New Devices for Retrieving and Broadcasting Quantum Information

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Quantum Mechanics and Quantum Information
Lecce, 6 May 2005

QUit
quantum information theory group

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- Super-broadcasting
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- Quantum calibration of measuring apparatuses
- Optimal phase estimation for mixed states

Superbroadcasting

Perinotti

Macchiavello

G. M. D'Ariano, C. Macchiavello, and P. Perinotti,
Superbroadcasting of mixed states,
Phys. Rev. Lett. (submitted)

Superbroadcasting

$$N \text{ inputs} \Rightarrow M \text{ outputs}$$

$$R_{out} = \quad \rho \otimes \rho \otimes \ldots \otimes \rho \qquad \text{"}cloning\text{"}$$

$$\mathrm{Tr}_{123\ldots M-1}[R_{out}] = \mathrm{Tr}_{23\ldots M}[R_{out}] = \rho \qquad \text{"}broadcasting\text{"}$$

- For pure states ideal broadcasting coincides with the *quantum cloning*.
- For mixed states there are infinitely many joint states that correspond to the same local state.

Superbroadcasting

- For mixed input states the ***no-cloning theorem*** is not logically sufficient to forbid ideal broadcasting

- The ***impossibility of ideal broadcasting*** has been proved in the case of <u>one input</u> copy and <u>two output</u> copies for ***non mutually commuting density operators*** [H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, Phys. Rev. Lett. **76** 2818 (1996)]

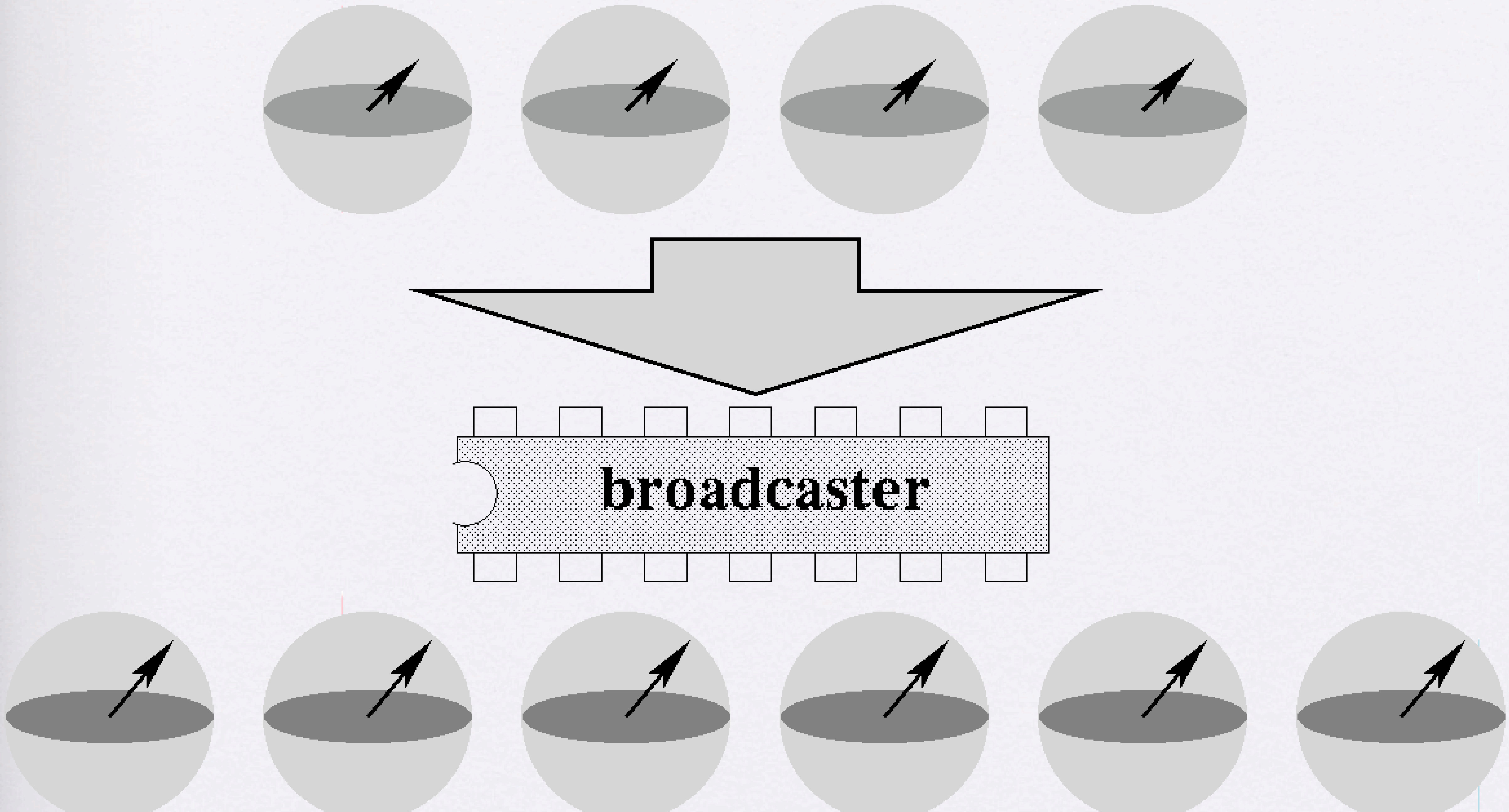
Is this a generalization of the no-cloning theorem to mixed states?

Superbroadcasting

The answer is no!

- We have shown that *the no broadcasting theorem does not generalize to more than three input copies!*
- For $N \geq 4$ input copies *it is even possible to purify the state while broadcasting!*
- ***broadcasting + cloning*** ***"superbraodcasting"***.

Superbroadcasting



$$\rho_{\boldsymbol{n}} = \frac{1}{2}(I + r\boldsymbol{n} \cdot \boldsymbol{\sigma})$$

Superbroadcasting

shrinking/stretching factor $p(r) = r'_{opt}(r)/r$

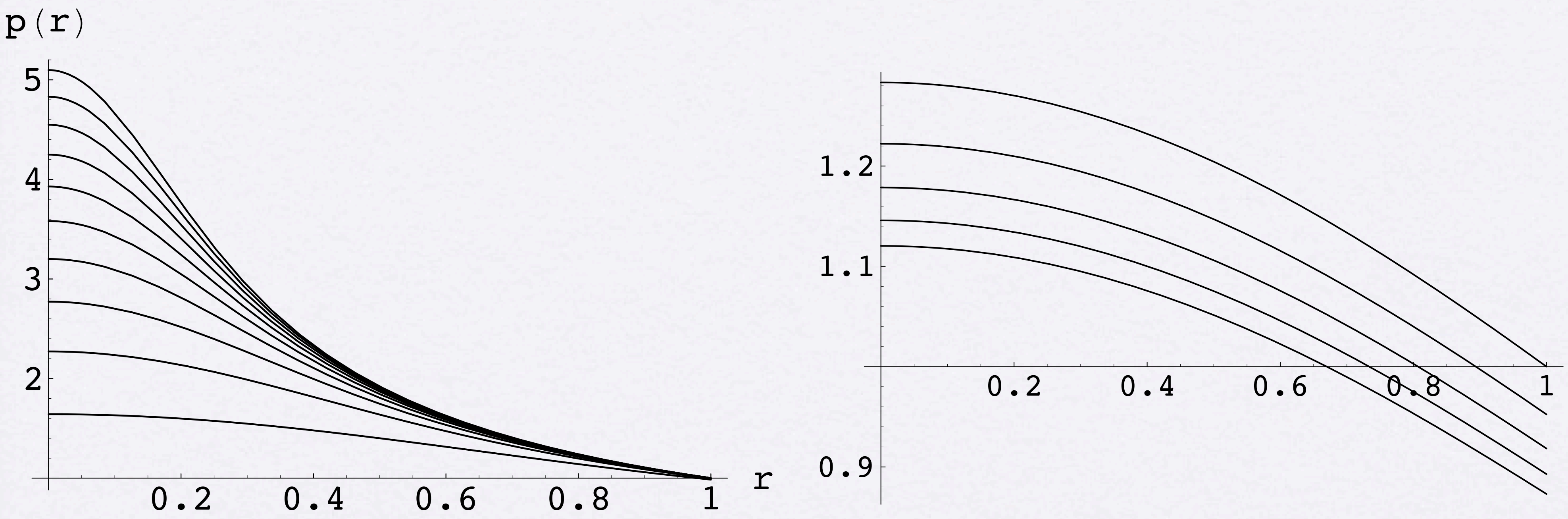
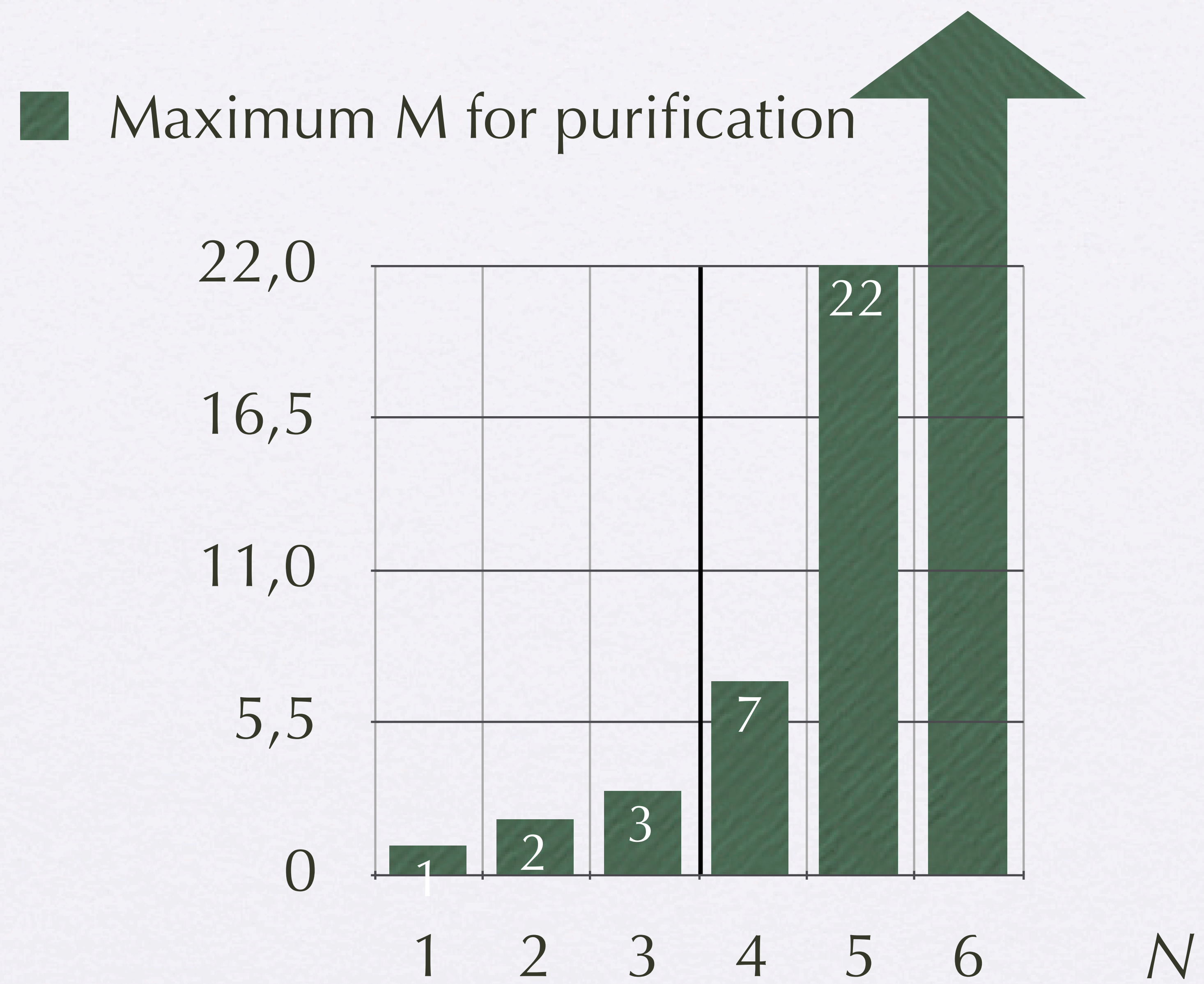


FIG. 2: The stretching factor $p(r)$ versus r. On the left: for $M = N + 1$ and $N = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ (from the bottom to the top. On the right: for $N = 5$ and $5 \leq M \leq 9$ (from the top to the bottom).

Superbroadcasting



Superbroadcasting

shrinking/stretching factor $p(r) = r'_{opt}(r)/r$

$r_*(N, M)$ *maximum purity for purification*

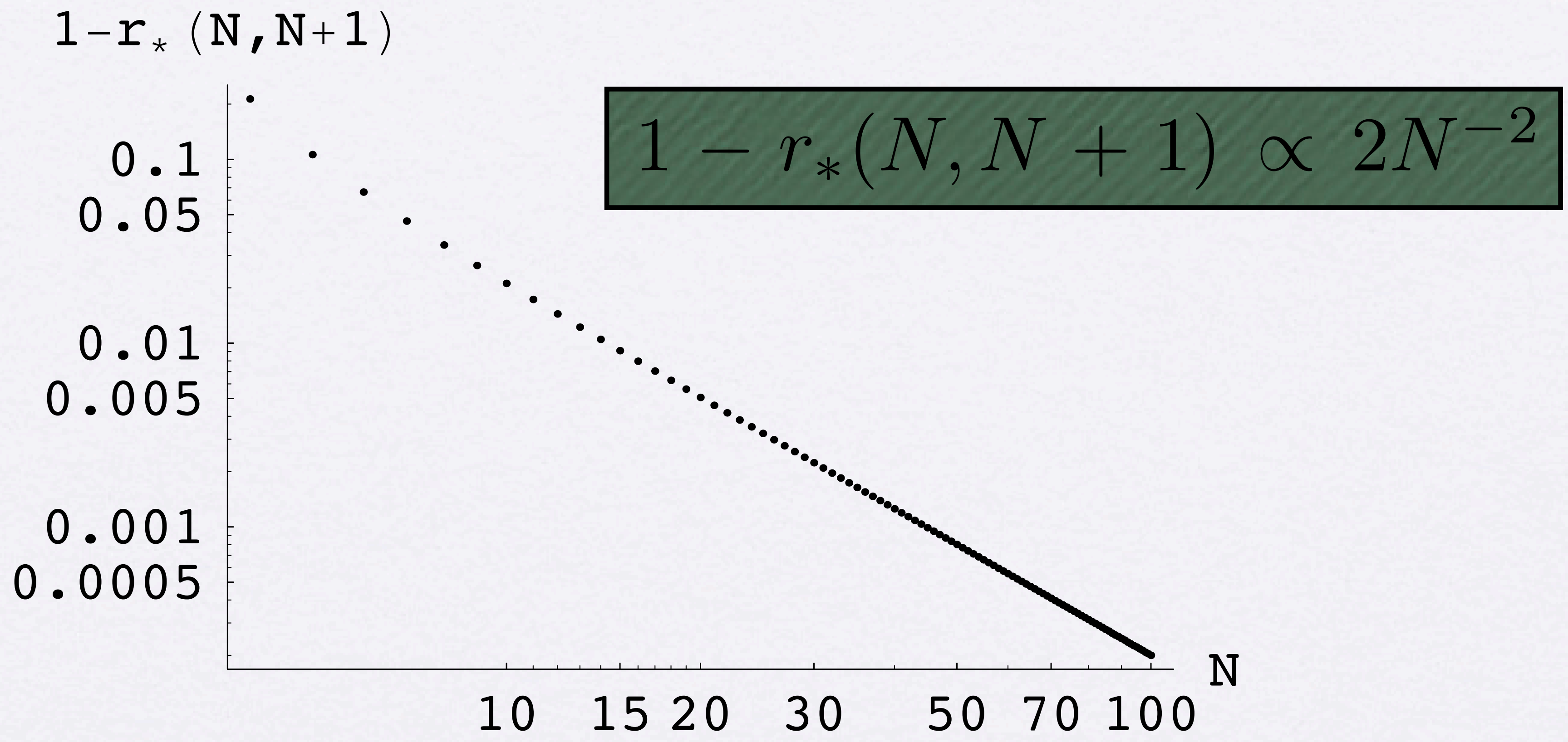


FIG. 3: Logarithmic plot of $1 - r_*(N, N+1)$ versus N. $r_*(N, M)$ denotes the maximum purity for which one can have superbroadcasting from N to M copies.

Superbroadcasting

- For pure states the optimal superbroadcasting map is the same as the optimal universal cloning [R. F. Werner, Phys. Rev. A **58** 1827 (1998)].
- For $M<N$ it corresponds to the optimal purification map [J. I. Cirac, A. K. Ekert, and C. Macchiavello, Phys. Rev. Lett. **82** 4344 (1999)].
- Therefore, the superbroadcasting map generalizes and ***interpolates optimal purification and optimal cloning***.

Superbroadcasting

- ***Superbroadcasting doesn't mean more available information about the original input state***.

- This is due to ***detrimental correlations between the broadcast copies***, which does not allow to exploit their statistics [this phenomenon was already noticed by M. Keyl and R. F. Werner, Ann. H. Poincaré **2** 1 (2001)].

- From the ***point of view of each single user*** our broadcasting protocol is a purification in all respects (for states sufficiently mixed). The process transfers noise from the local states to the correlations between them.

Programmable detectors

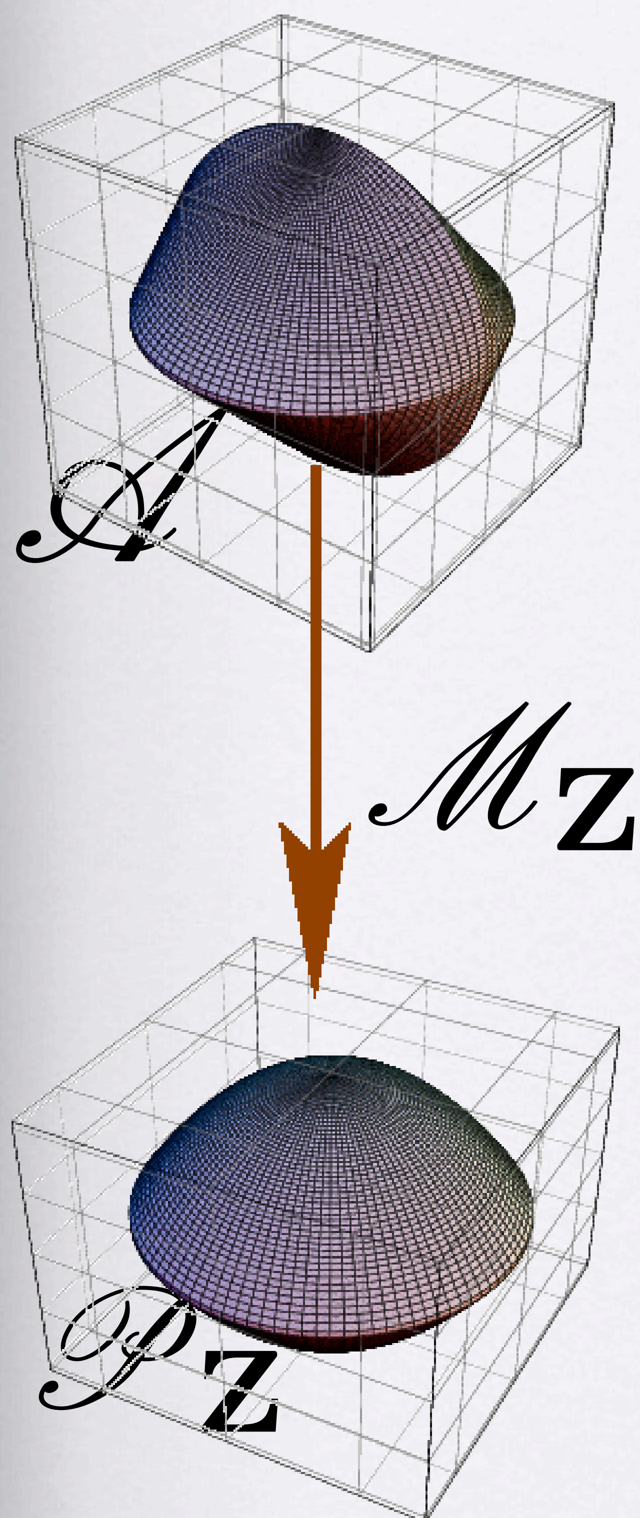
Perinotti

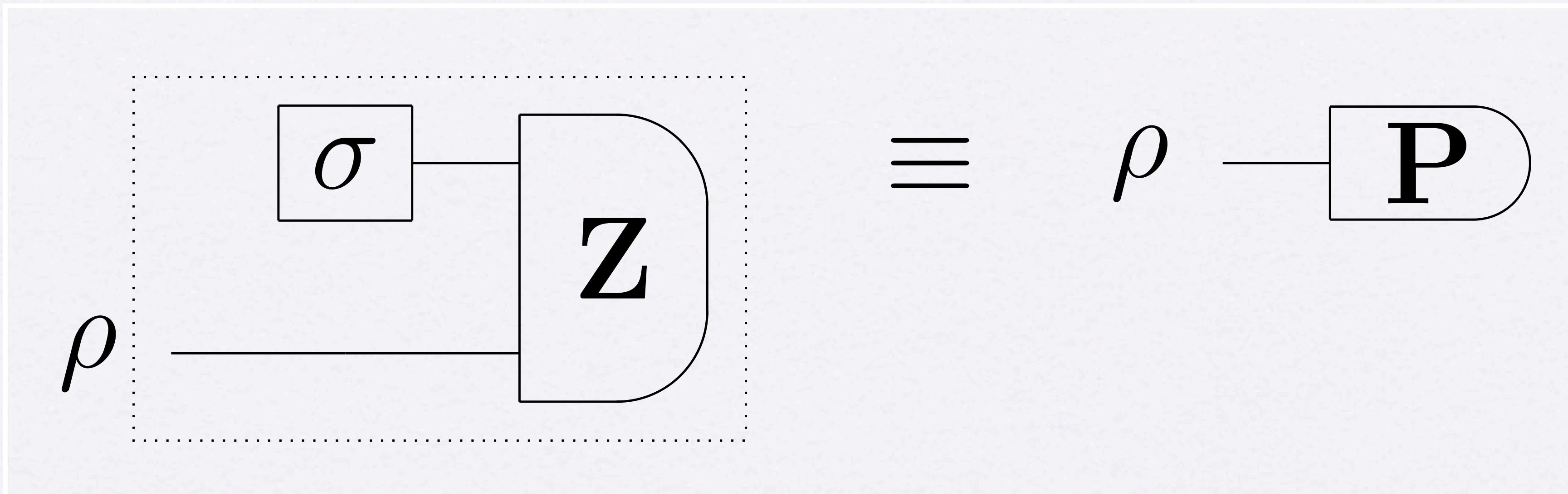
G. M. D'Ariano, and P. Perinotti,

Efficient Universal Programmable Quantum Measurements,

Phys. Rev. Lett. **94** 090401 (2005)

Programmability of POVM's





$$\mathbf{Z} \doteq \{Z_1, Z_2, \ldots, Z_N\},\ \mathbf{P} \doteq \{P_1, P_2, \ldots, P_N\}$$

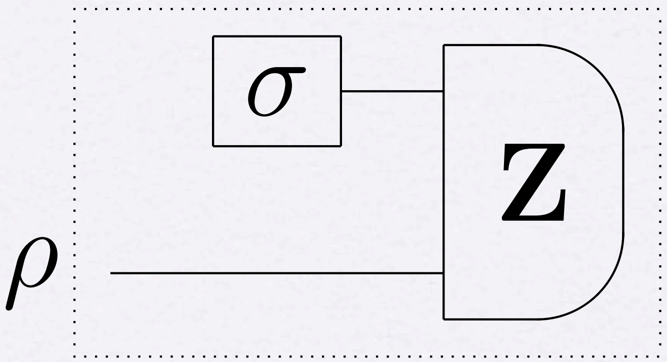
$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

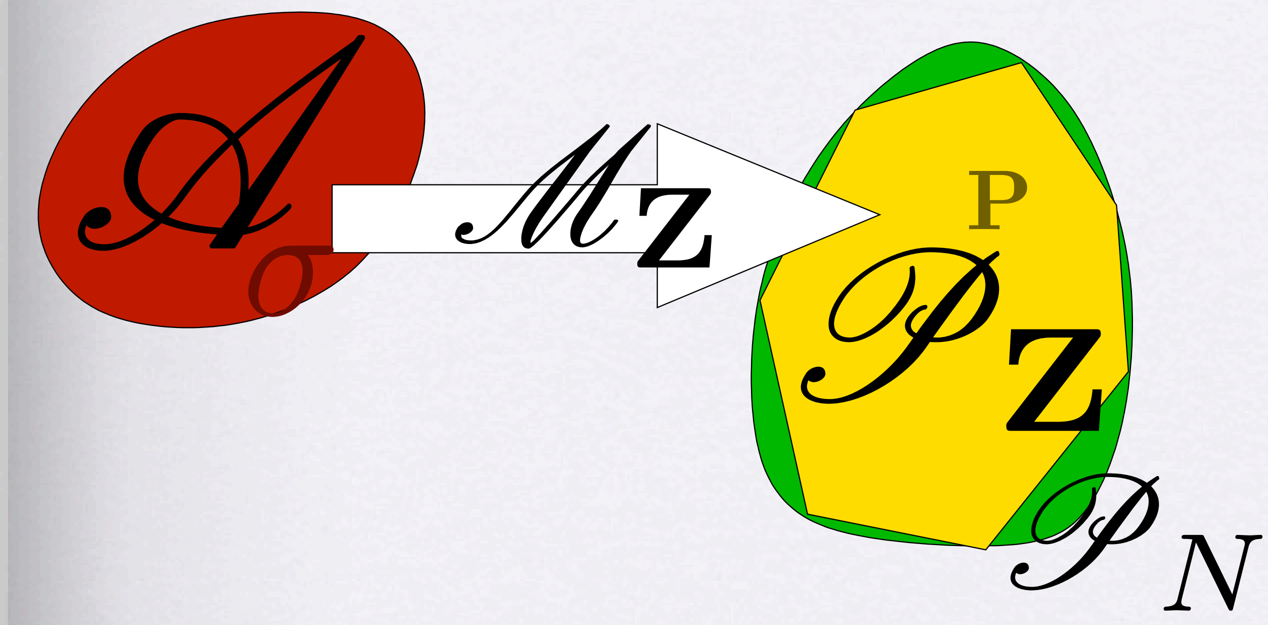
$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$

Programmability of POVM's

$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$





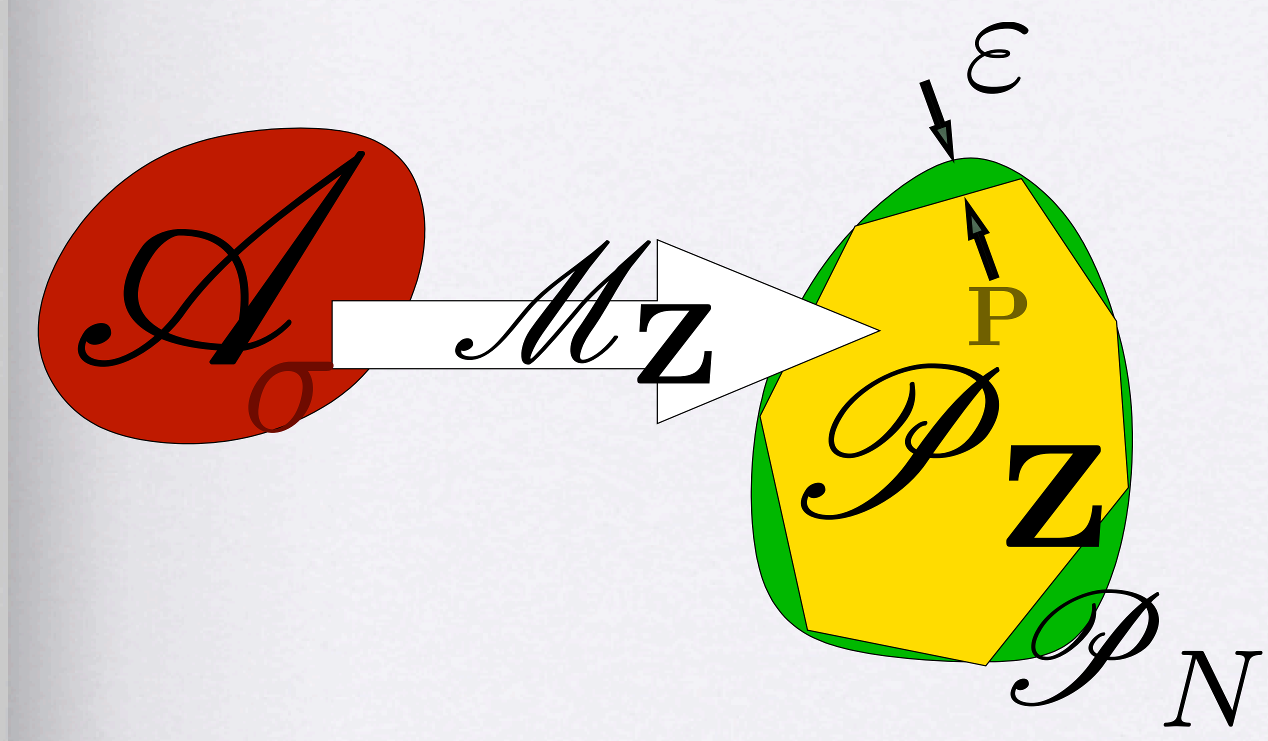
No go theorem

It is impossible to program all observables with a single ***Z*** and a finite-dimensional ancilla

Programmability of POVM's

$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$



Problem: *The "big* $\mathbf{Z}$*"*

For given $d = \dim(\mathcal{A})$ and $N = |\mathbf{Z}| = |\mathbf{P}|$, find the observables $\mathbf{Z}$ that are the most efficient in programming POVM's, namely which minimize the largest distance of each POVM from the programmable set:

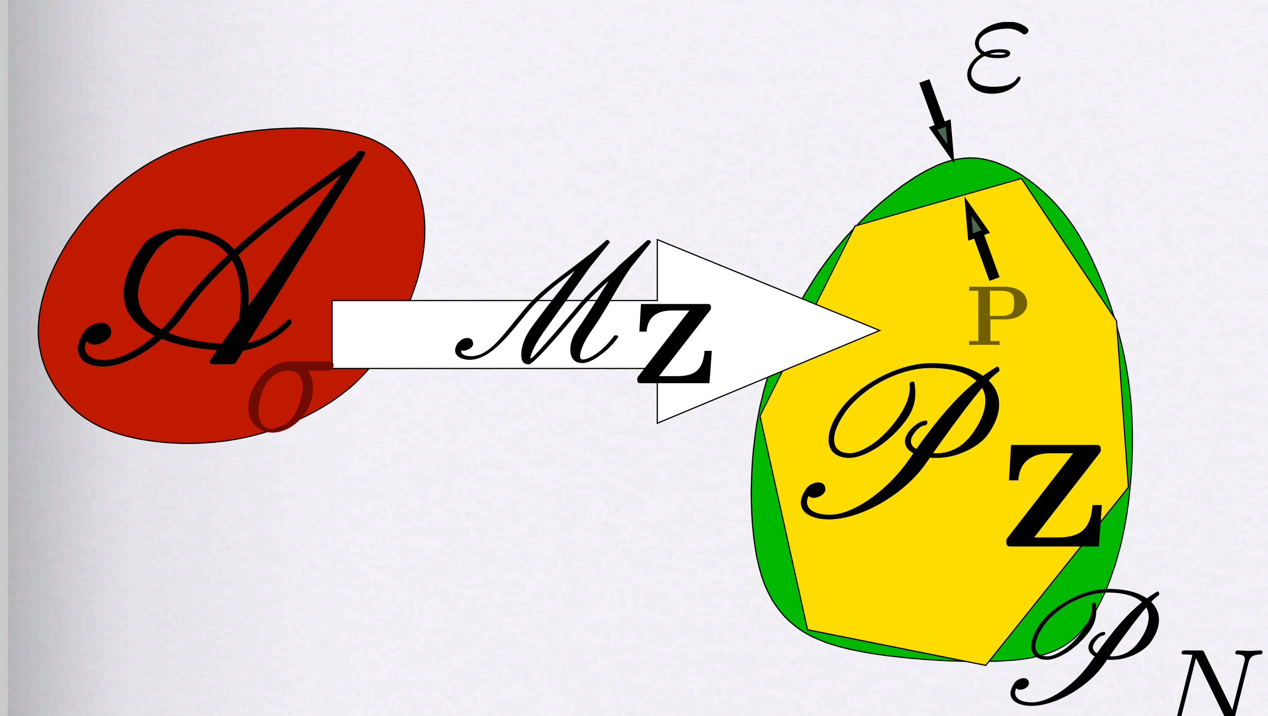
$$\varepsilon(\mathbf{Z}) \doteq \max_{\mathbf{P}\in\mathscr{P}_N} \min_{\mathbf{Q}\in\mathscr{M}_{\mathbf{Z},\mathscr{A}}} \delta(\mathbf{P},\mathbf{Q})$$

.

Programmability of POVM's

$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$



In the literature it was found that

$$\varepsilon \sim \exp(\kappa \dim(\mathscr{A}))$$

We found that polynomial (and even linear!) precision is achievable

Programmability of POVM's

programmability with **accuracy** ε^{-1}:

$$\varepsilon \doteq \max_{\mathbf{P}\in\mathscr{P}_N} \min_{\mathbf{Q}\in\mathscr{P}_{\mathbf{Z}}} \delta(\mathbf{P},\mathbf{Q})$$

$$\delta(\mathbf{P},\mathbf{Q}) = \max_{\rho} \sum_i |\operatorname{Tr}[\rho(P_i - Q_i)]|$$

Controlled-U

Using a joint observable $\mathbf{Z}$ of the form

$$Z_i = U^\dagger(|\psi_i\rangle\langle\psi_i| \otimes I_A)U, \qquad U = \sum_{k=1}^{\dim(\mathcal{A})} W_k \otimes |\phi_k\rangle\langle\phi_k|$$

with $\{\psi_i\}$ and $\{\phi_k\}$ orthonormal sets and W_k unitary, we can program observables with accuracy ε^{-1} using an ancilla with **polynomial** growth

$$\dim(\mathcal{A}) \leqslant \kappa(N)\left(\frac{1}{\varepsilon}\right)^{N(N-1)}$$

Programmability of POVM's

For qubits: *linear growth!*

Program for the observable $\mathbf{P} = \{U_g^{(1/2)}|\pm\frac{1}{2}\rangle\langle\pm\frac{1}{2}|U_g^{(1/2)\dagger}\}$

$$\sigma = U_g^{(j)}|jj\rangle\langle jj|U_g^{(j)\dagger}$$

in dimension $\dim(\mathcal{A}) = 2j+1$, with joint observable

$$\mathbf{Z} = \{\Pi^{(j\pm\frac{1}{2})}\}$$

gives the programmability accuracy

$$\varepsilon = \delta(\mathbf{P},\mathbf{Q}) = \frac{2}{2j+1} \Rightarrow \boxed{\dim(\mathcal{A}) = 2\varepsilon^{-1}}$$

Transmission of reference frames

Chiribella

Perinotti

Sacchi

G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. F. Sacchi,
Efficient use of quantum resources for the transmission of a reference frame,
Phys. Rev. Lett. **93** 180503 (2004)

Transmission of reference frames

Transmission of reference frames

- Use N spins that can carry information about the rotation g_* that connects the two frames
- Alice prepares N spins in $|A\rangle$
- She sends the spins to Bob who receives

$$|A_{g^*}\rangle = U_{g*}^{\otimes N}|A\rangle$$

- Bob performs a measurement to infer g_* and rotates his frame by the estimated rotation g

Transmission of reference frames

The deviation between estimated and true axes is

$$e(g, g_*) = \sum_{\alpha=x,y,z} |g n_\alpha^B - g_* n_\alpha^B|^2$$

The state and the measurement are chosen in order to minimize the average transmission error

$$\langle e \rangle = \int dg_* \int dg \, p(g|g_*) \, e(g, g_*)$$

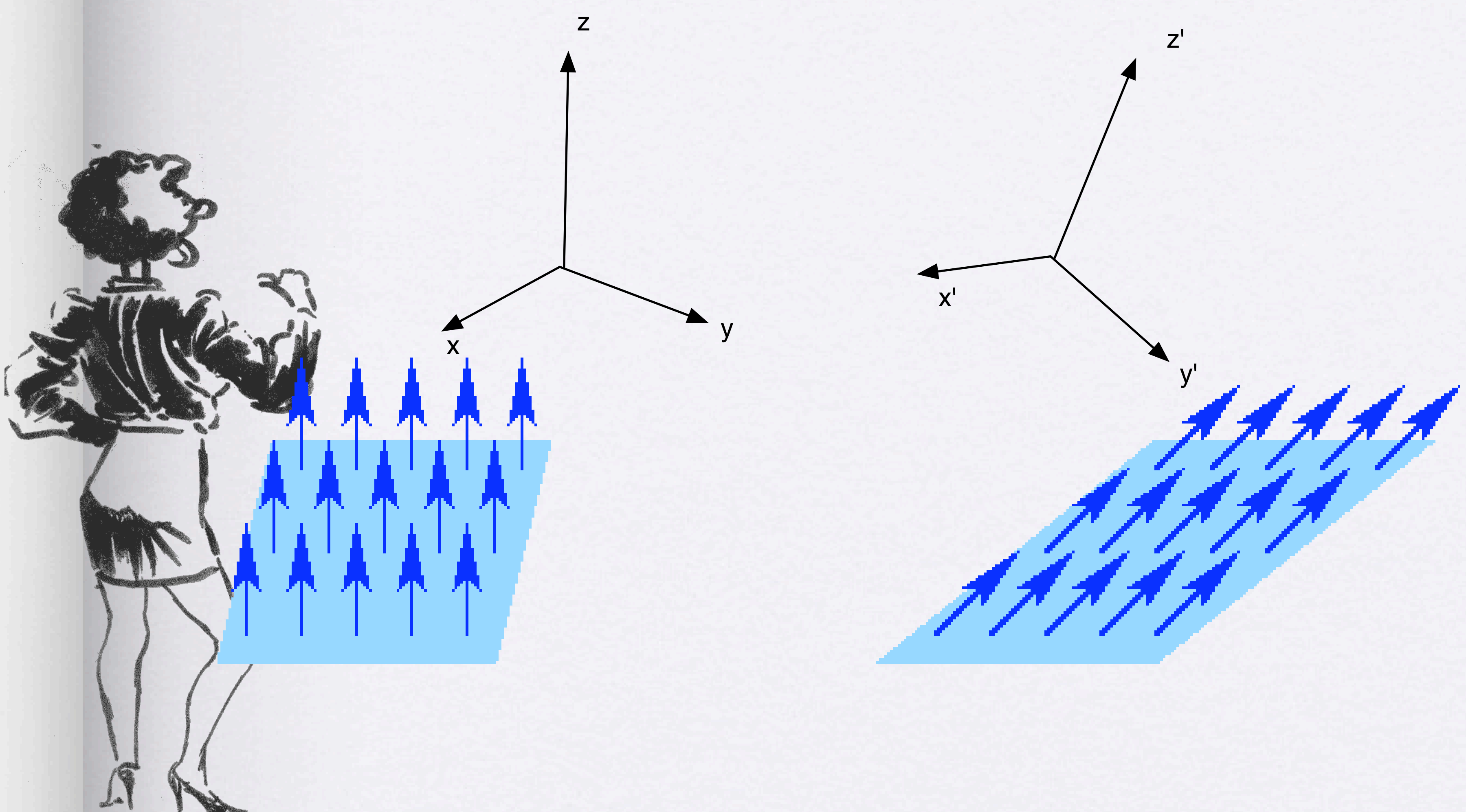
The previous literature claimed that equivalent representations are of no use, and the optimal achievable asymptotic sensitivity is $\propto 1/N$

BUT... the use of equivalent irreducible representations dramatically improves the sensitivity up to $\propto 1/N^2$!

Use entanglement with the multiplicity space!

Transmission of reference frames

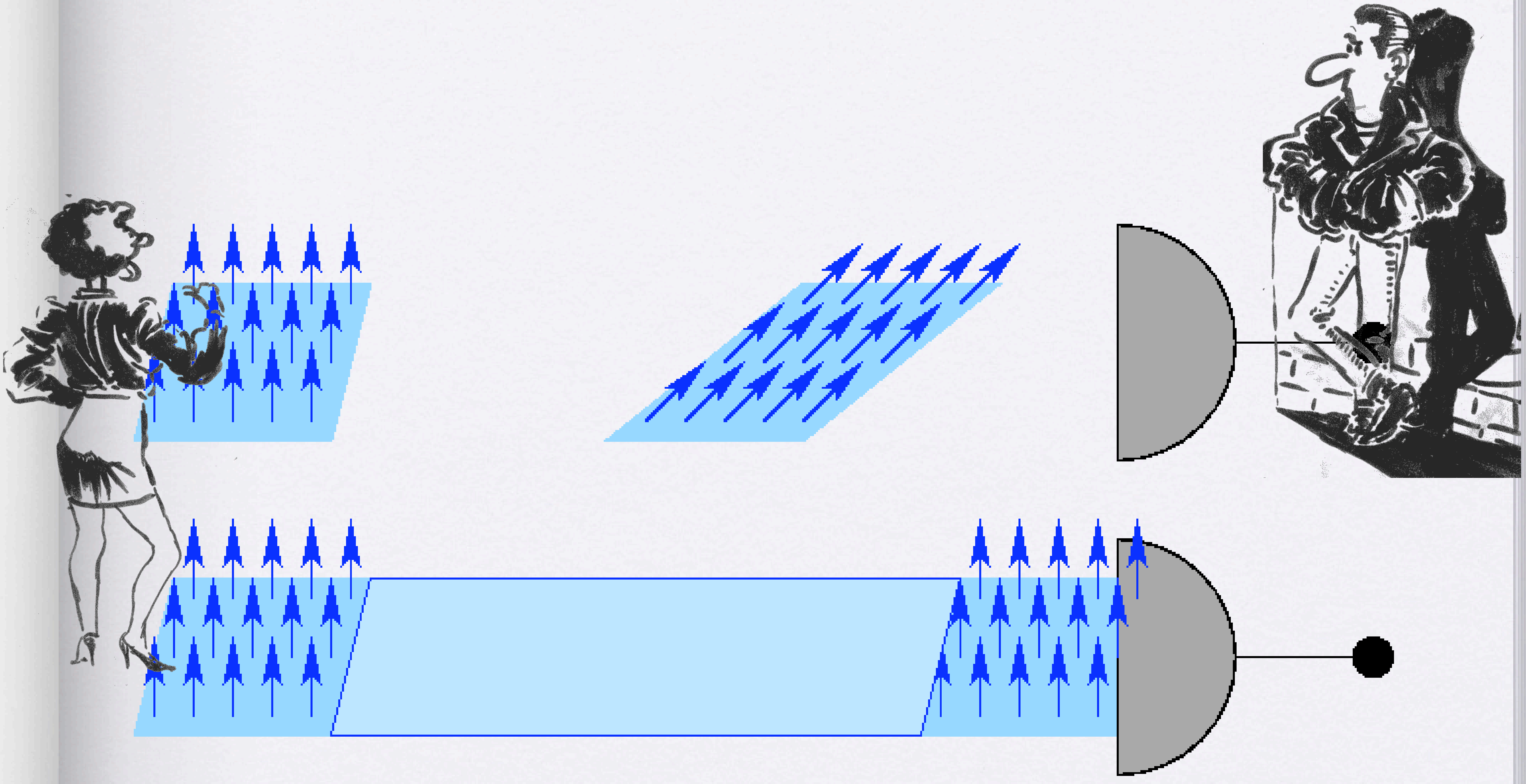
Sensitivity N^{-2} instead of N^{-1}



$$\mathrm{H}^{\otimes N} = \oplus_\nu (\mathrm{H}_\nu \otimes \mathbb{C}^{m_\nu})$$

Transmission of reference frames

No need of shared entanglement!



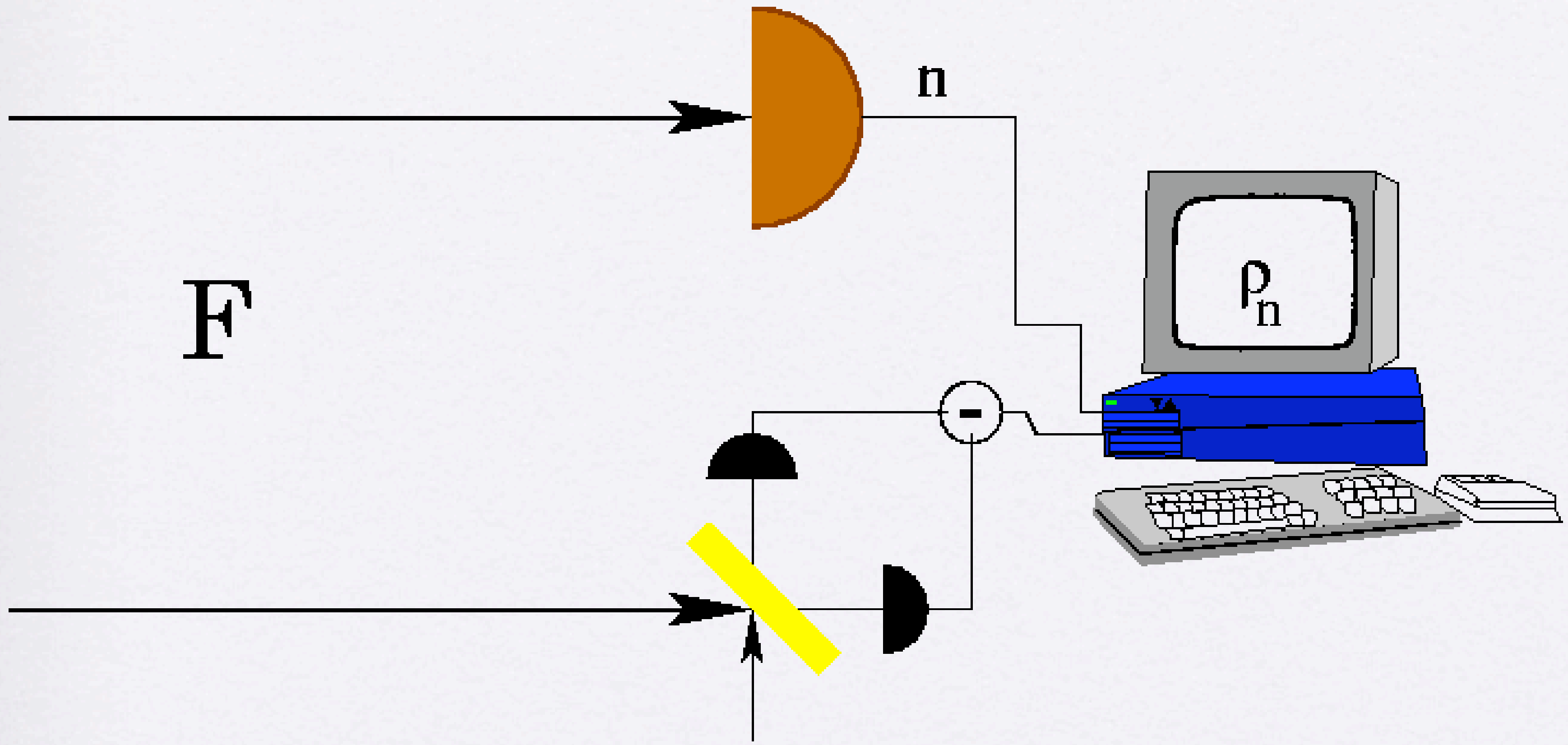
Quantum Calibration

Maccone

Lo Presti

G. M. D'Ariano, P. Lo Presti, and L. Maccone,
Quantum Calibration of Measurement Instrumentation,
Phys. Rev. Lett. **93** 250407 (2004)

Quantum Calibration

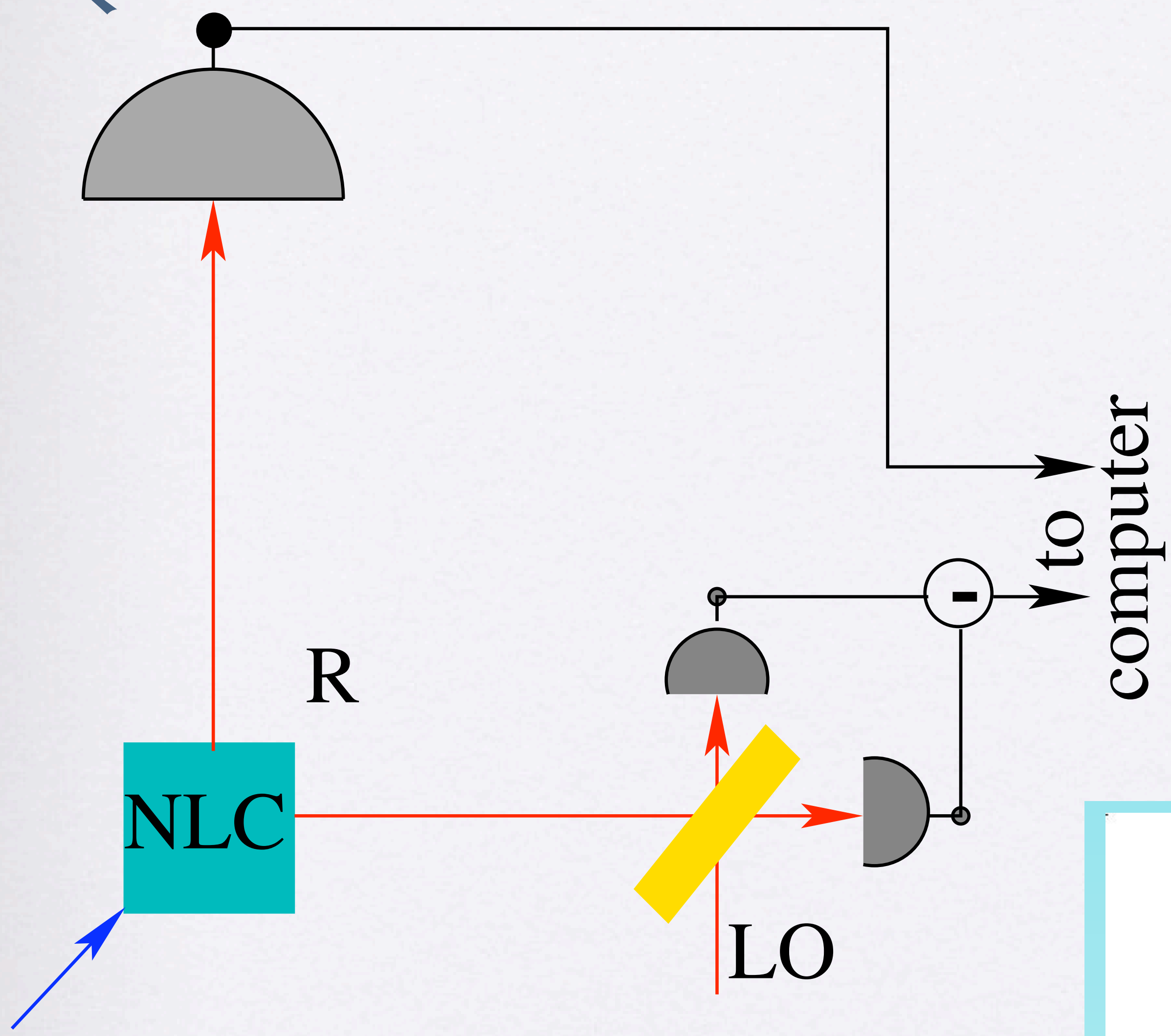


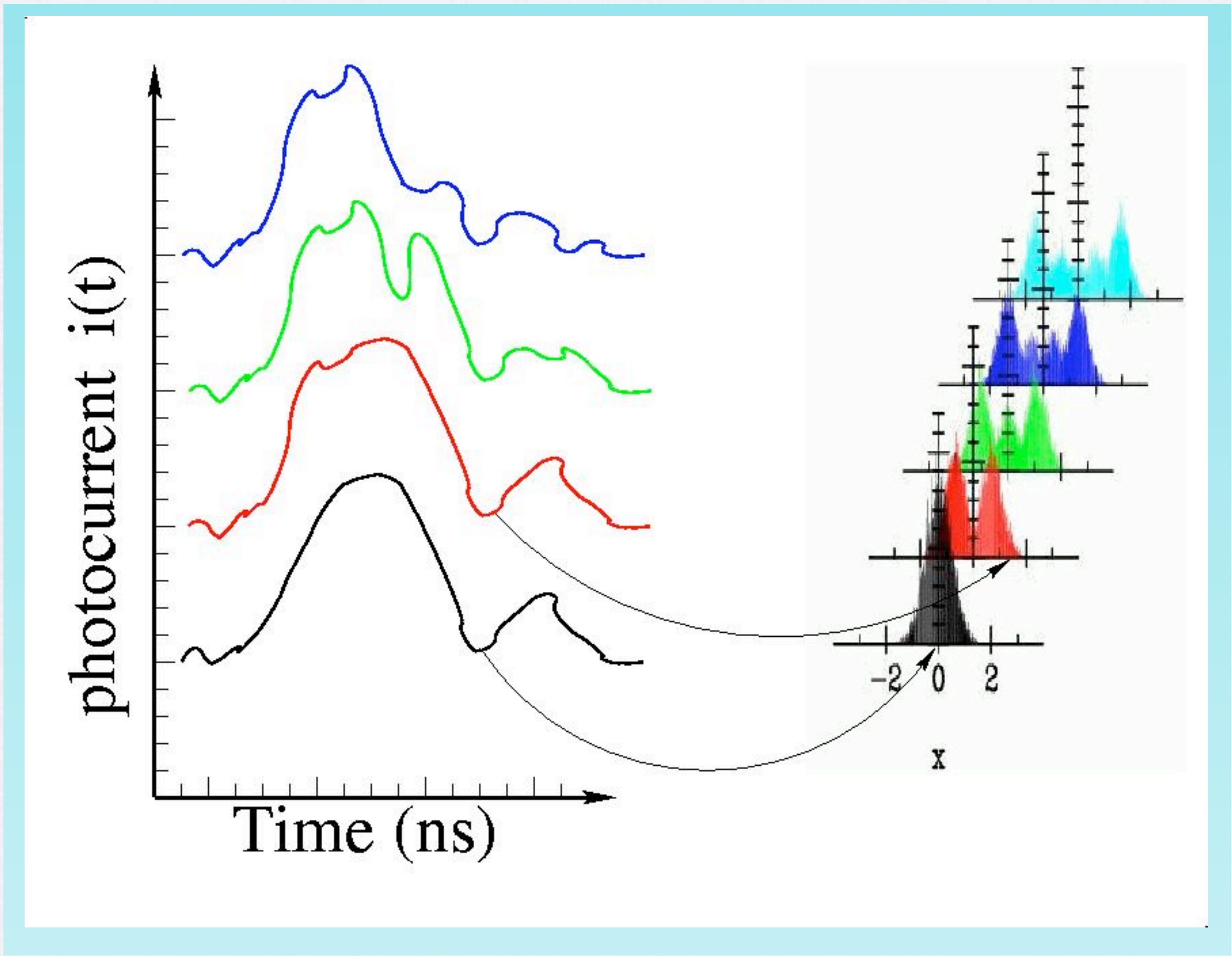
$$p_n\rho_n = \mathscr{F}(P_n), \quad P_n = \mathscr{F}^{-1}(p_n\rho_n),$$

$$\mathscr{F}(X) = \mathrm{Tr}_2[(I \otimes X)F]$$

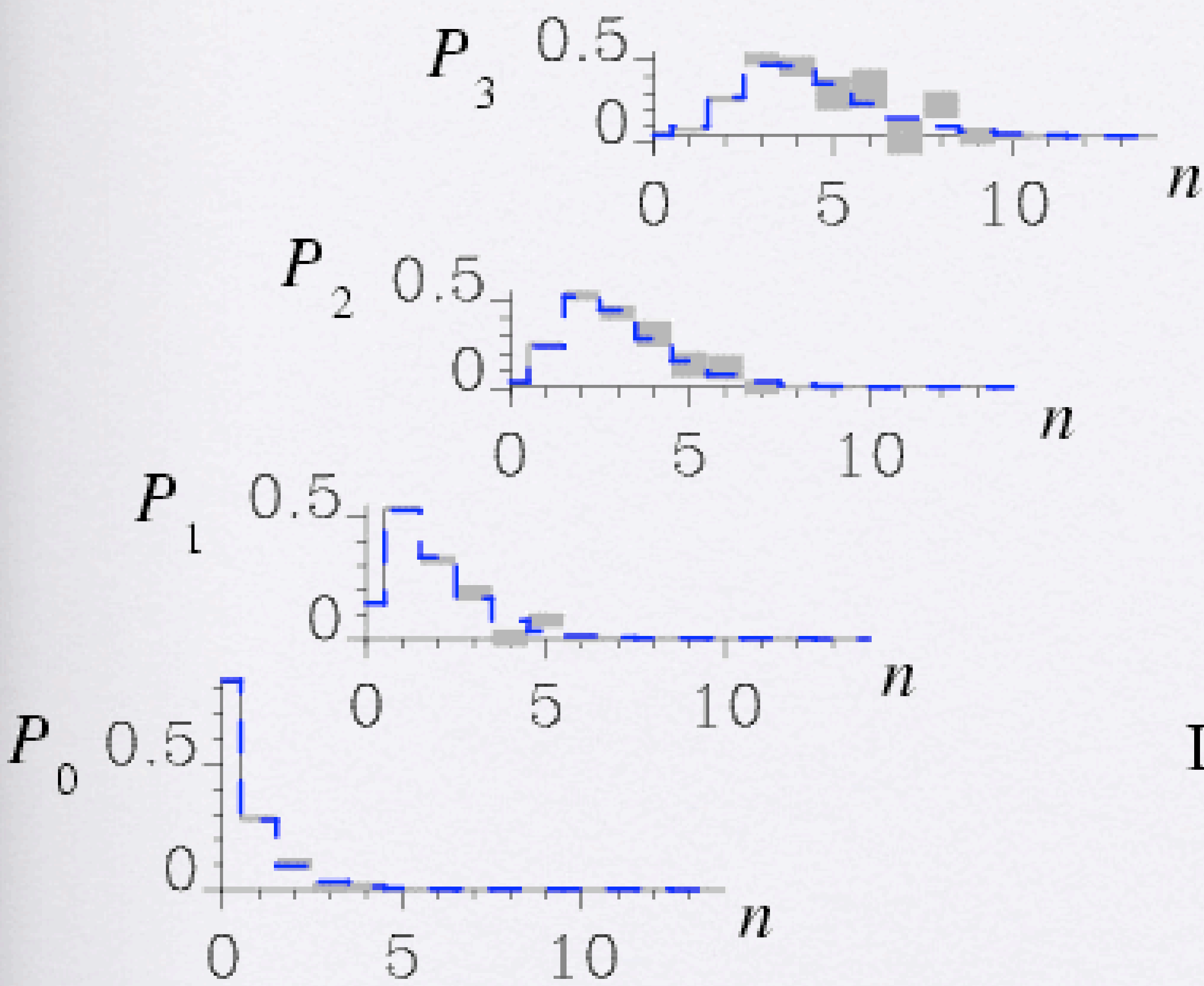
- p_n probability of the outcome n,
- ρ_n conditioned state, to be determined by quantum tomography,
- $\mathscr{F}$ associated map of the faithful state F.

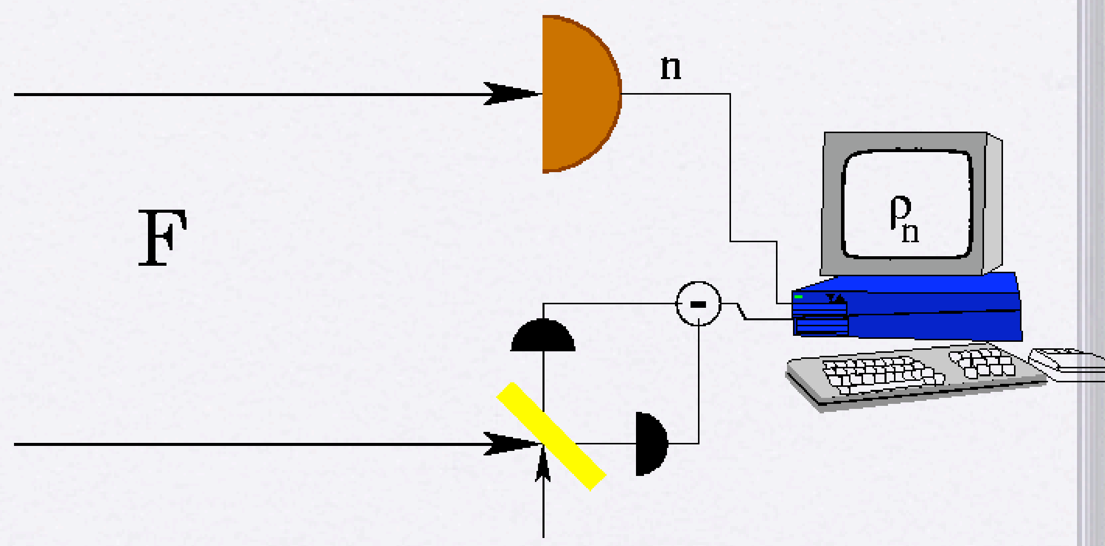
Quantum calibration of a photocounter

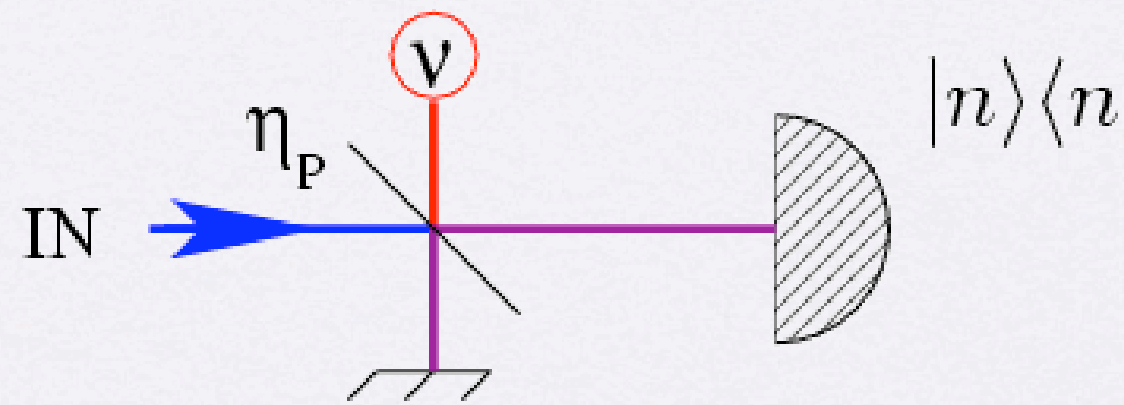




Quantum Calibration







Phase estimation with mixed states

Perinotti

Macchiavello

G. M. D'Ariano, C. Macchiavello, and P. Perinotti,
Optimal phase estimation for qubit in mixed states,
Phys. Rev. Lett. (submitted)

Phase estimation with mixed states

N qubits in the same mixed state

$$R_{\vec{n}} = \rho_{\vec{n}}^{\otimes N}$$

with

$$\rho_{\boldsymbol{n}} = \tfrac{1}{2}(I + r\boldsymbol{n}\cdot\boldsymbol{\sigma})$$

experiencing the same phase shift

$$R_{\vec{n}}(\phi) = U_\phi R_{\vec{n}} U_\phi^\dagger = \left[e^{-i\frac{\phi}{2}\sigma_z}\rho_{\vec{n}} e^{i\frac{\phi}{2}\sigma_z}\right]^{\otimes N}$$

Problem: *estimate the phase shift* ϕ *optimally, minimizing the average cost*

$$\langle C\rangle = \int_0^{2\pi}\frac{\mathrm{d}\,\phi}{2\pi}\int_0^{2\pi} C(\phi,\phi')\,\mathrm{Tr}[U_\phi R_{\vec{n}} U_\phi^\dagger P(\mathrm{d}\,\phi')]$$

Phase estimation with mixed states

Solution:

$$\langle m{+}1, j\alpha|R_{\vec{n}}|m, j\alpha\rangle = |\langle m{+}1, j\alpha|R_{\vec{n}}|m, j\alpha\rangle|e^{i\chi(m+1,m,j\alpha)}$$

Since only the elements on the first over-diagonal and under-diagonal are involved, one can choose phases in such a way that

$$\chi(m+1, m, j\alpha) = \theta(m, j\alpha) - \theta(m+1, j\alpha)$$

The optimal POVM is of the covariant form

$$P(\mathrm{d}\,\phi) = U_\phi \xi U_\phi^\dagger \frac{\mathrm{d}\,\phi}{2\pi} \qquad \xi = \sum_{j,\alpha} |e(j,\alpha)\rangle\langle e(j,\alpha)|$$

with the generalized Susskind-Glogower vector

$$|e(j,\alpha)\rangle = \sum_{m=-j}^{j} e^{i\theta(m,j\alpha)}|m, j, \alpha\rangle$$

Phase estimation with mixed states

The optimal POVM achieves the ***Quantum Cramer Rao bound:***

$$\Delta\phi^2 \geq \frac{1}{N}\frac{1}{r^2\cos^2\theta}$$

r: purity

θ: tilt angle

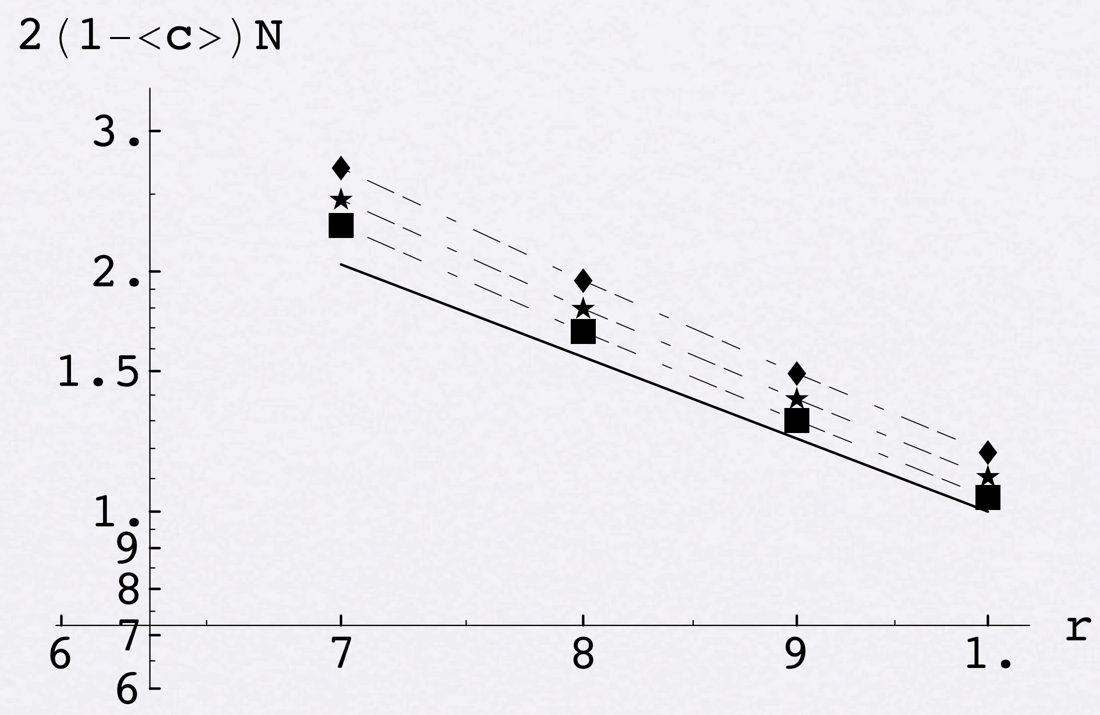


FIG. 3: The logarithmic plot of $2N(1-\langle c\rangle)$ vs r, for $N = 16, 18, 20$ and $\theta = 0$. The line on the bottom represents the bound given by the Cramer-Rao inequality, namely $1/r^2$

Summary

- Super-broadcasting
- Efficiently universally programmable measuring apparatuses
- Optimal transmission of reference frames
- Quantum calibration of measuring apparatuses
- Optimal phase estimation for mixed states

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