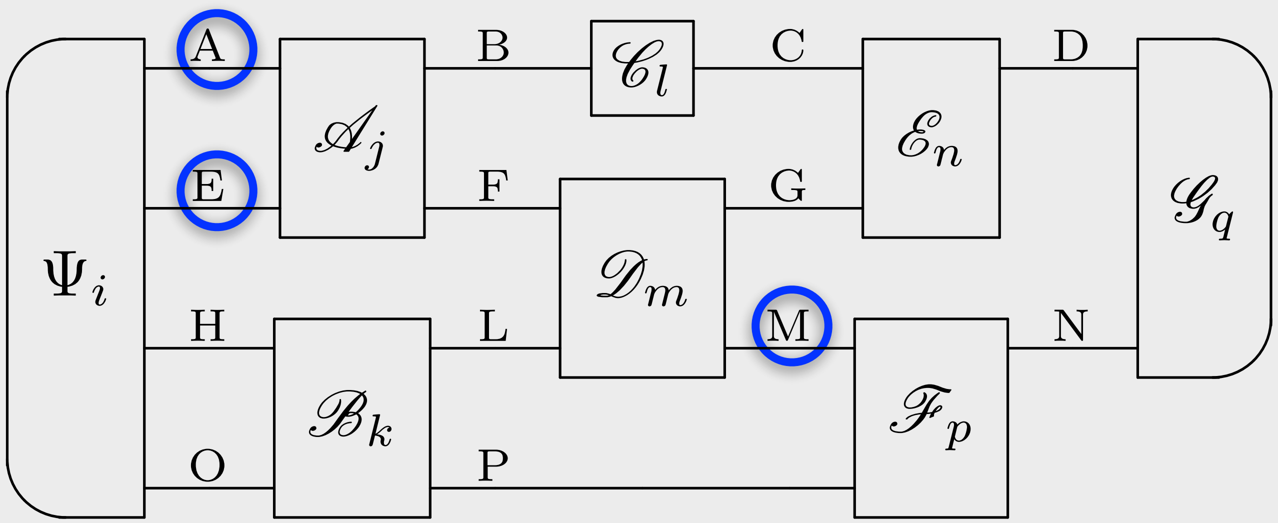
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, ...|\text{circuit})$$

NOT independent systems

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

The framework

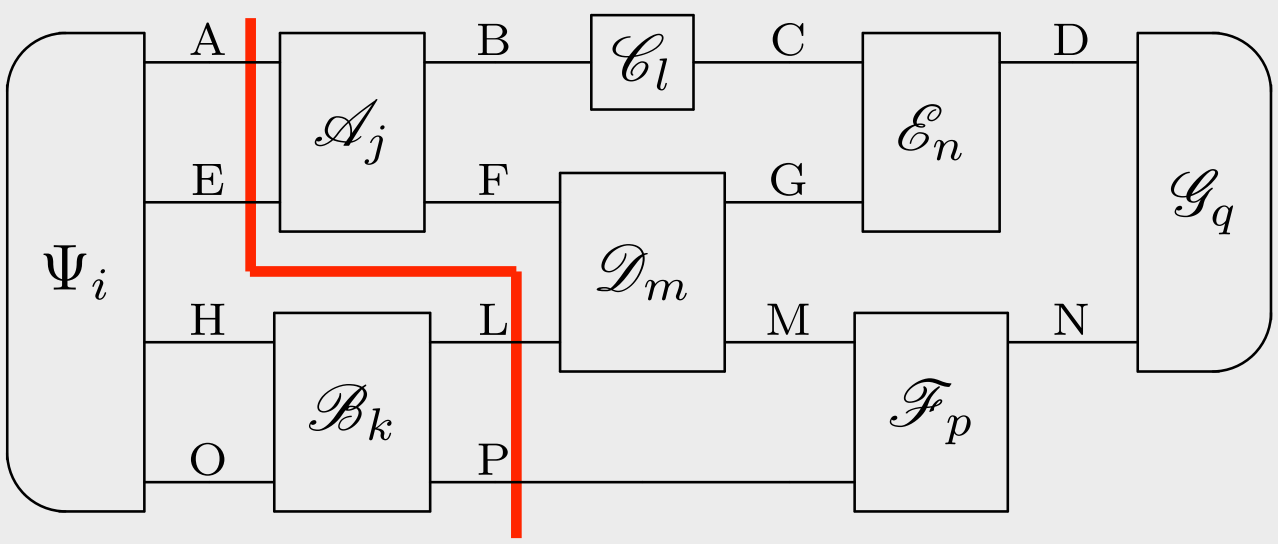
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Maximal set of
independent systems
= *"leaf"*

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

The framework

Logic ⊂ Probability ⊂ OPT

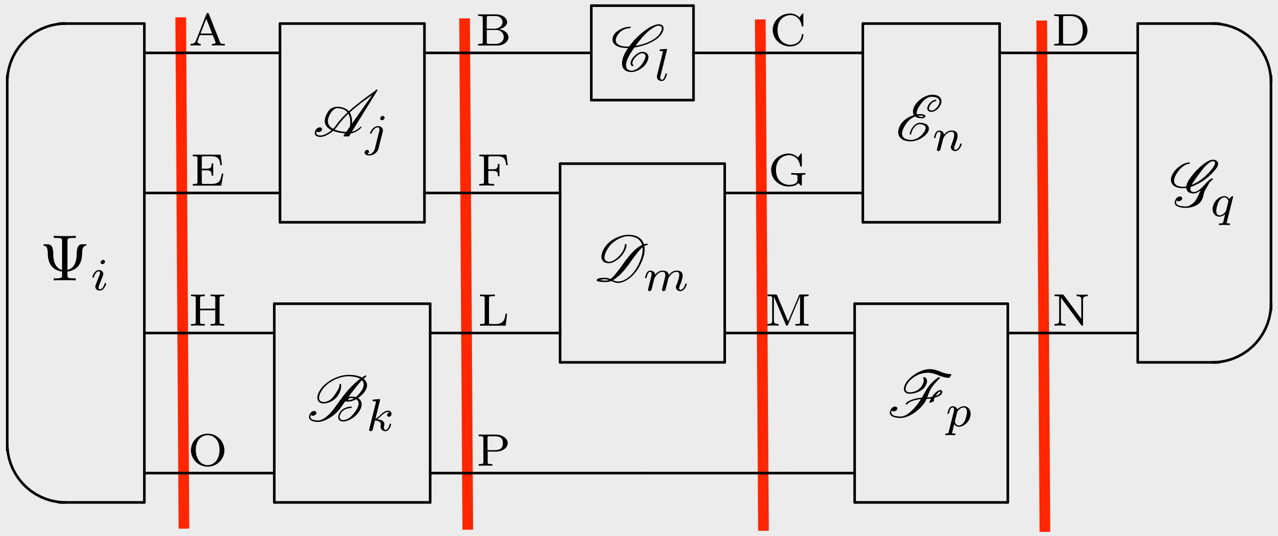
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Maximal set of
independent systems
= *"leaf"*

Foliation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

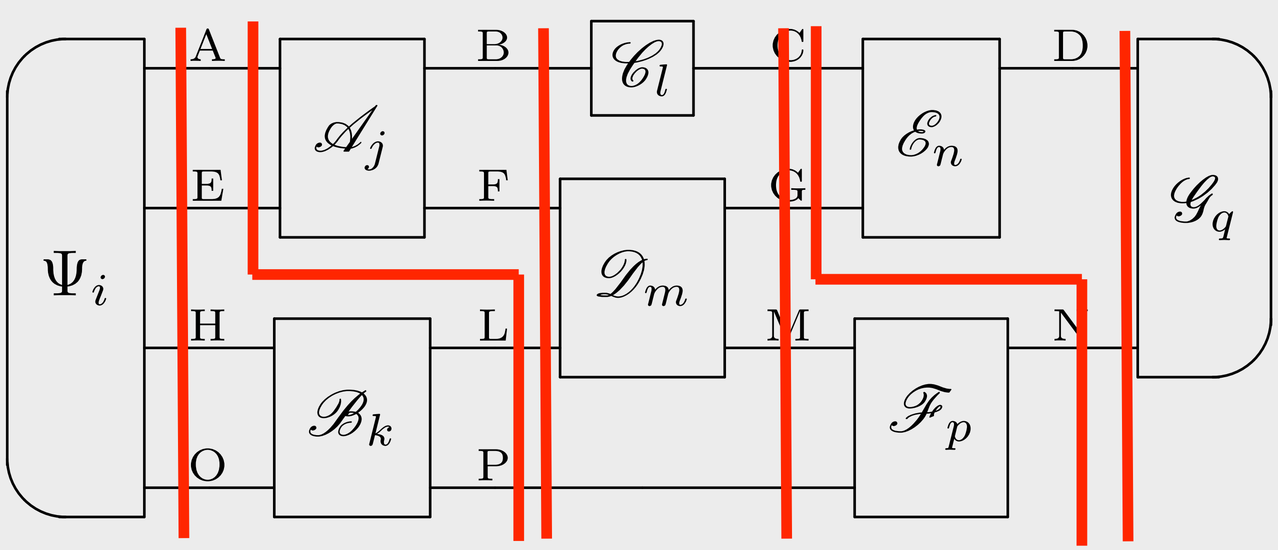
The framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, ...|\text{circuit})$$

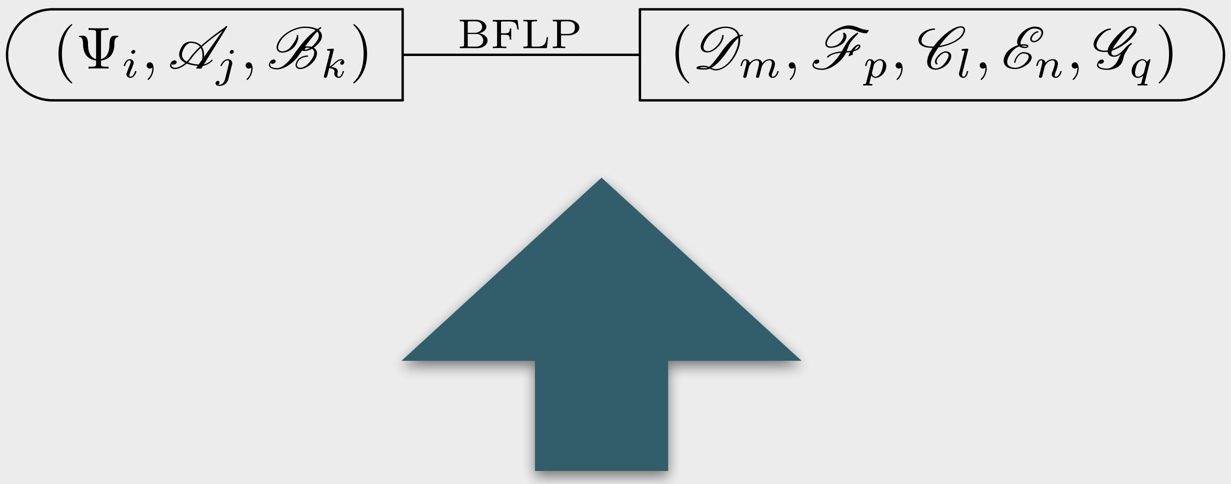
$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



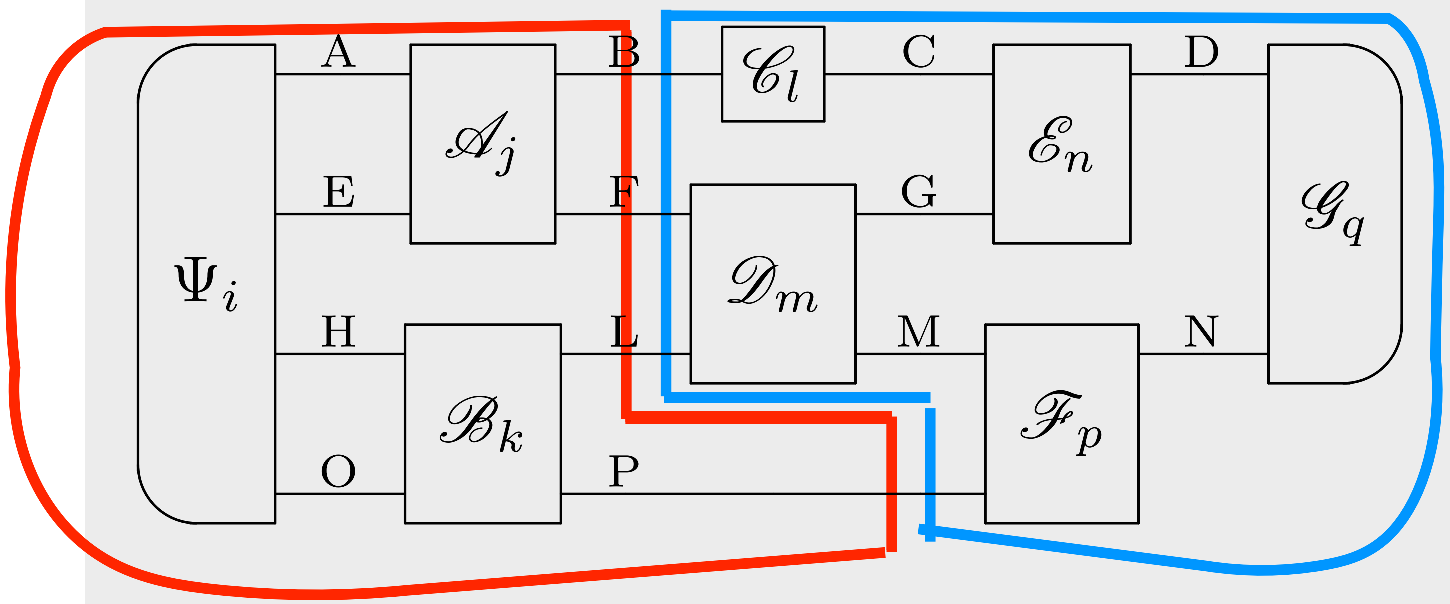
Maximal set of independent systems = *"leaf"*

Foliation

Operational Probabilistic Theory



$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

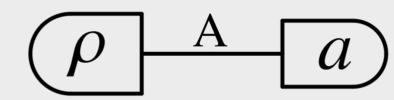
States are functionals for effects

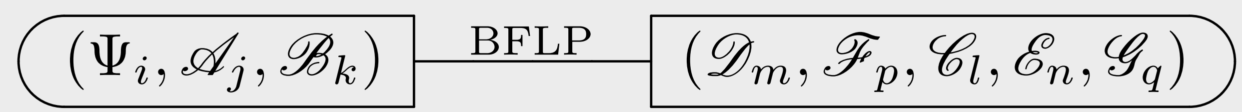
States are separating for effects

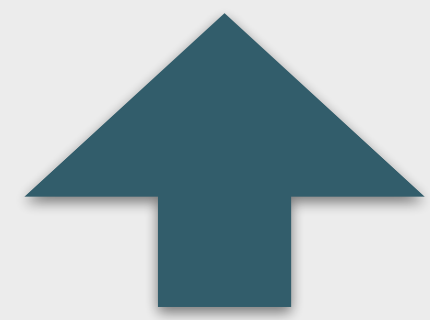
Effects are functionals on states

Effects are separating for states

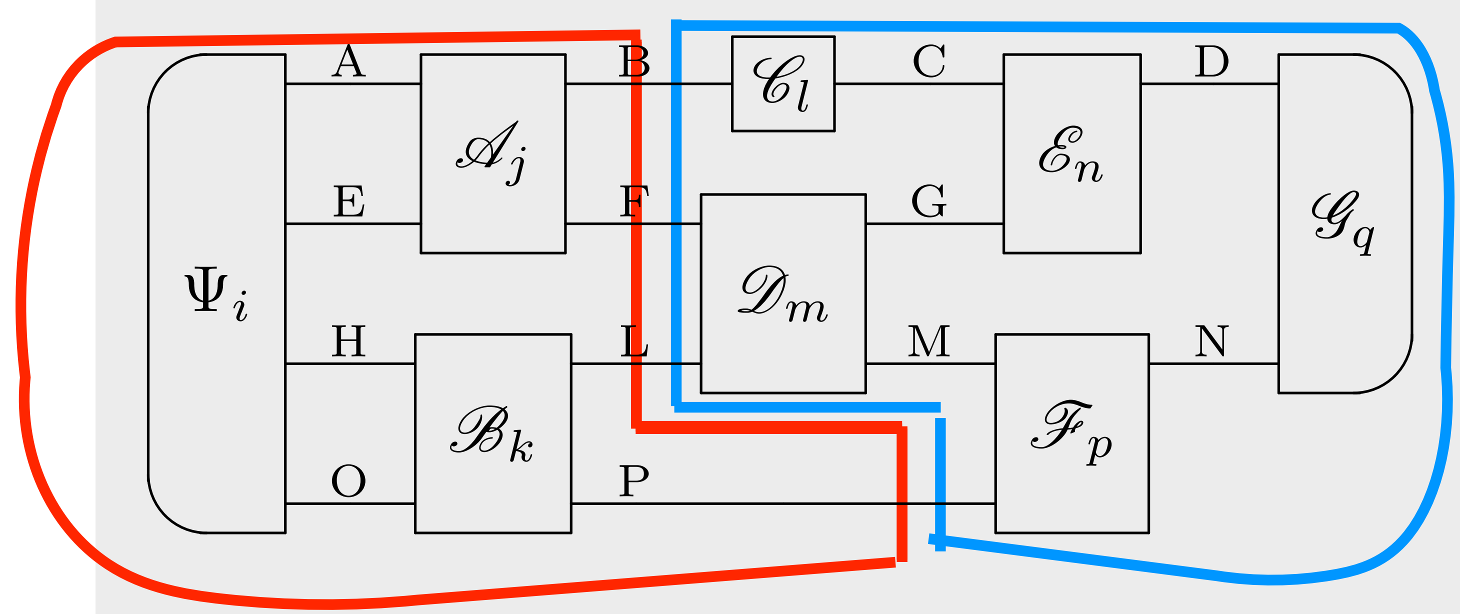
Paring notation:

$\rho \in \mathrm{St}(\mathrm{A}), a \in \mathrm{Eff}(\mathrm{A}),$  $= (a|\rho)$





$p(i, j, k, l, m, n, p, q|\mathrm{circuit})$



Operational Probabilistic Theory

States are functionals for effects

States are separating for effects

Effects are functionals on states

Effects are separating for states

Embedding in real vector spaces

$\mathrm{St(A)},\ \mathrm{St}_1\mathrm{(A)},\ \mathrm{St}_{\mathbb{R}}\mathrm{(A)}$

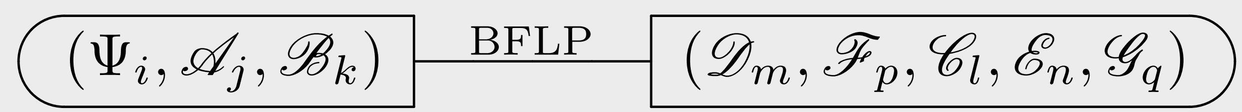
$\mathrm{Eff(A)},\ \mathrm{Eff}_1\mathrm{(A)},\ \mathrm{Eff}_{\mathbb{R}}\mathrm{(A)}$

Dimension D_{A}

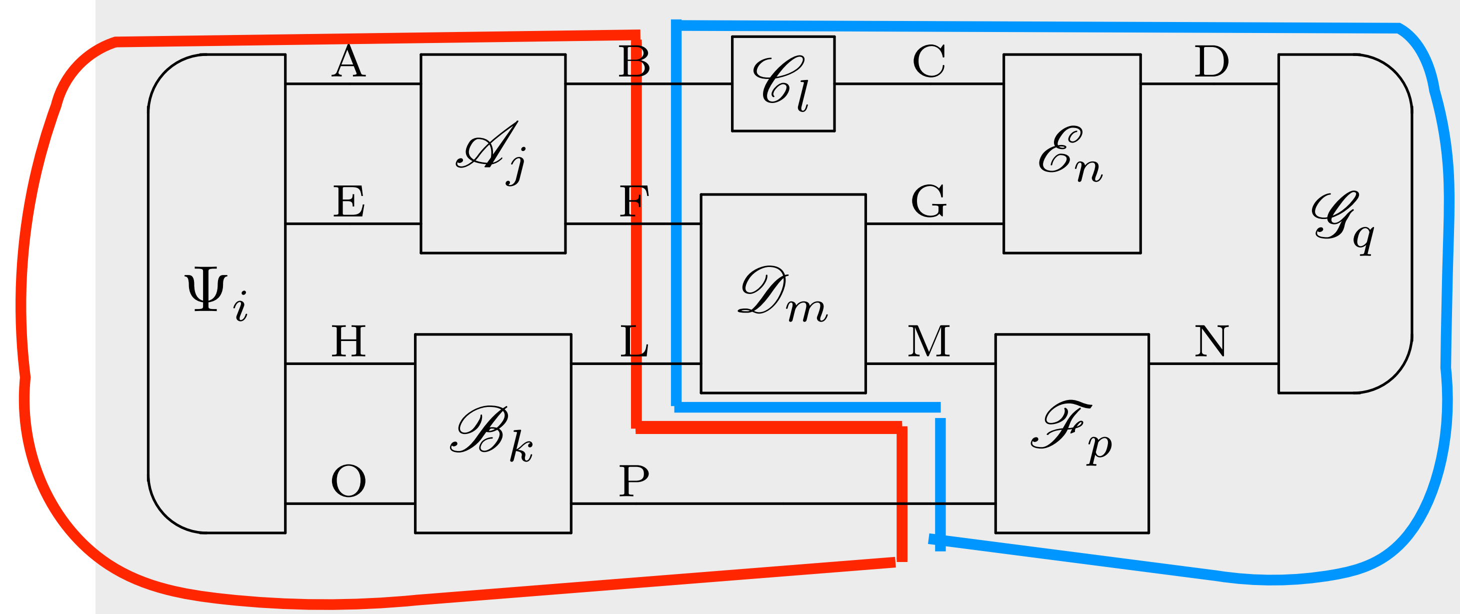
$$\boxed{\begin{aligned}\mathrm{Eff}_{\mathbb{R}}\mathrm{(A)} &= \mathrm{St}_{\mathbb{R}}\mathrm{(A)}^{\vee} \\ \mathrm{St}_{\mathbb{R}}\mathrm{(A)} &= \mathrm{Eff}_{\mathbb{R}}\mathrm{(A)}^{\vee}\end{aligned}}$$

Paring notation:

$\rho \in \mathrm{St(A)}, a \in \mathrm{Eff(A)},\quad (\rho\, \xrightarrow{\mathrm{A}}\, a) = (a|\rho)$



$p(i,j,k,l,m,n,p,q|\mathrm{circuit})$



Operational Probabilistic Theory

$$\{\mathscr{T}_i\}_{i\in\{\underbrace{i_1,i_2,\ldots,i_n}_{j_1},\underbrace{i_{n+1},i_{n+2},\ldots}_{j_2},\underbrace{\ldots}_{\ldots}\}}$$

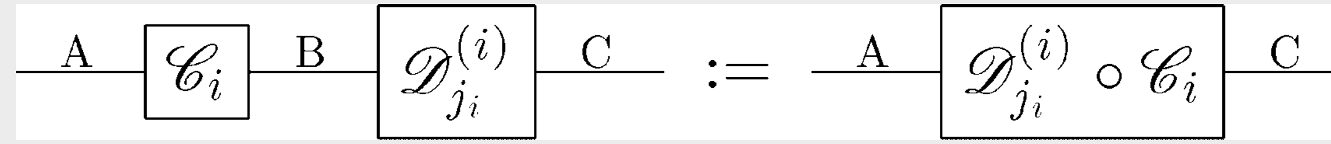
Coarse-graining $\downarrow$ $\uparrow$ Refinement

$$\{\hat{\mathscr{T}}_j\}_{j\in\{j_1,j_2,\ldots\}}$$

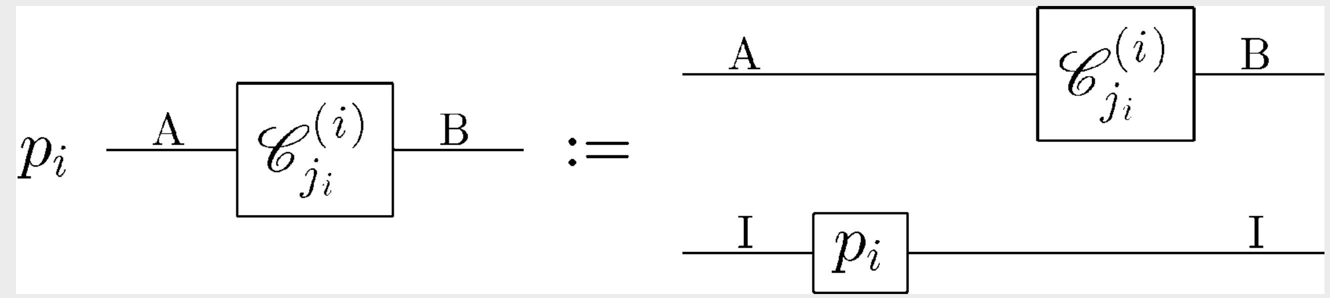
$$\hat{\mathscr{T}}_S = \sum_{i\in S} \mathscr{T}_i$$

Partial ordering

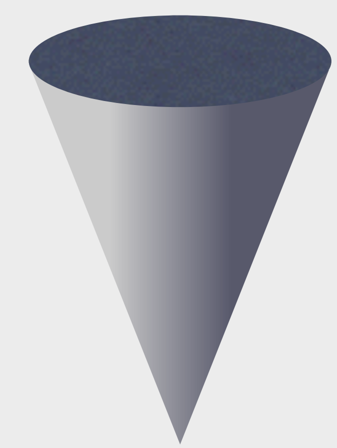
Conditioned test (needs causality)



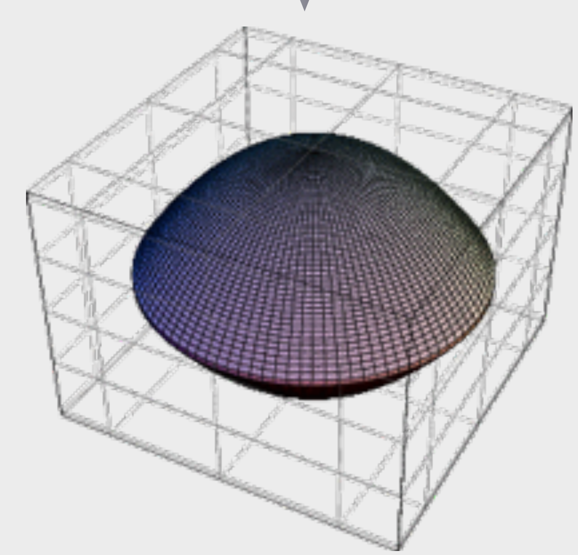
Circuit multiplication: randomize tests



Cone structure



Convex structure



Principles for Quantum Theory

$\{\rho_0, \rho_1\} \subseteq \mathsf{St}(\mathrm{A})$ preparation test

$\{a_0, a_1\}$ observation test

success probability of discrimination

$$p_{\mathrm{succ}} = (a_0|\rho_0) + (a_1|\rho_1)$$
$$= (a|\rho_0) + (a_1|\rho_1 - \rho_0)$$
$$= (a|\rho_1) + (a_0|\rho_0 - \rho_1)$$
$$= \frac{1}{2}[1 + (a_1 - a_0|\rho_1 - \rho_0)]$$

$$a := a_0 + a_1$$

Metric

$$p_{\mathrm{succ}}^{(\mathrm{opt})} = \frac{1}{2}[1 + ||\rho_1 - \rho_0||]$$

$$||\delta|| := \sup_{\{a_0, a_1\}} (a_0 - a_1|\delta),$$

$$||\delta|| = \sup_{a_0 \in \mathsf{Eff}(\mathrm{A})} (a_0|\delta) - \inf_{a_1 \in \mathsf{Eff}(\mathrm{A})} (a_1|\delta)$$

monotonicity

$$\mathscr{C} \in \mathsf{Transf}_1(\mathrm{A}, \mathrm{B})$$

$$||\mathscr{C}\delta||_{\mathrm{B}} \le ||\delta||_{\mathrm{A}}$$

Principles for Quantum Theory

P1. Causality

P2. Local discriminability

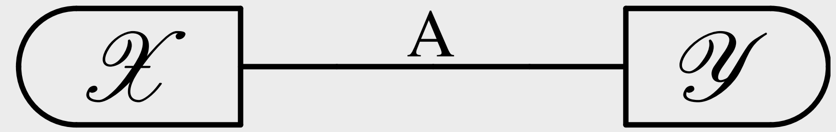
P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations



$$p(i, j|\mathscr{X}, \mathscr{Y}) := (a_j|\rho_i)$$

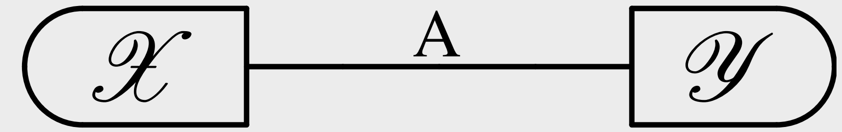
$$p(i|\mathscr{X}, \mathscr{Y}) = p(i|\mathscr{X}, \mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique; b) states are "normalizable"

Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

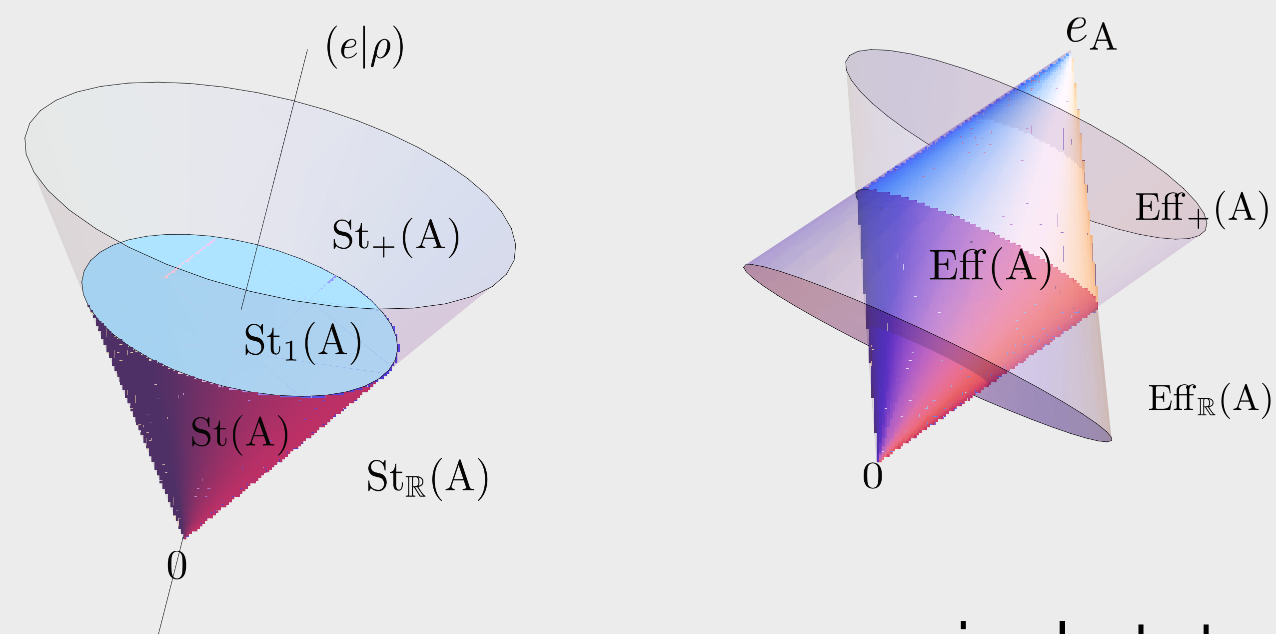
The probability of preparations is independent of the choice of observations



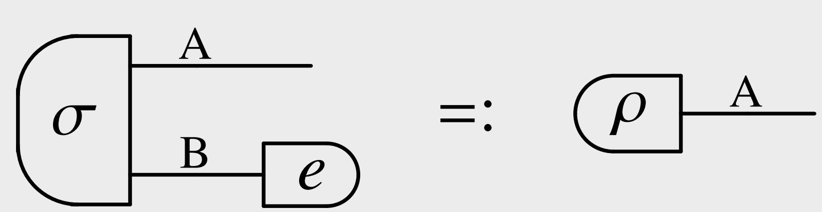
$$p(i,j|\mathscr{X},\mathscr{Y}) := (a_j|\rho_i)$$

$$p(i|\mathscr{X},\mathscr{Y}) = p(i|\mathscr{X},\mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique; b) states are “normalizable”



marginal state

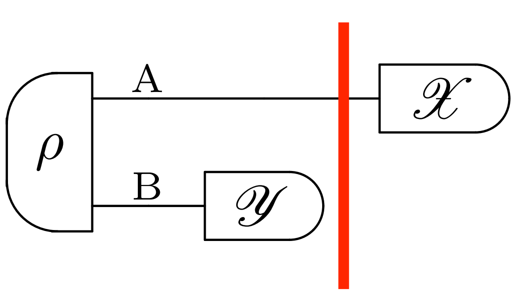


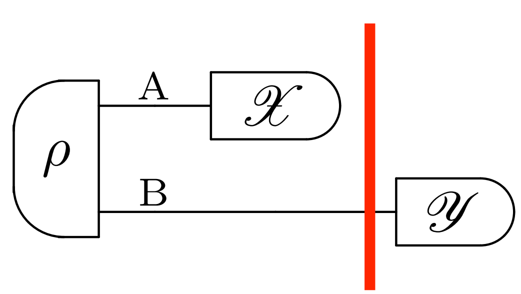
Principles for Quantum Theory

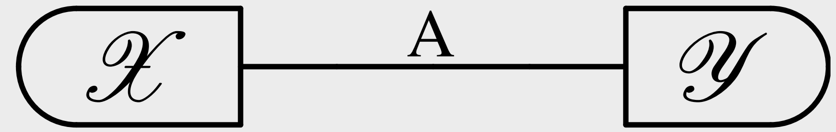
P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

no signaling without interaction



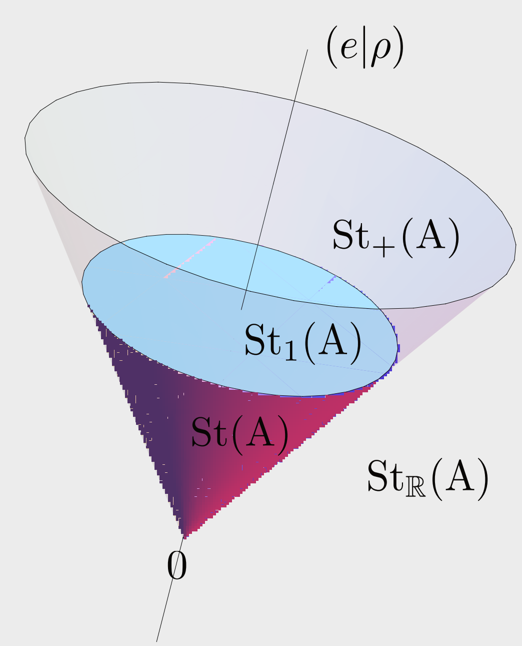


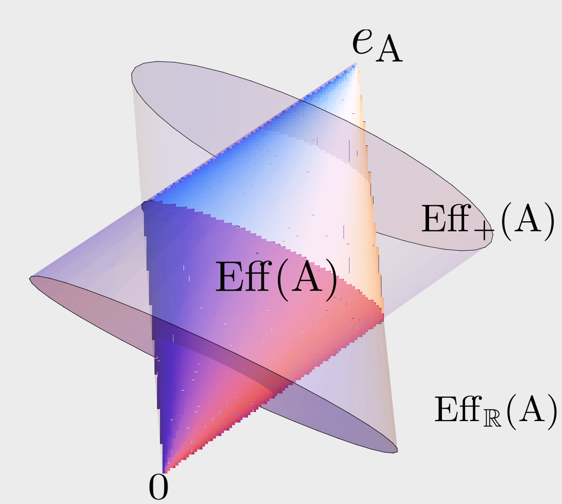


$$p(i, j|\mathscr{X}, \mathscr{Y}) := (a_j|\rho_i)$$

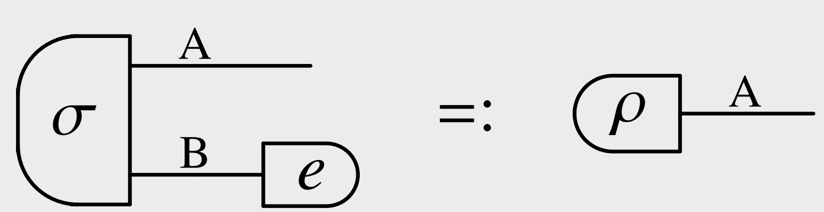
$$p(i|\mathscr{X}, \mathscr{Y}) = p(i|\mathscr{X}, \mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique; b) states are "normalizable"





marginal state



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

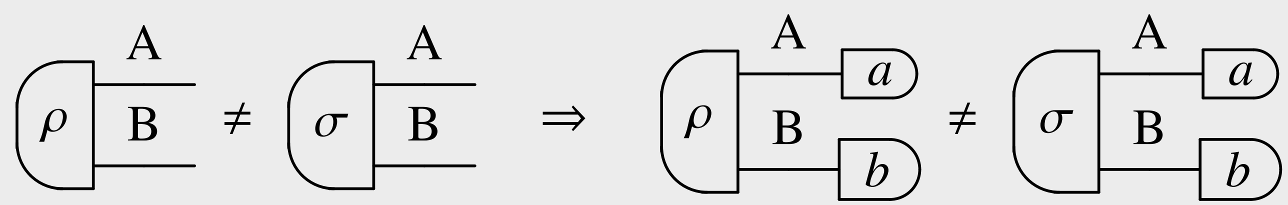
P4. Atomicity of composition

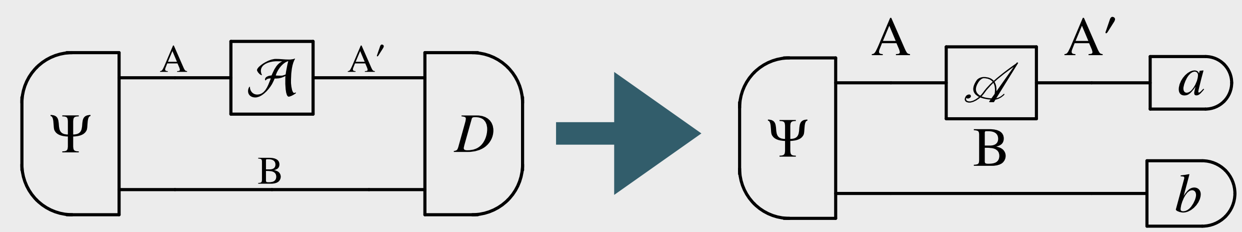
P5. Perfect distinguishability

P6. Lossless Compressibility

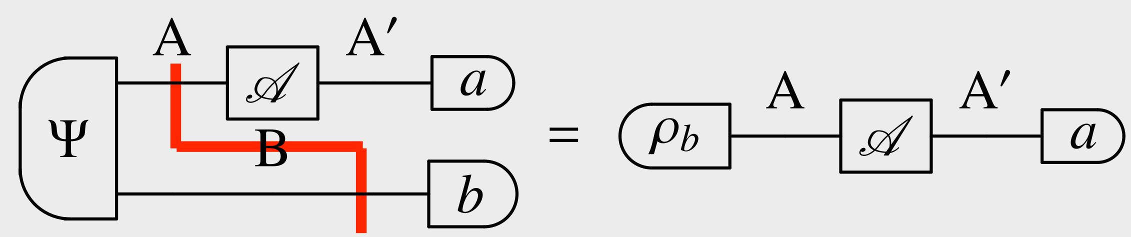
It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product





Local characterization of transformations

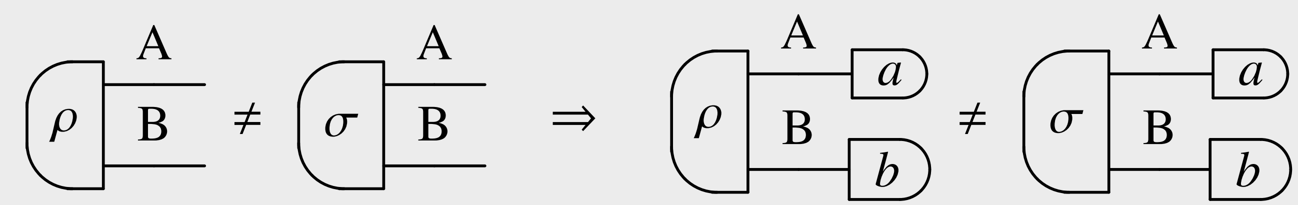


Principles for Quantum Theory

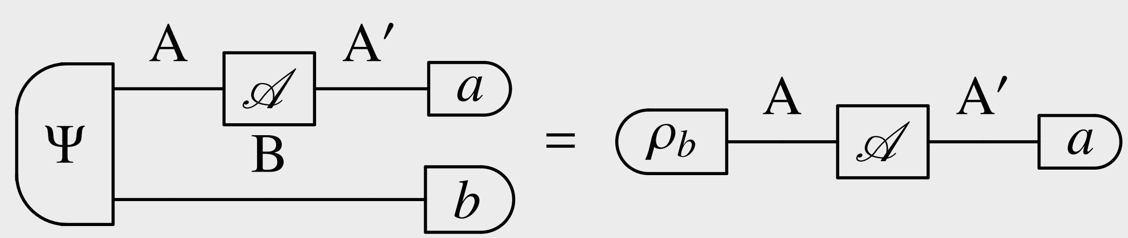
P1. Causality
P2. $\boxed{\text{Local discriminability}}$
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product



Local characterization of transformations

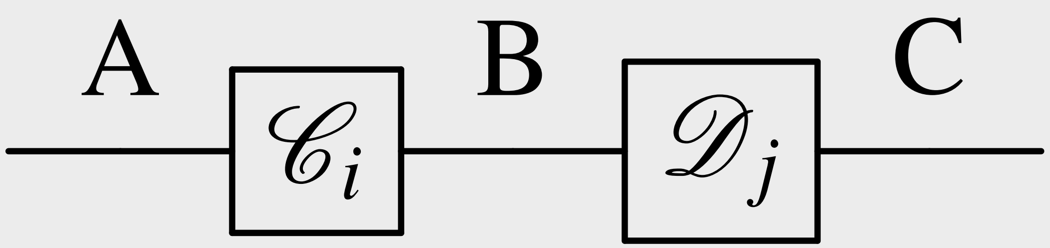


Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis

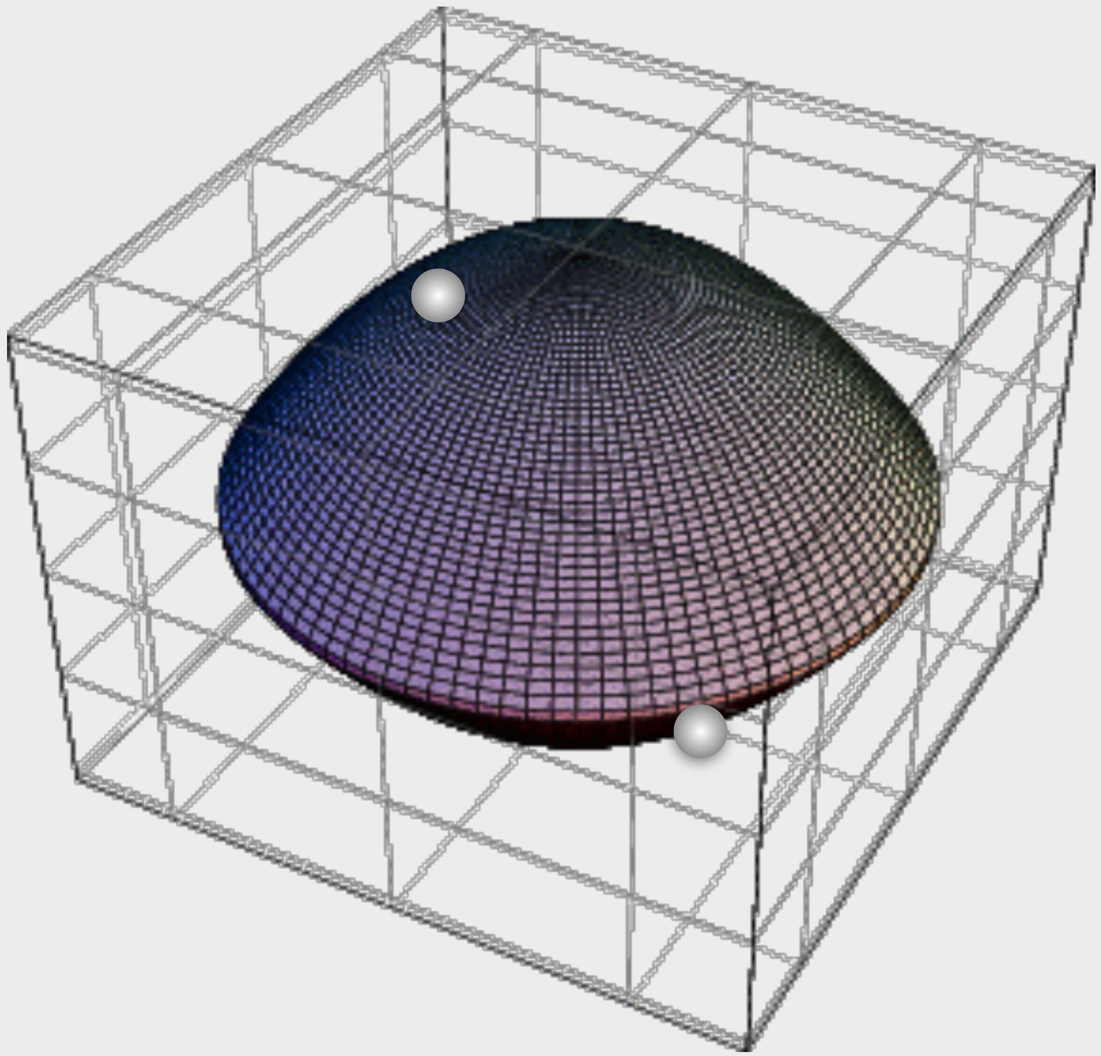


Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

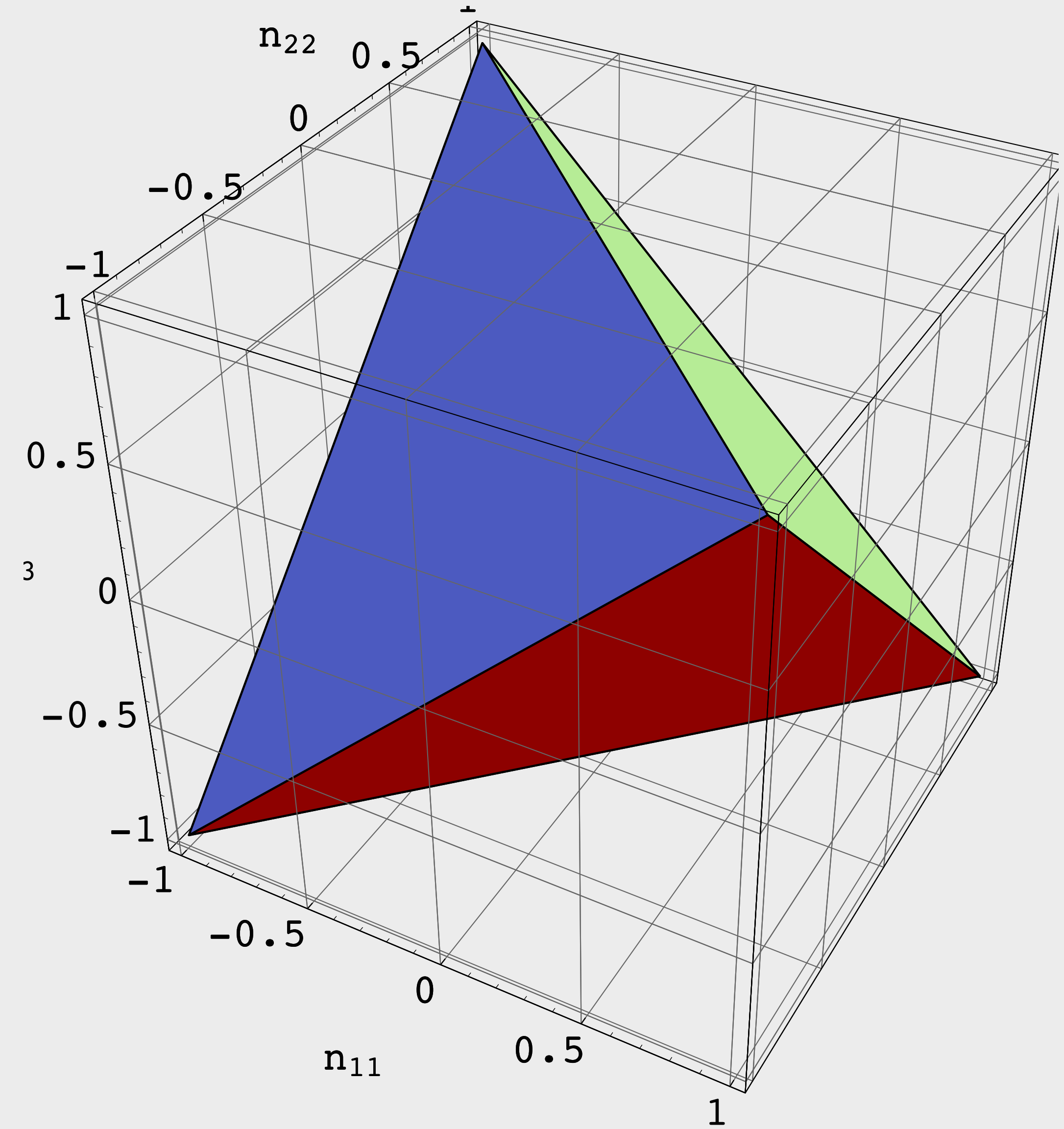
P4. Atomicity of composition

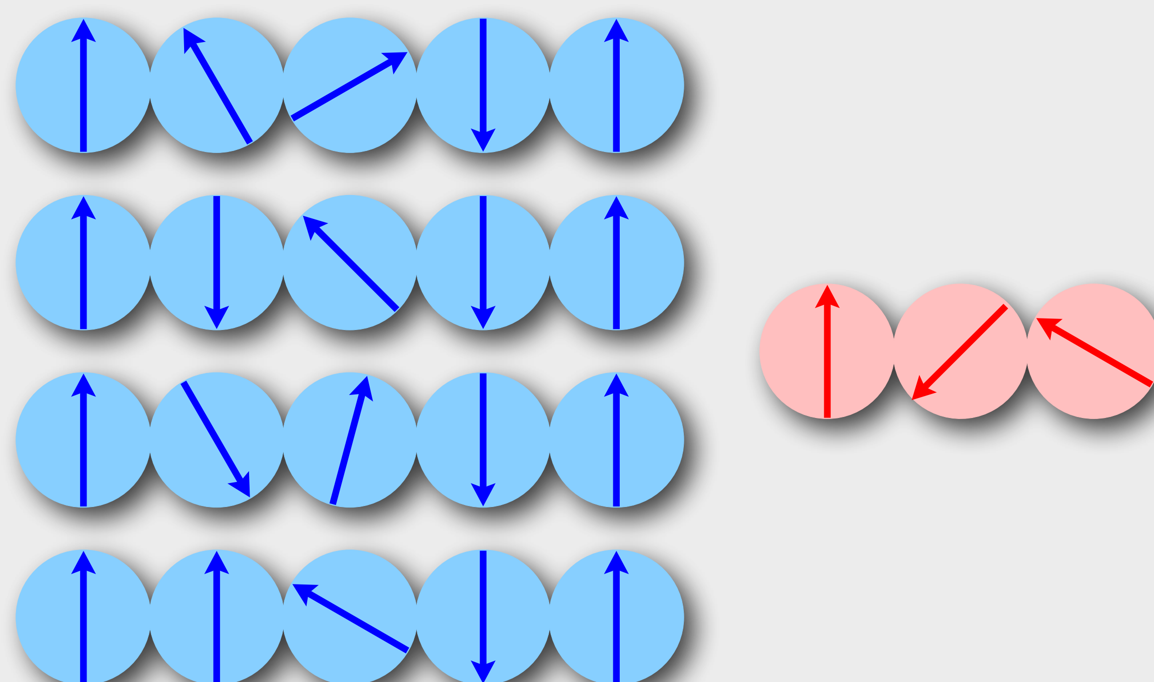
P5. Perfect distinguishability

P6. Lossless Compressibility

For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system





Principles for Quantum Theory

P1. Causality

P2. Local discriminability

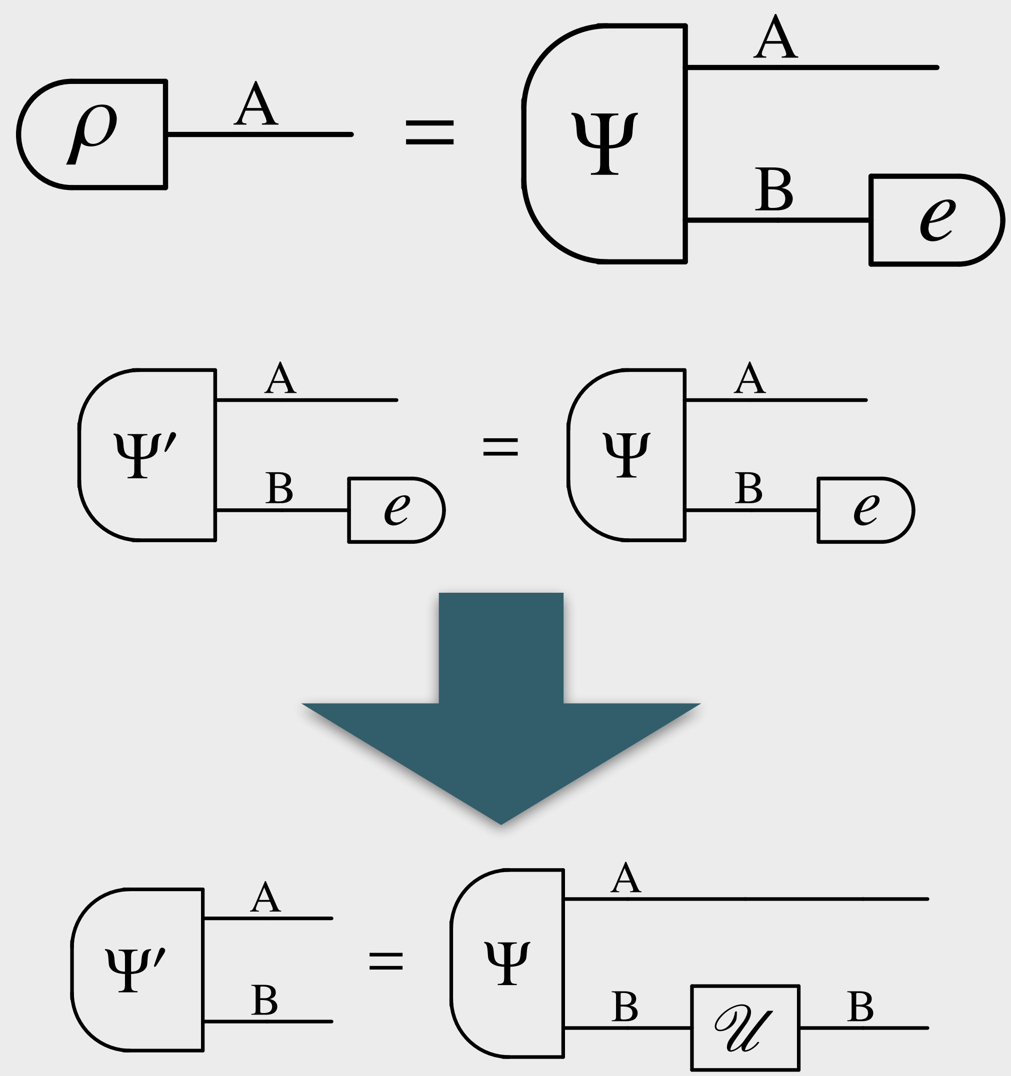
P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

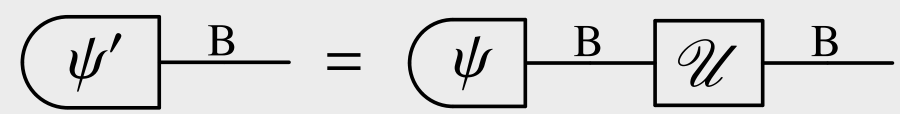
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

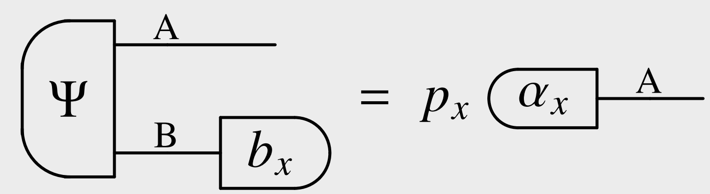
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

2. *Every two normalized pure states of the same system are connected by a reversible transformation*

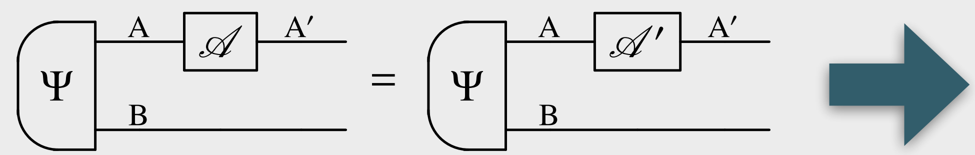


3. ***Steering:*** Let $\boldsymbol{\Psi}$ purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



$\forall x \in X$

4. ***Process tomography (faithful state):***



$$\mathscr{A}\rho = \mathscr{A}'\rho \quad \forall\rho$$

5. ***No information without disturbance***

Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. $\boxed{\text{Purification}}$

P4. Atomicity of composition

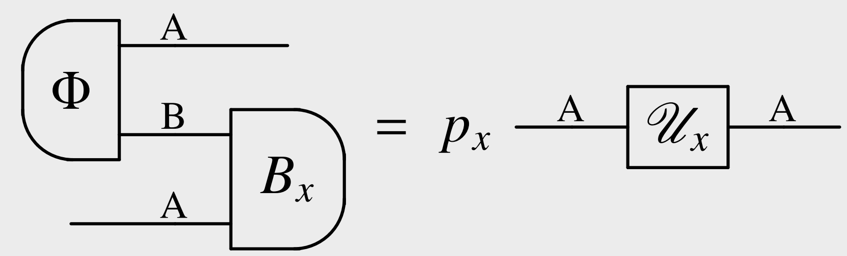
P5. Perfect distinguishability

P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

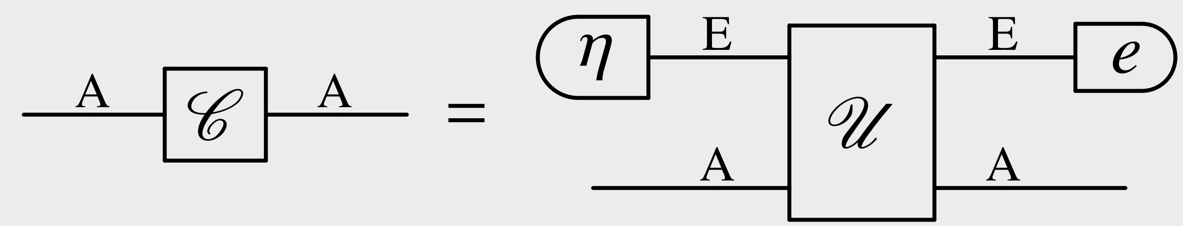
Consequences

6. ***Teleportation***

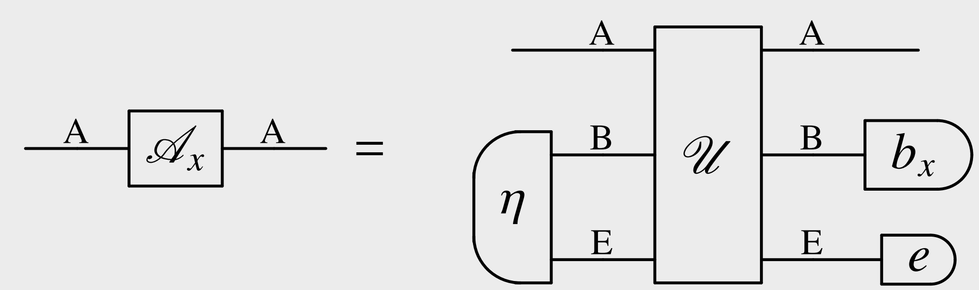


$\forall x \in$ X

7. ***Reversible dilation of "channels"***



8. ***Reversible dilation of "instruments"***



$\forall x \in$ X

9. ***State-transformation cone isomorphism***

10. ***Rev. transform. for a system make a compact Lie group***

This is more or less what I wanted to say

Thank you for your attention