JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

Project: *A Quantum-Digital Universe*, Grant ID: 43796

Physics without physics

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

Quantum Theory: from foundations to technologies – QTFT
Linnaeus University, Växjö, June 12-17 2016

Program

Deriving the whole Physics axiomatically

from “principles” stated in form of purely mathematical axioms without physical primitives,

but having a thorough physical interpretation.

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from “principles” stated in form of purely mathematical axioms without physical primitives,

but having a thorough physical interpretation.

Examples of physical primitives:

mass, force, clocks, rods, …

Physical interpretation from where?

from experience and from un-axiomatized physics …

Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

Giulio Chiribella*

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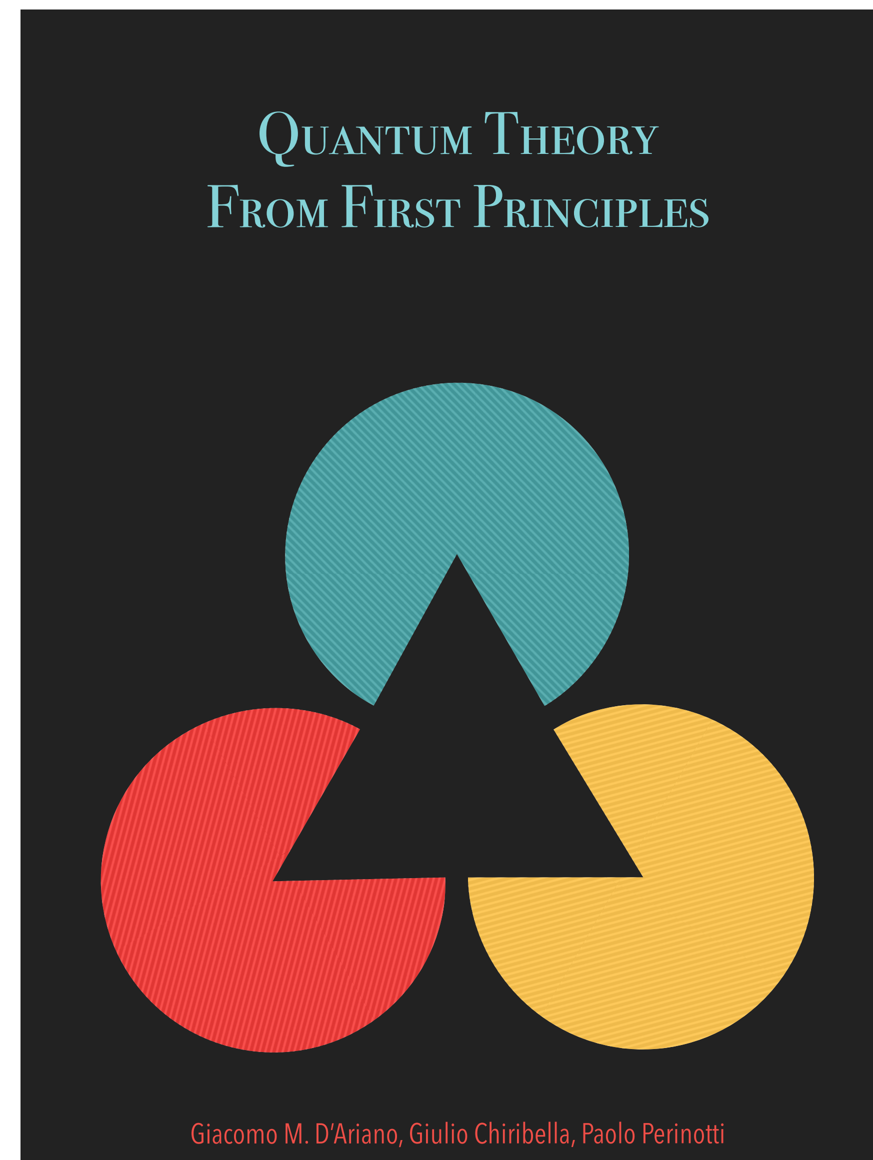
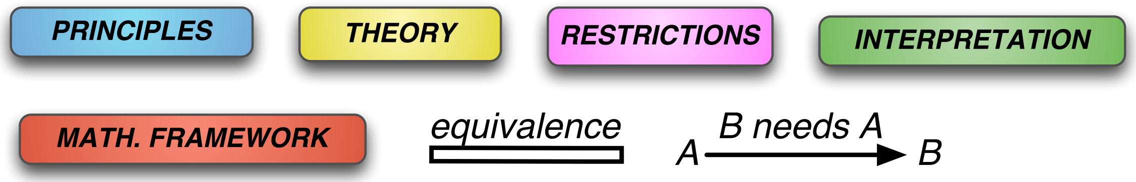
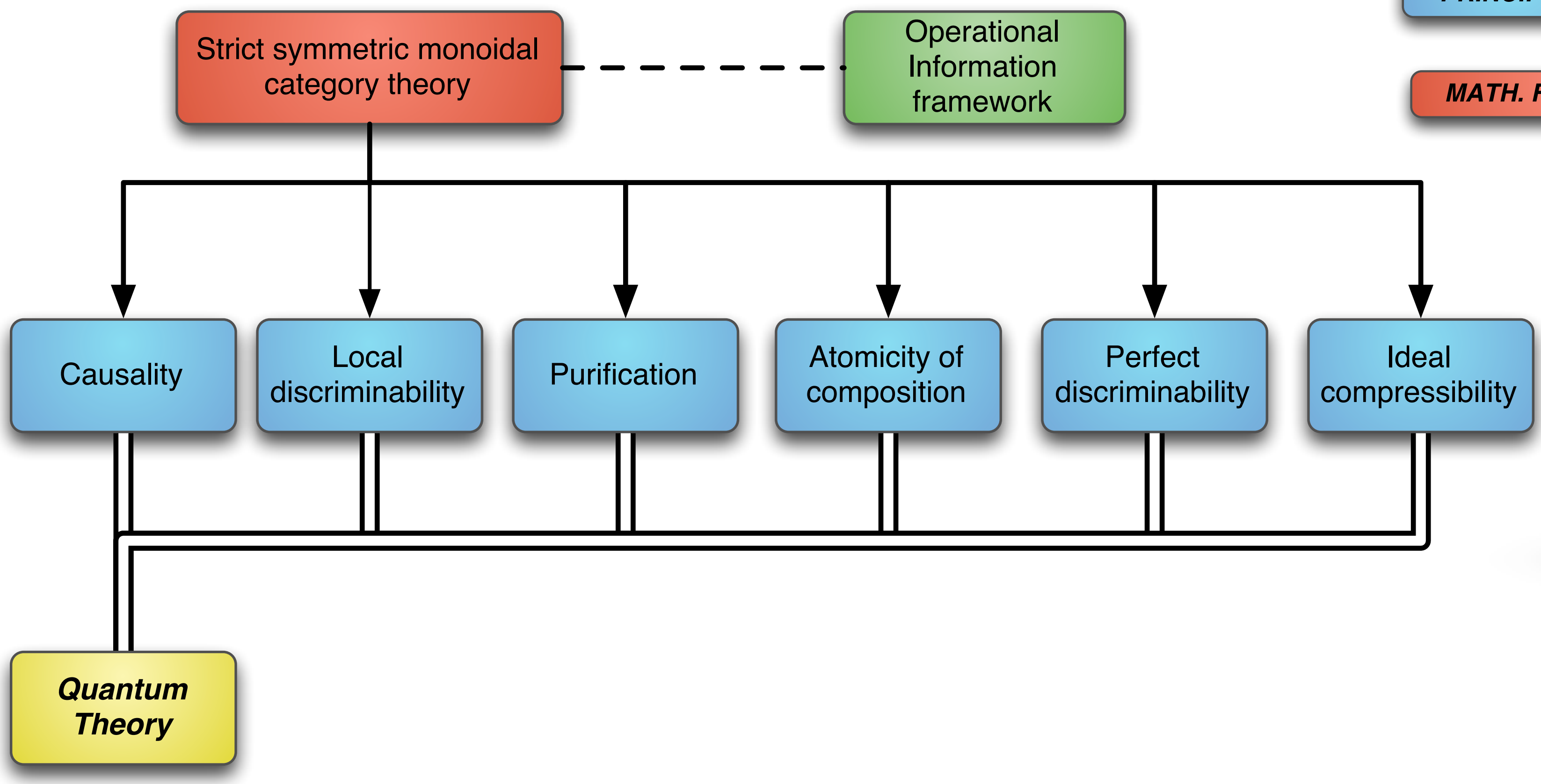
(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03.67.Ac, 03.65.Ta







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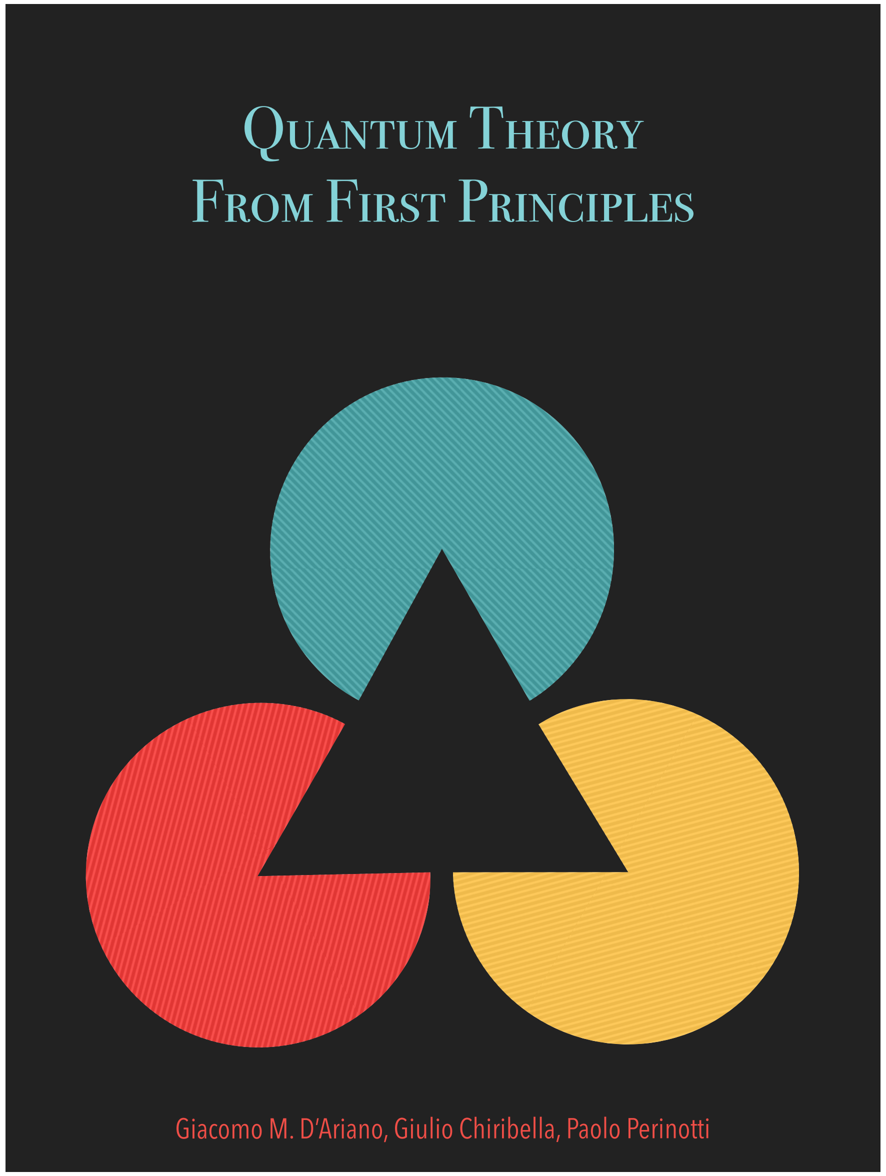

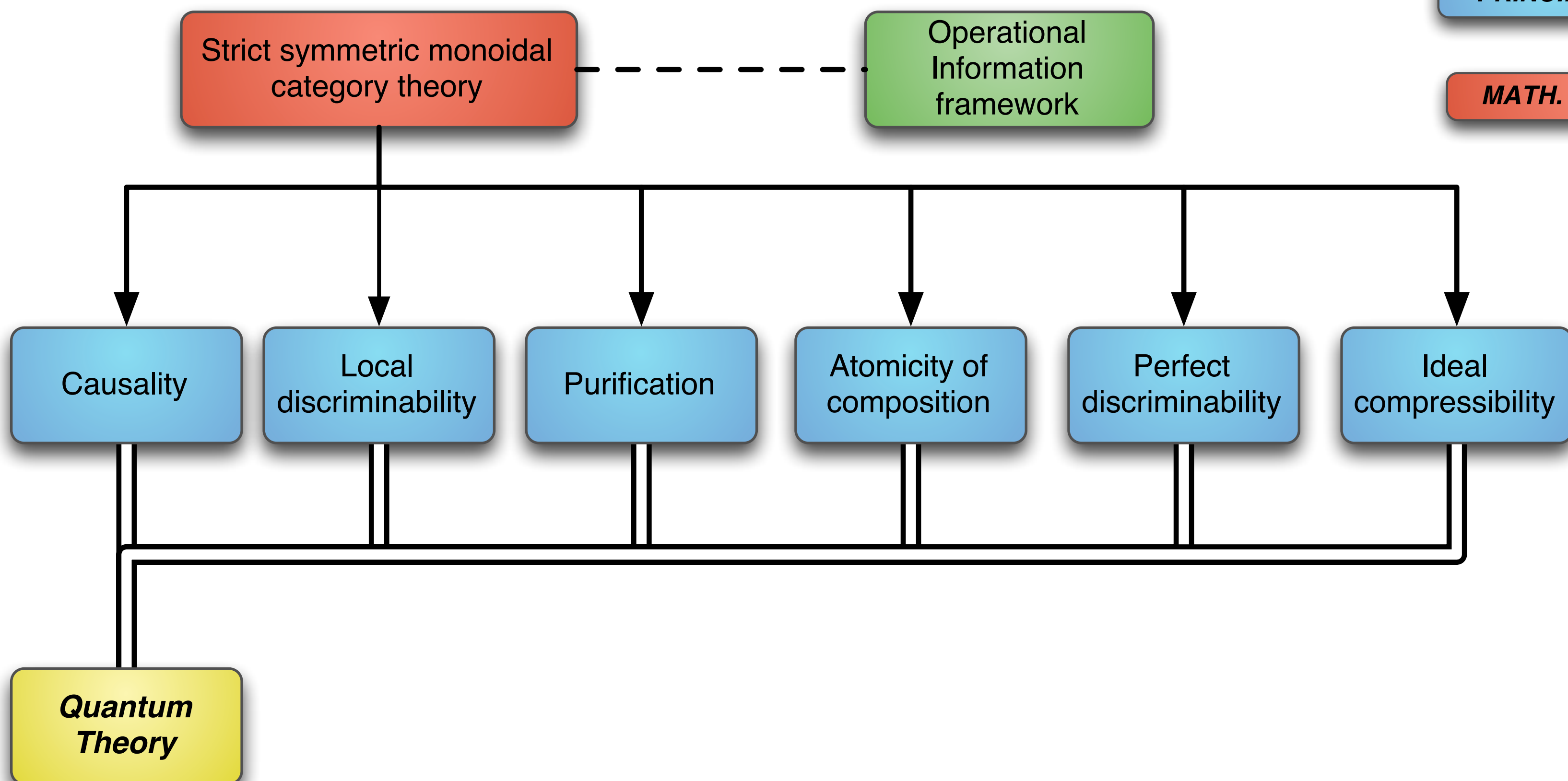

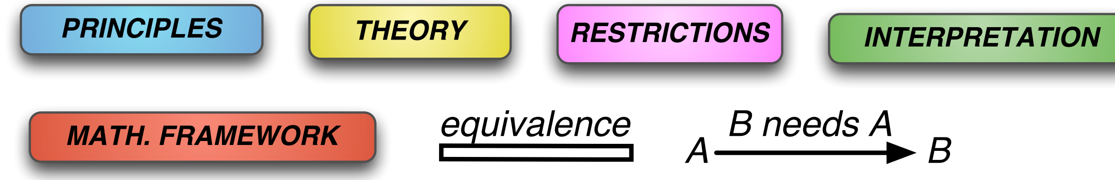

Principles for Mechanics

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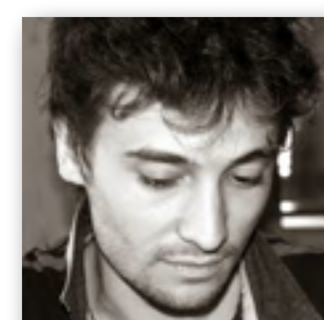
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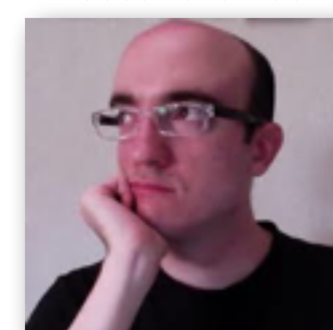
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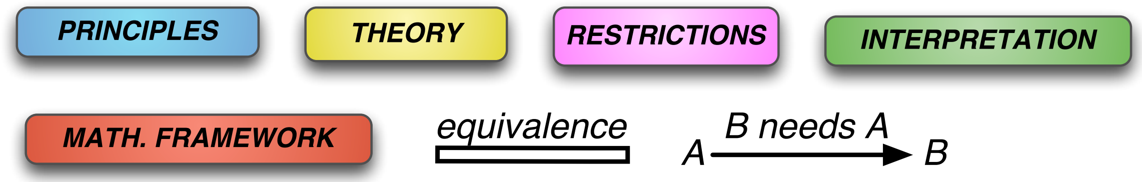

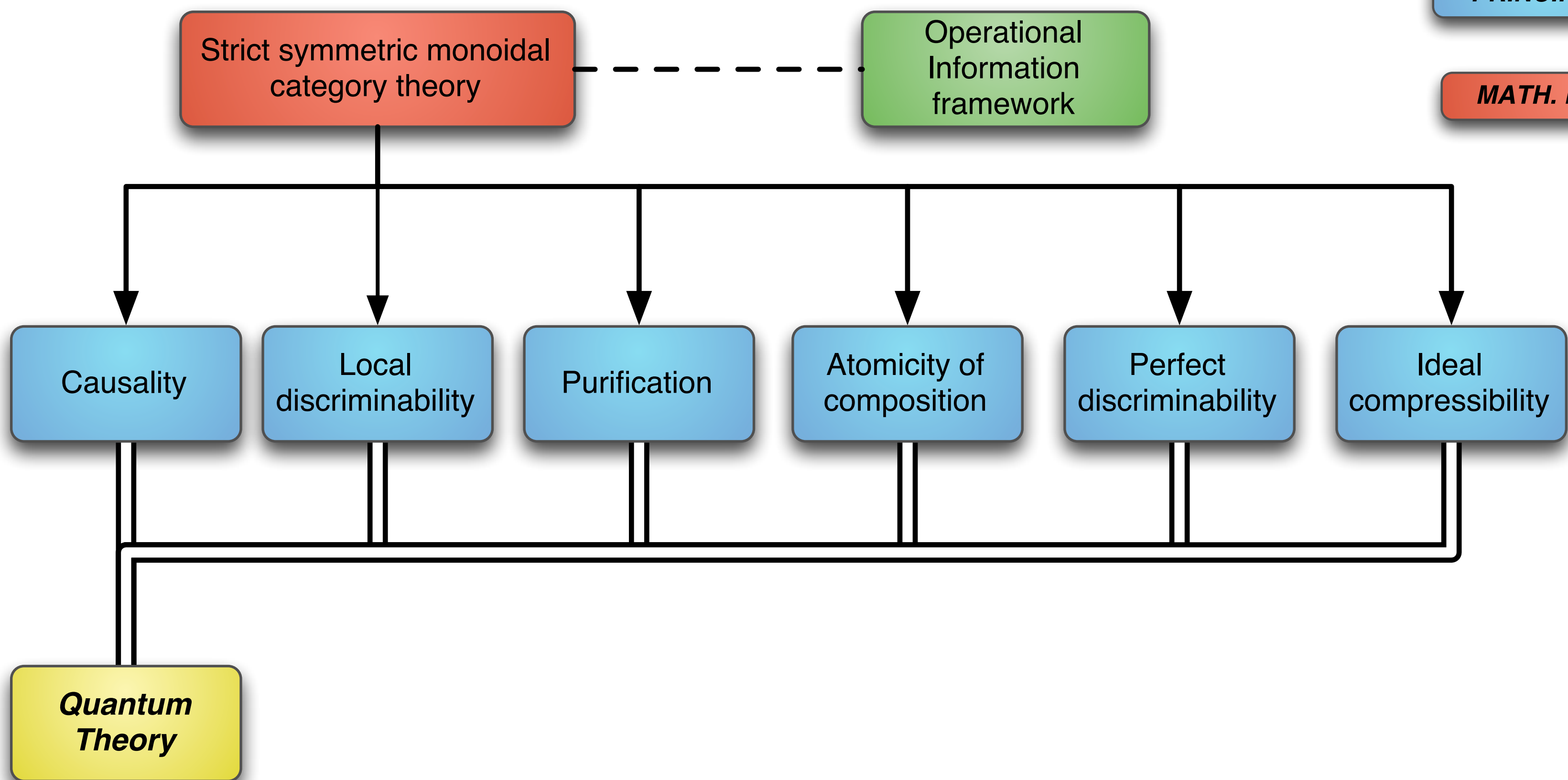

Principles for Mechanics

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

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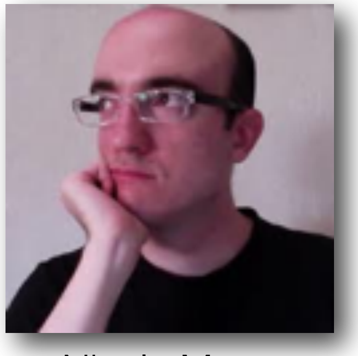
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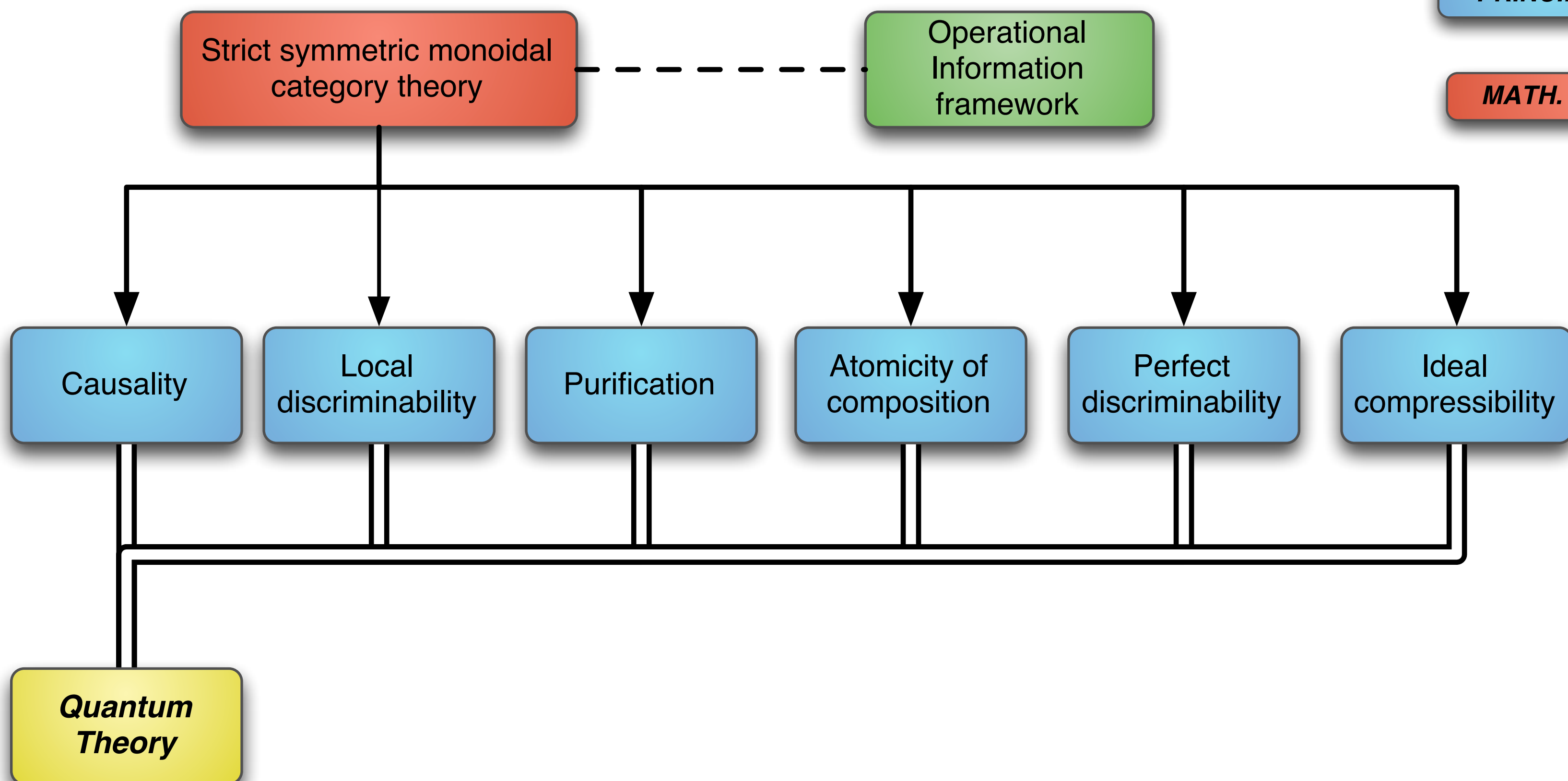

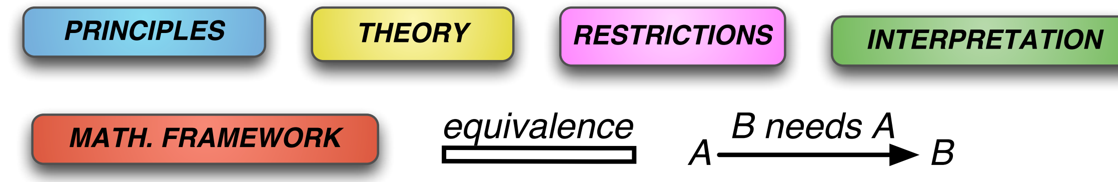

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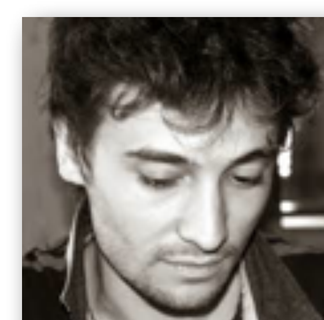

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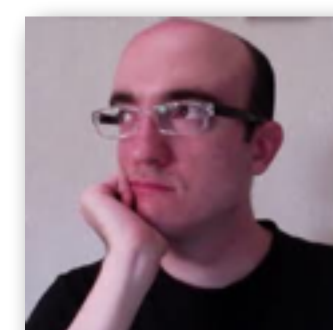

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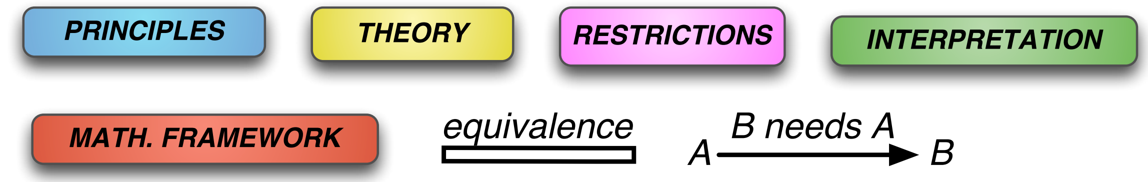
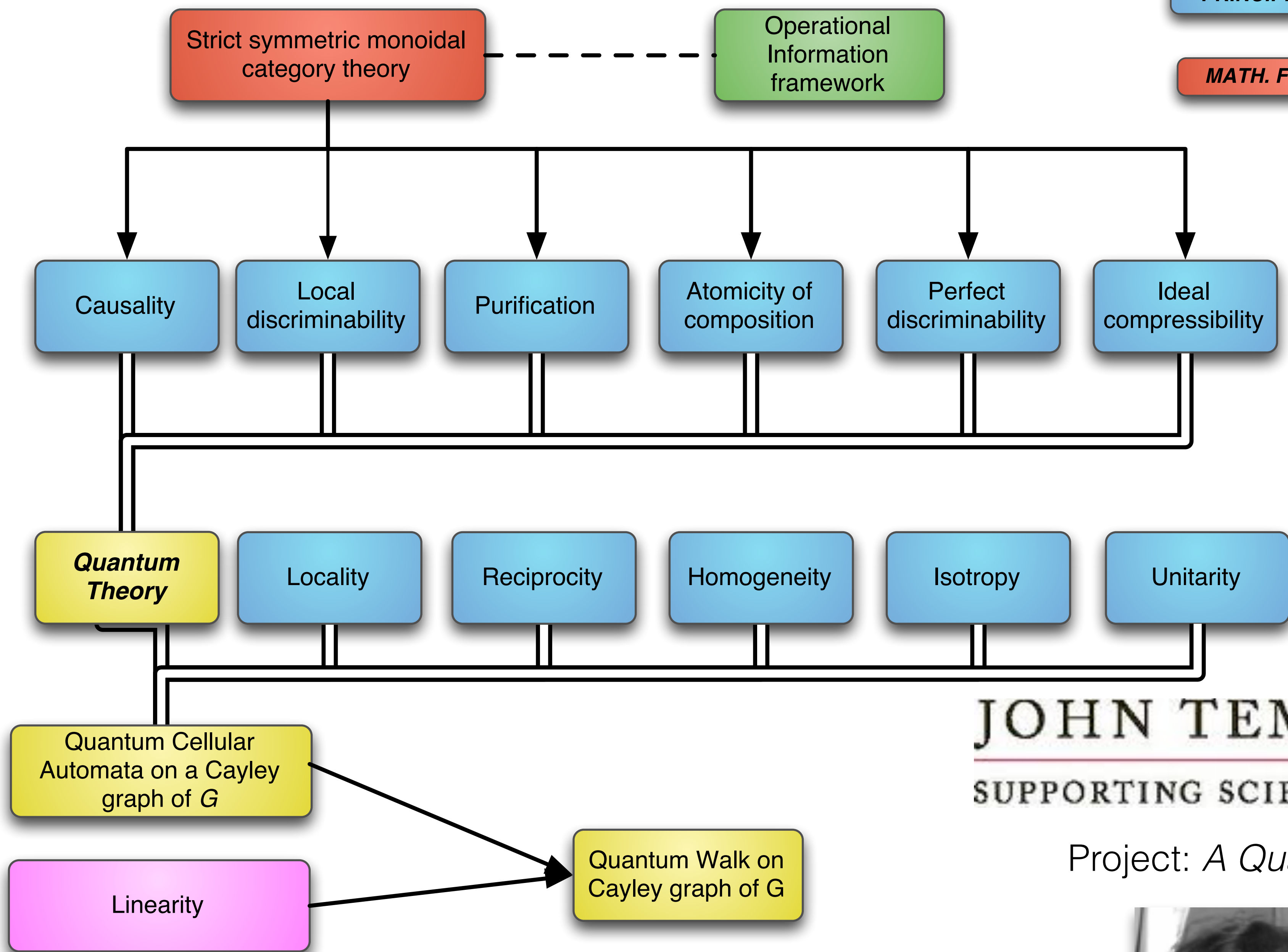
- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

Principles for Mechanics





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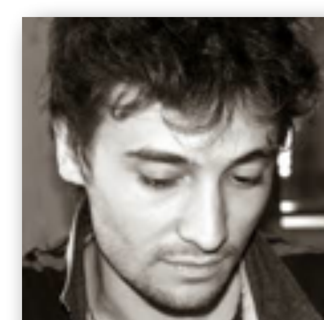

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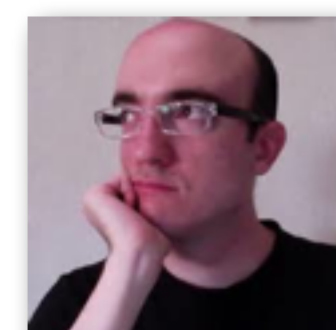

Nicola Mosco

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \bigoplus_{g\in G}\mathbb{C}^{s_g} \quad |G| \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$

1) *Locality*: S_g uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent

$\Big\}$ = Quantum Walk on Cayley graph

D'Ariano, Perinotti,
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The following operator over the Hilbert space $\ell^2(G)\otimes\mathbb{C}^s$ is unitary

$$A = \sum_{h\in S} T_h \otimes A_h$$

where T is the right regular representation of G on $\ell^2(G)$ acting as

$$T_g|g'\rangle = |g'g^{-1}\rangle$$

1) Locality: S_g uniformly bounded

2) Reciprocity: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

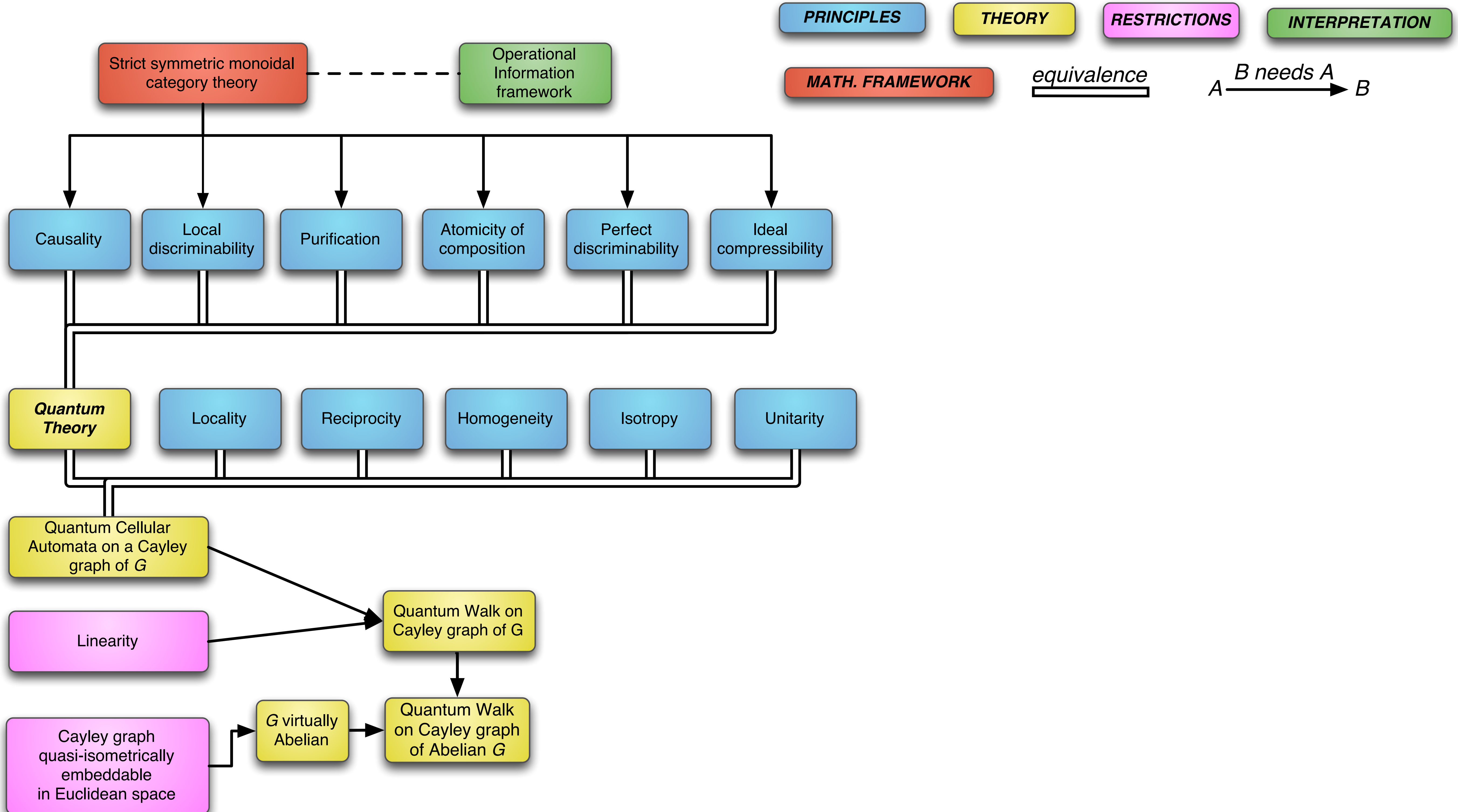
3) Homogeneity: all $g \in G$ are equivalent

4) Isotropy:

There exist:

- a group L of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:

$$A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh} \otimes L_l A_h L_l^\dagger$$


PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
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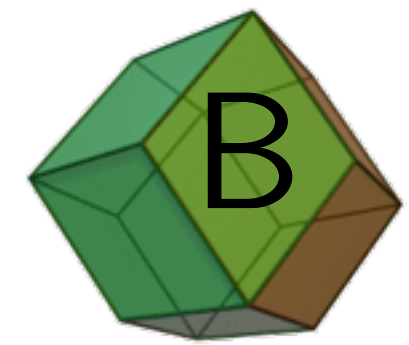
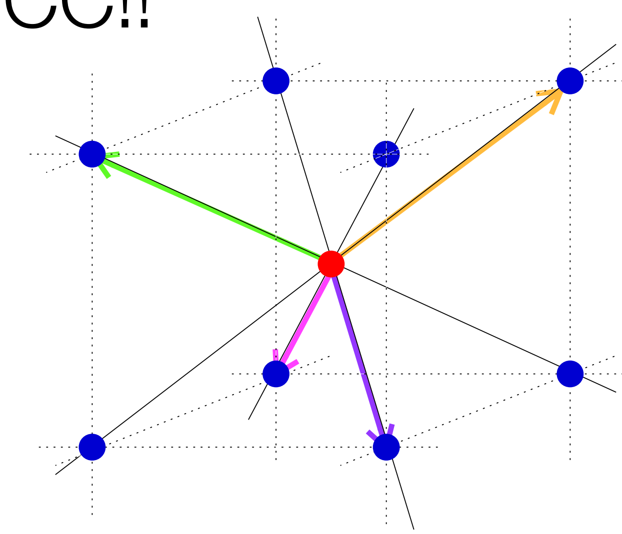
The Weyl QW

☞ Minimal dimension for nontrivial unitary *A:* s=2



Unitarity + isotropy ⇒ for d=3 the only Cayley is the BCC!!



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QWs connected by P

$$\boxed{\begin{aligned} A^{\pm}_{\mathbf{k}} = & -i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ & \mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ & - i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ & + I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin \frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos \frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

Physical interpretation: the Weyl Fermions

$$i\partial_t\psi(t) \simeq \tfrac{i}{2}[\psi(t+1)-\psi(t-1)] = \tfrac{i}{2}(A-A^\dagger)\psi(t)$$

$$\begin{aligned}\tfrac{i}{2}(A^{\pm}_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+\sigma_x(s_x c_y c_z \pm c_x s_y s_z)\\ &\pm\sigma_y(c_x s_y c_z \mp s_x c_y s_z)\\ &+\sigma_z(c_x c_y s_z \pm s_x s_y c_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$

$$\boxed{i\partial_t\psi = \tfrac{1}{\sqrt{3}}\boldsymbol{\sigma}^{\pm}\cdot\mathbf{k}\,\psi \quad \text{☜ Weyl equation!}}$$

$\boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$

Two QCAs connected by P

$$\boxed{\begin{aligned}A^{\pm}_{\mathbf{k}} = &-i\sigma_x(s_x c_y c_z \pm c_x s_y s_z)\\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z)\\ &-i\sigma_z(c_x c_y s_z \pm s_x s_y c_z)\\ &+I(c_x c_y c_z \mp s_x s_y s_z)\end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QW $\oplus$

<u>Local</u> coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$\boxed{\begin{gathered} E^{\pm}_{\mathbf{k}} = \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix} \\ n^2 + m^2 = 1 \qquad n, m \in \mathbb{R} \end{gathered}}$$

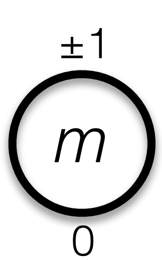
$E^{\pm}_{\mathbf{k}}$ CPT-connected!

$$\omega^{E}_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $\quad k \ll m \ll 1$

m: mass, $m^2 \leq 1$

n^{-1}: refraction index



!

Bisio, D'Ariano, Perinotti, Ann. Phys. **368** 177 (2016)

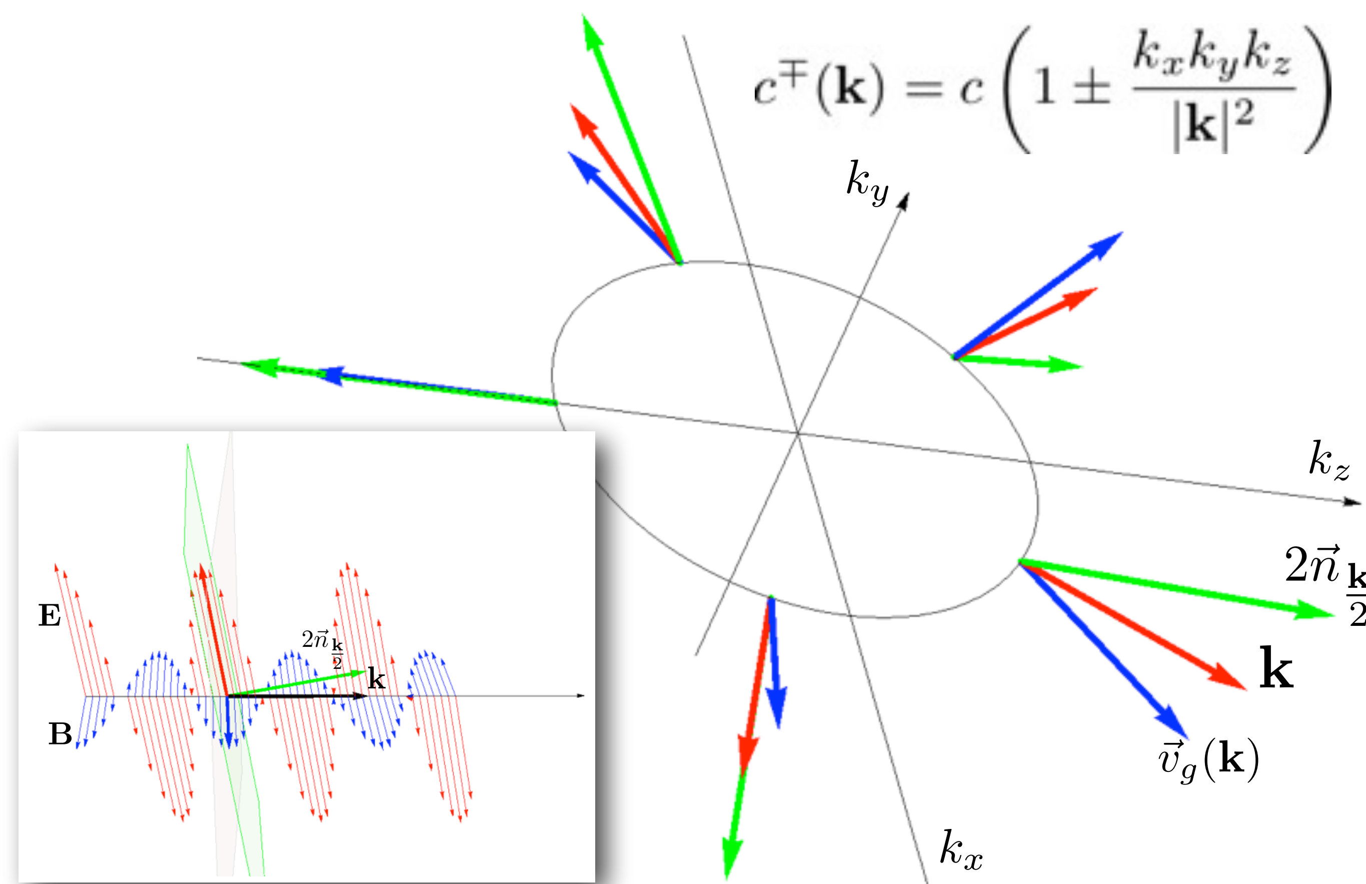
Maxwell QW $\otimes$

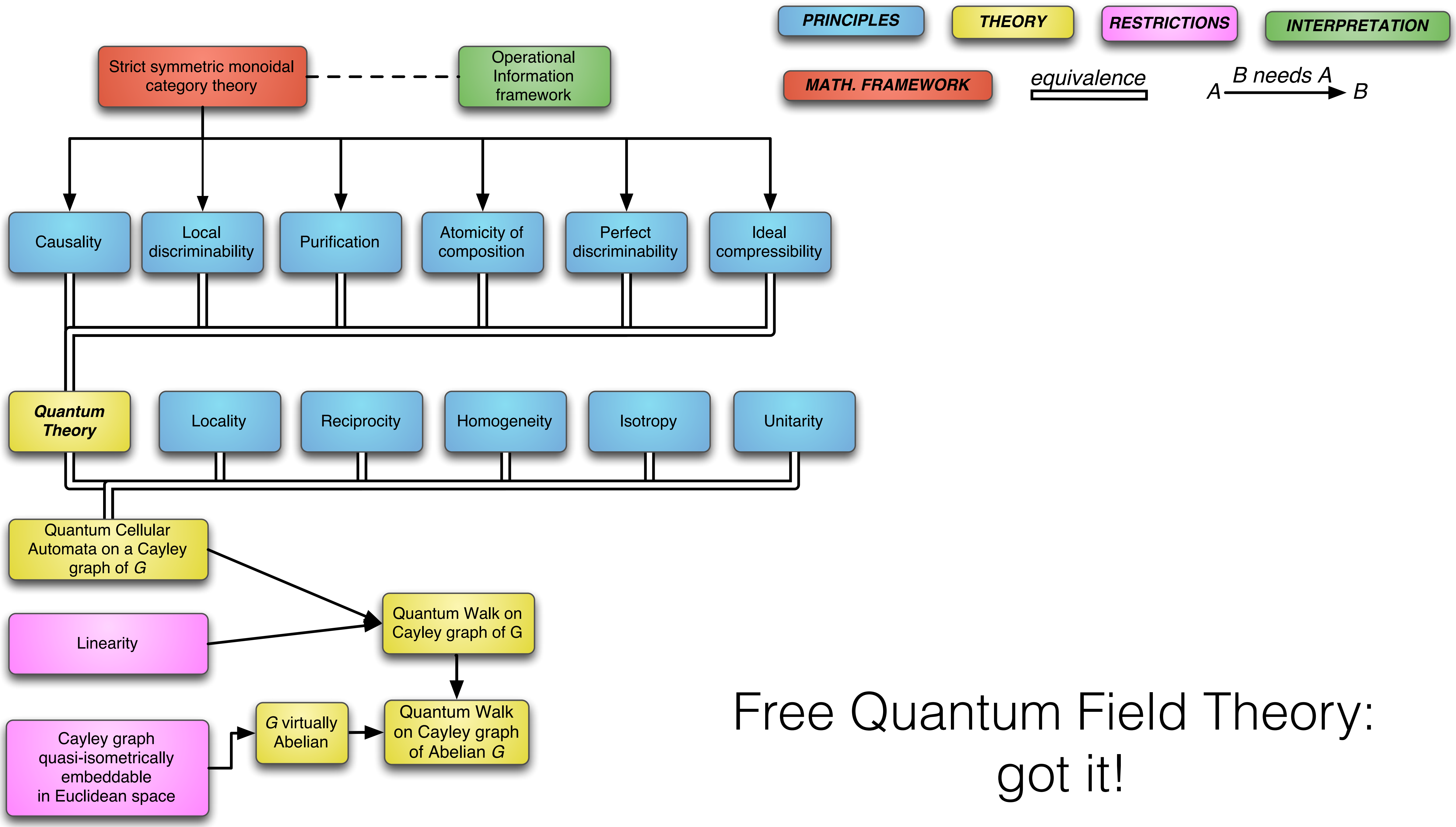
$$\boxed{\begin{gathered} M^{\pm}_{\mathbf{k}} = A^{\pm}_{\mathbf{k}} \otimes A^{\pm *}_{\mathbf{k}} \\ F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q}) \end{gathered}}$$

Maxwell in relativistic limit $\quad k \ll 1$

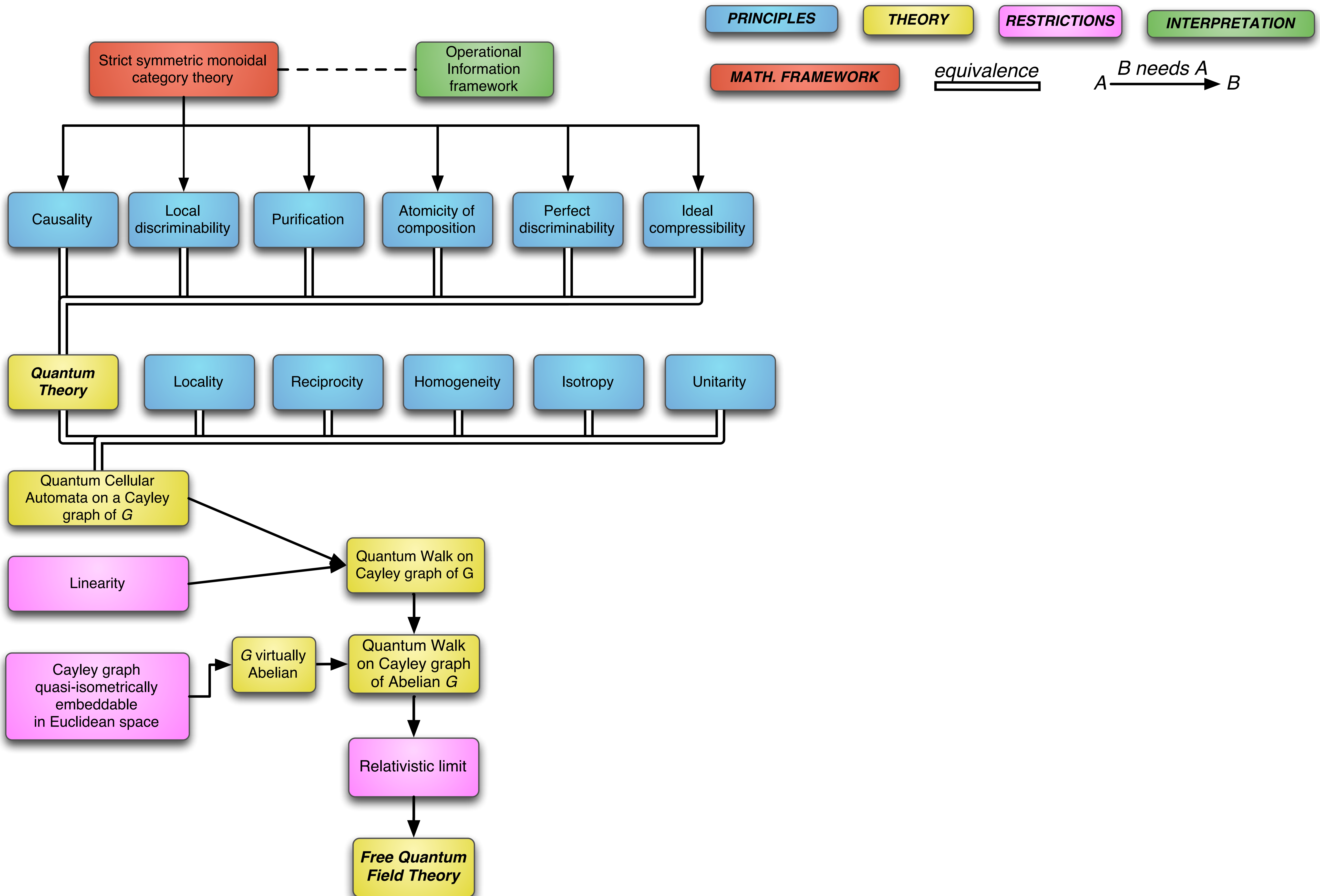
Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)





Free Quantum Field Theory: got it!


PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
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B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
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Atomicity of composition
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Ideal compressibility
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Locality
Reciprocity
Homogeneity
Isotropy
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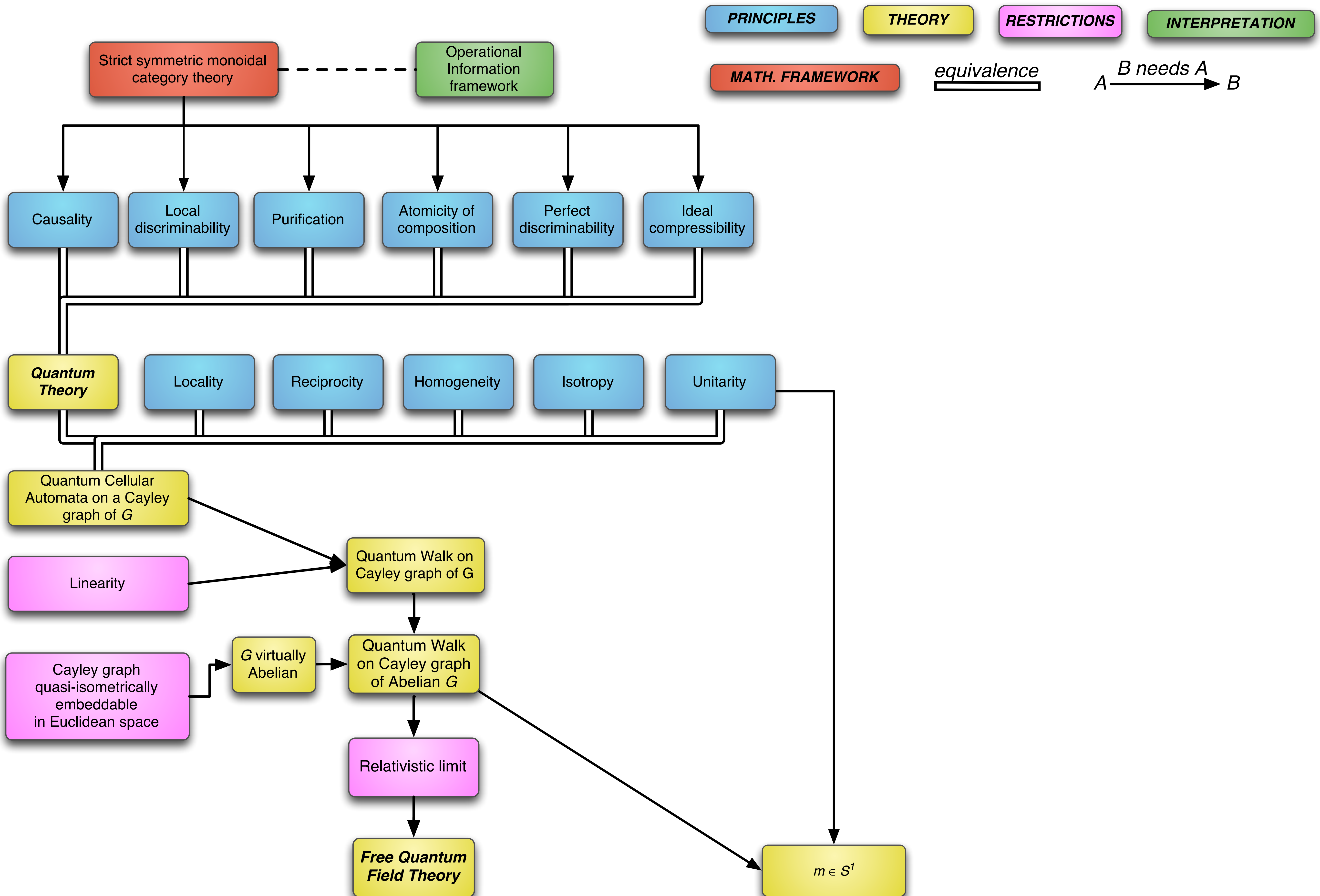

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THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
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A B needs A B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
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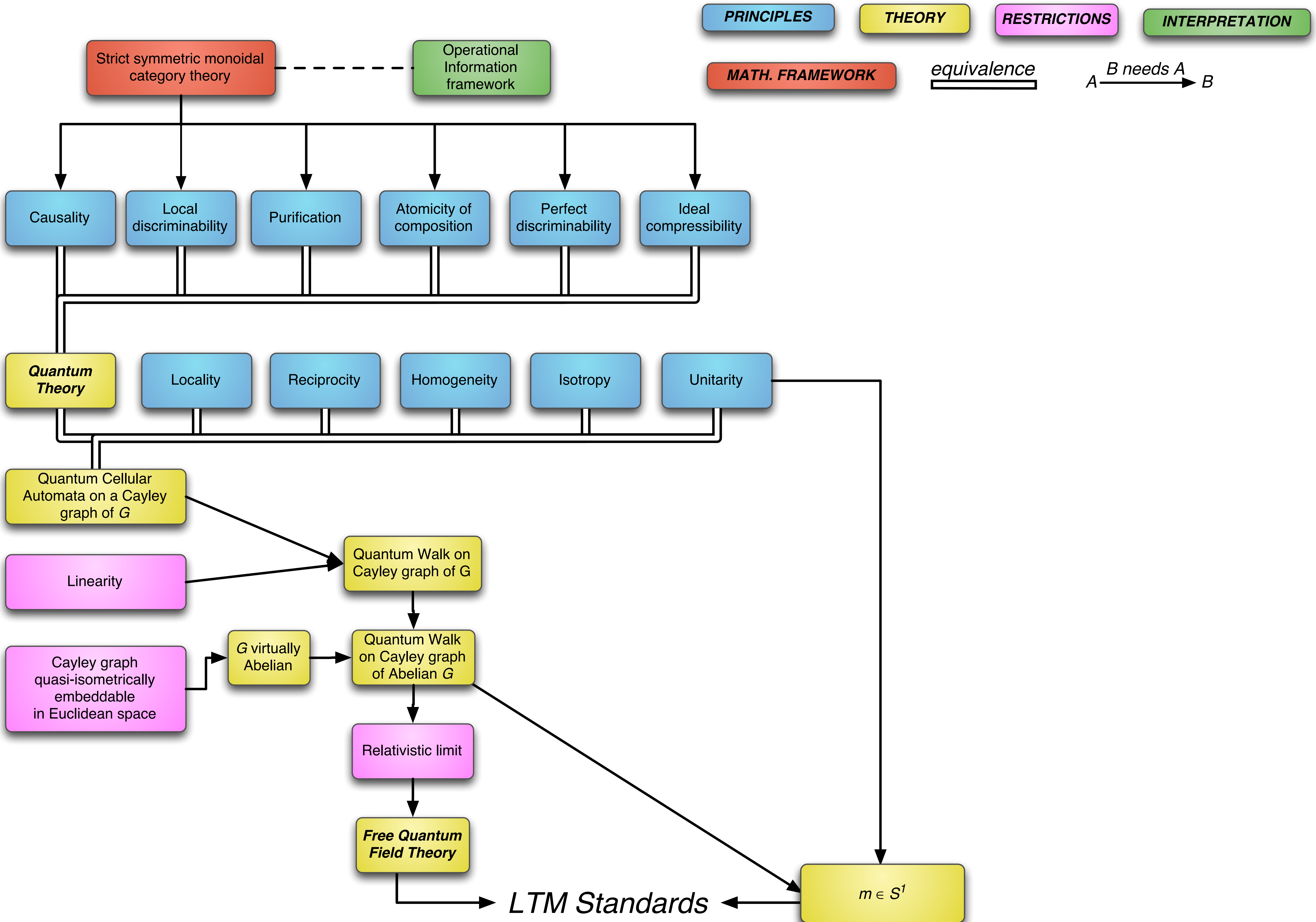
m ∈ S1

The theory contains its own LTM standards!

$$M \simeq \frac{1}{\sqrt{3}} \frac{\hbar k}{c(k) - c(0)}$$

$$\begin{cases} c \equiv c(0) = \dfrac{a}{\tau} \\ \\ \hbar = Mac \end{cases}$$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LTM Standards
$m \in S^1$

Bisio, D'Ariano, Perinotti, unpublished

Case of study 1: Special Relativity recovered

- Mathematical statement:
  invariance of eigenvalue equation under change of representation.

- Physical interpretation:
  invariance of the physical law under change of inertial reference frame.

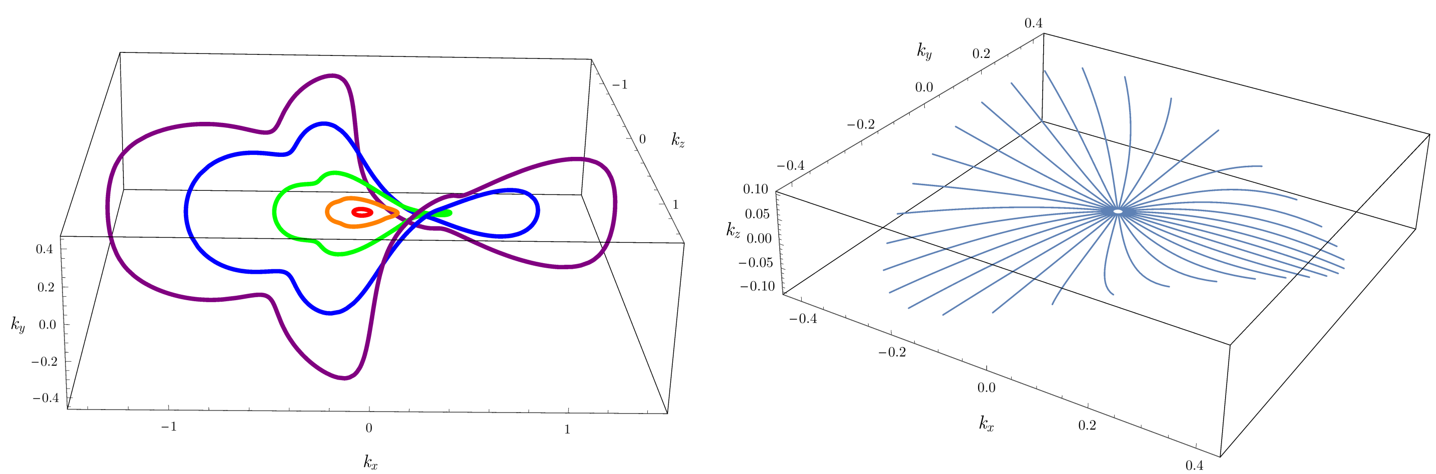


FIG. 2: The distortion effects of the Lorentz group for the discrete Planck-scale theory represented by the quantum walk in Eq. (6). Left figure: the orbit of the wavevectors $\mathbf{k} = (k_x, 0, 0)$, with $k_x \in \{.05, .2, .5, 1, 1.7\}$ under the rotation around the z axis. Right figure: the orbit of wavevectors with $|\mathbf{k}| = 0.01$ for various directions in the (k_x, k_y) plane under the boosts with $\boldsymbol{\beta}$ parallel to $\mathbf{k}$ and $|\boldsymbol{\beta}| \in [0, \tanh 4]$.

m=0

Deformed Poincaré group

- Lorentz transformations are perfectly recovered for *k≪1.*
- For *k~1*:
  - *Double Special Relativity* (Camelia-Smolin).
  - *Relative locality* (in addition to relativity of simultaneity)

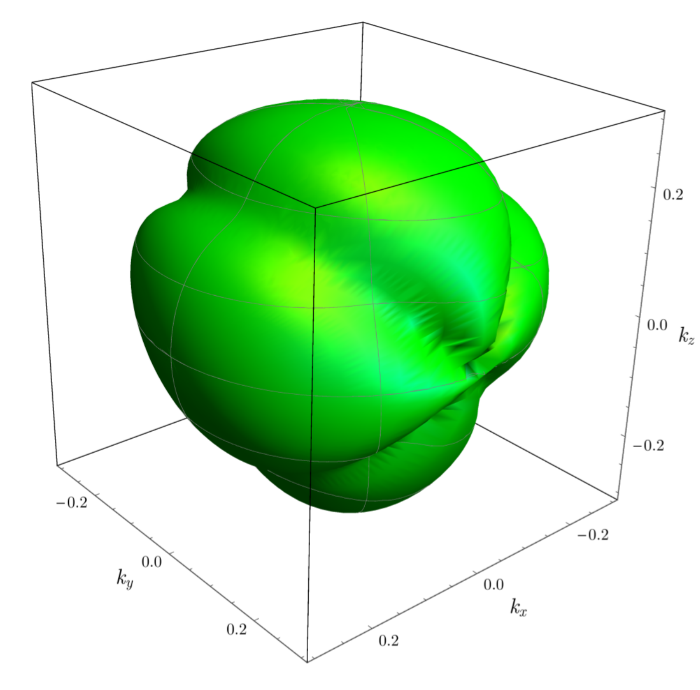


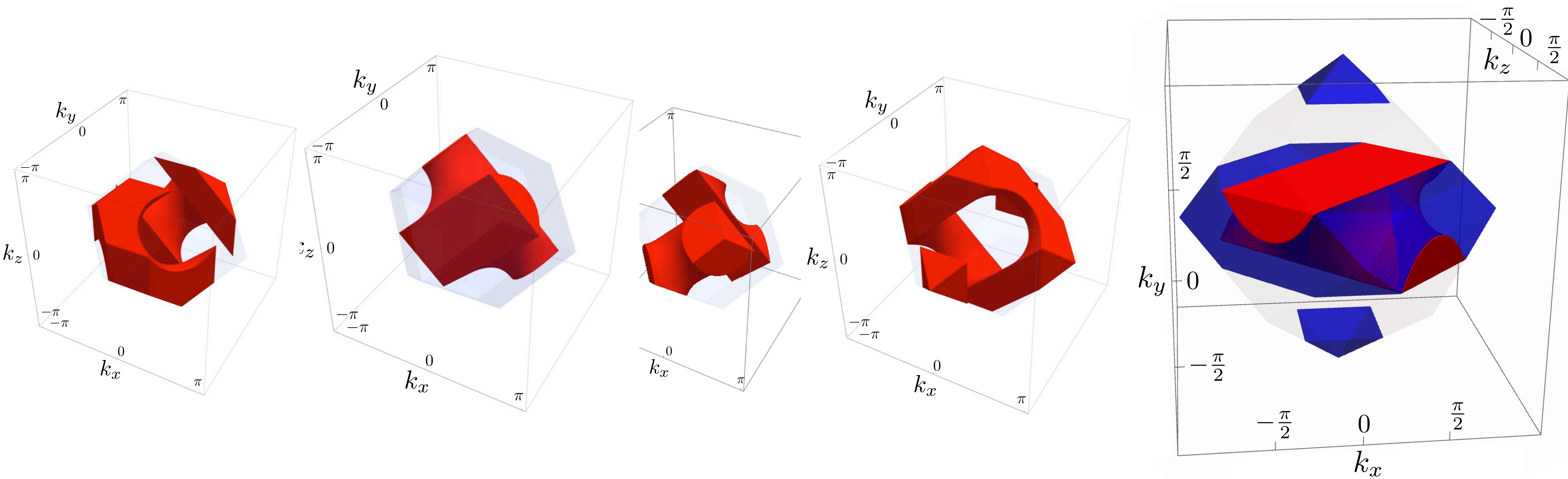
FIG. 3: The green surface represents the orbit of the wavevector $\mathbf{k} = (0.3, 0, 0)$ under the full rotation group $SO(3)$.

Bisio, D'Ariano, Perinotti, unpublished

Case of study 2: particle notion

- Mathematical statement:
  irreducible representation of deformed Poincaré group.

- Physical interpretation: particle!



- The Brillouin zone separates into four Poincaré-invariant regions diffeomorphic to balls, corresponding to four different particles.

- *m≠0* De Sitter *SO(1,4)*

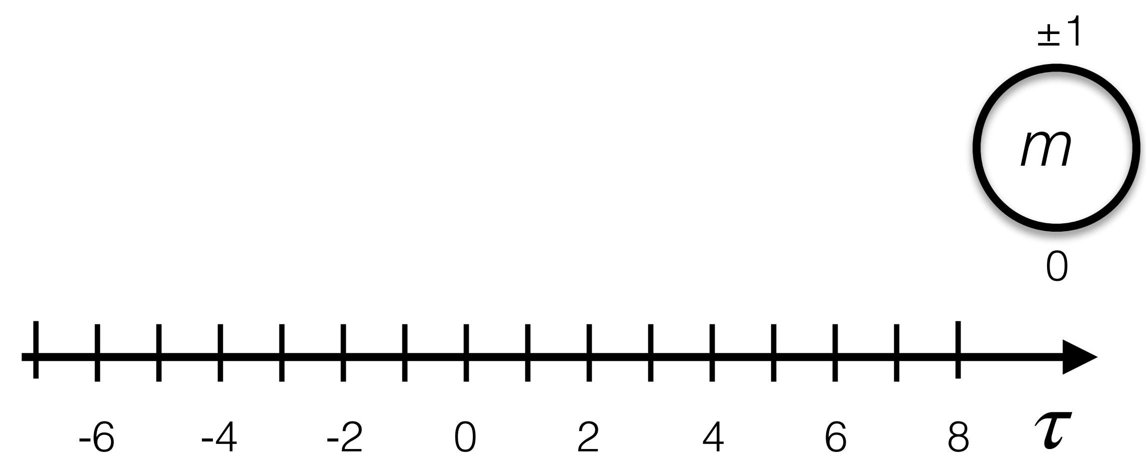
Bisio, D'Ariano, Perinotti,
unpublished

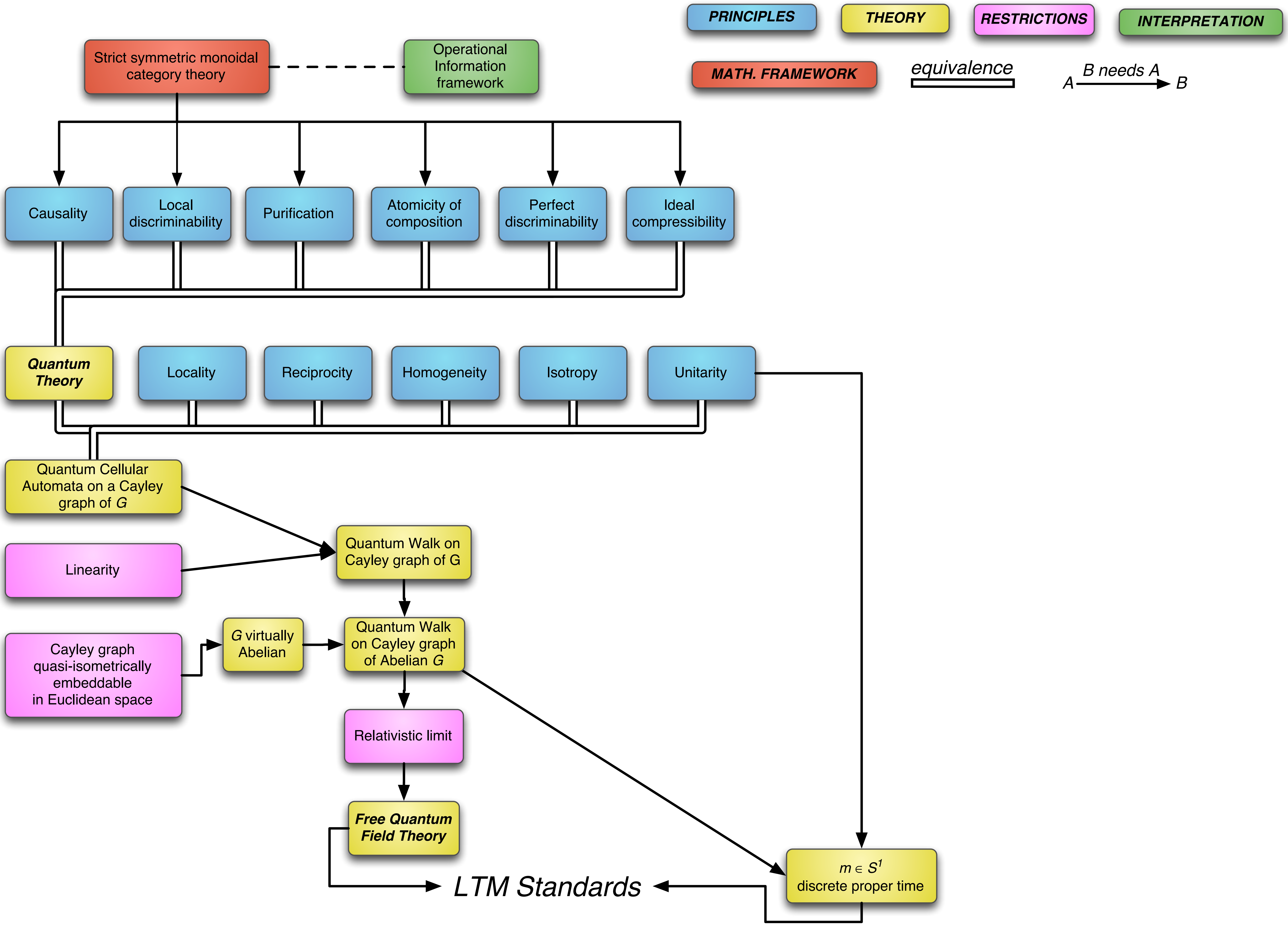
Case of study 3: proper time

- Mathematical statement:
  topology of the particle mass domain

- Physical interpretation: proper time is discrete!

$$H(q_\alpha, p_\alpha, \tau, m) = \sum_\alpha p_\alpha \dot{q}_\alpha + c^2 m \dot{\tau} - L$$





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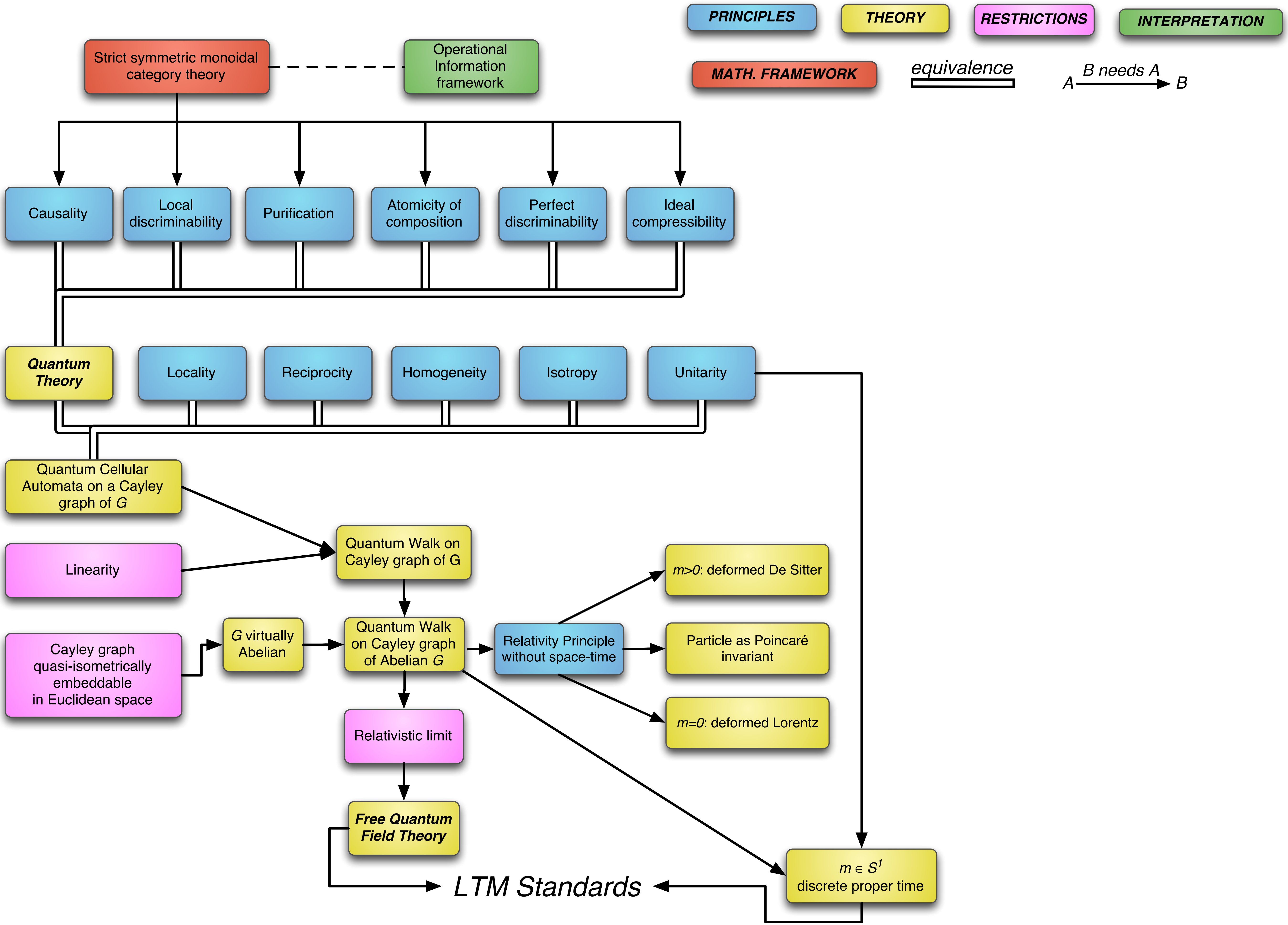

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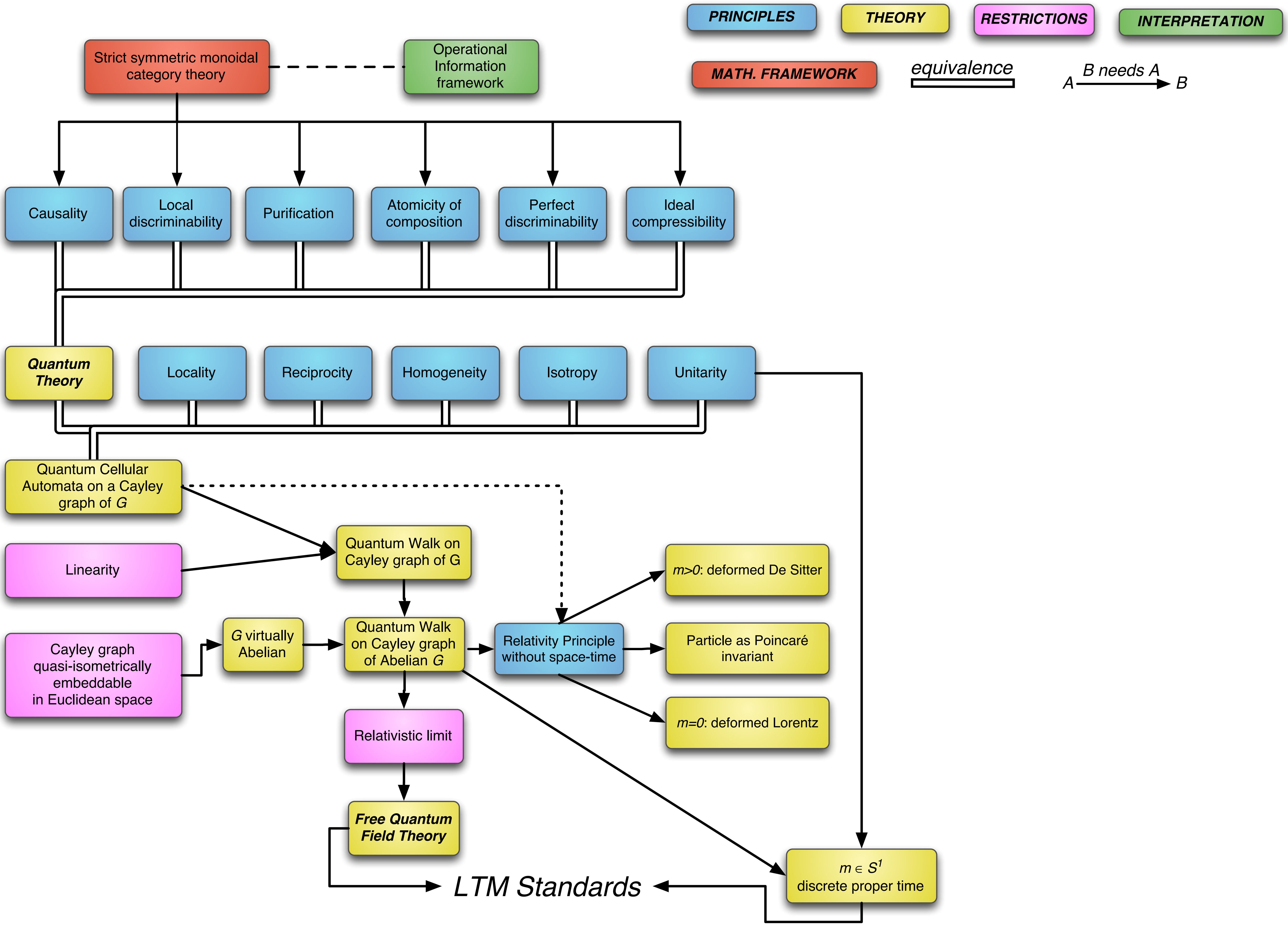
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
Relativistic limit
Free Quantum Field Theory
LTM Standards
$m \in S^1$
discrete proper time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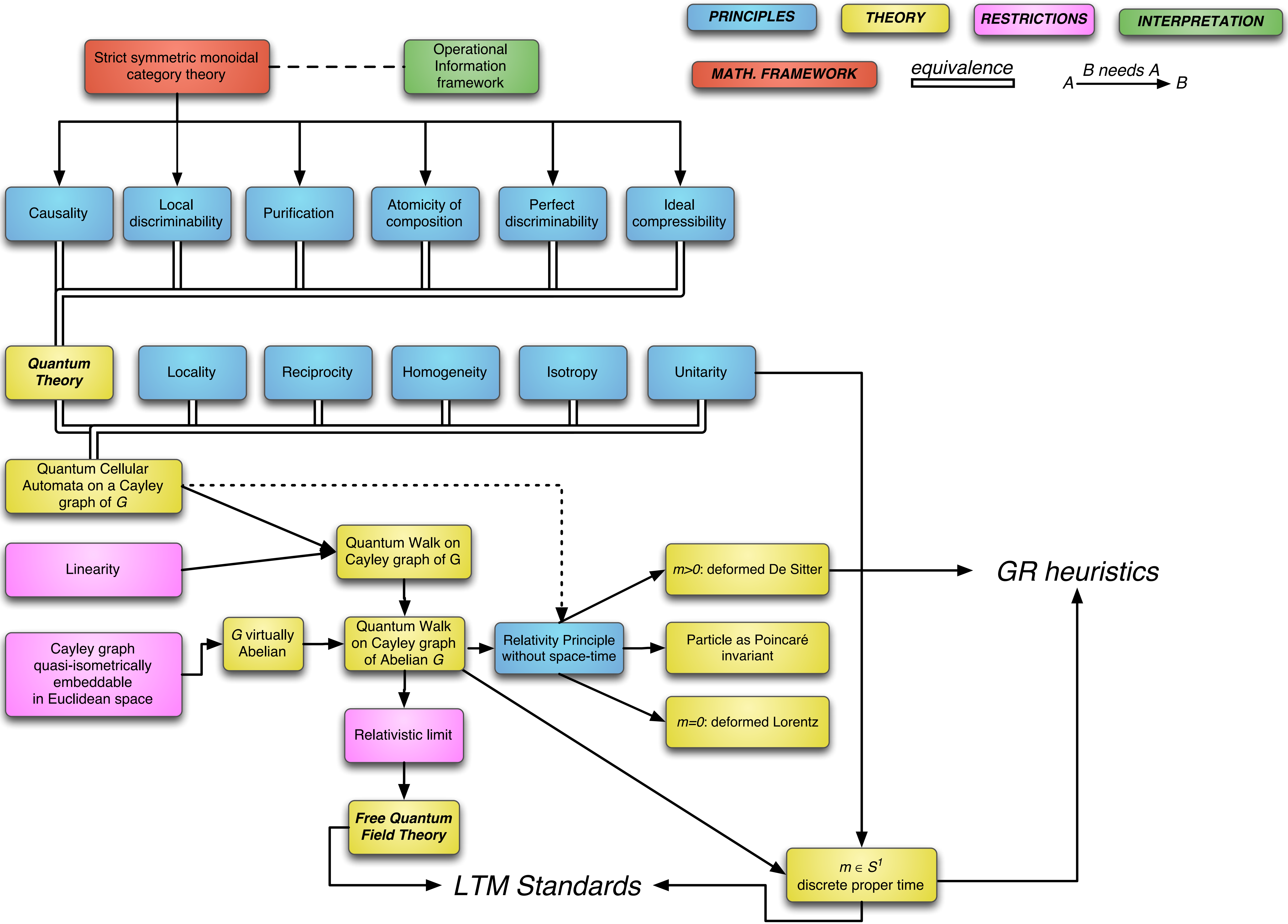
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A priori principles?


Hans Reichenbach

Conventionalism:
- Homogeneity
- Isotropy

Symmetries


Adolf Grünbaum

Theory simplicity

Minimization of algorithmic complexity

Homogeneity = clocks, roads …

This is more or less what I wanted to say

Thank you for your attention